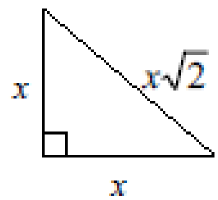
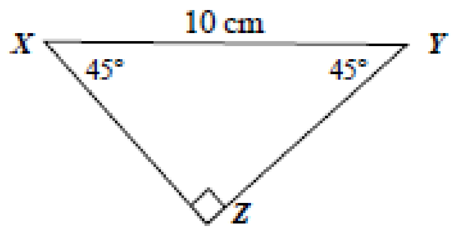
Special Right Triangles

SOL G.8b (2016)

For a right, isosceles triangle (45°-45°-90°), the length of the hypotenuse is equal to the length of a leg times $\sqrt{2}$.	
$a^2 + b^2 = c^2$ $x^2 + x^2 = c^2$ $2x^2 = c^2$ $\sqrt{2x^2} = \sqrt{c^2}$ $x\sqrt{2} = c$	

Example 1: Find the measure of the legs of the triangle.



$$10 = x\sqrt{2}$$

$$\frac{10}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$$

$$\frac{10}{\sqrt{2}} = x$$

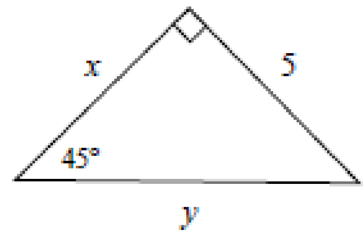
$$x = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

1. **According to the theorem, the hypotenuse, 10 cm, is equal to the leg length, x, times $\sqrt{2}$.**
2. **Use division to solve for x.**
3. **Rationalize the radical by multiplying by 1 ($\frac{\sqrt{2}}{\sqrt{2}}$). Simplify.**

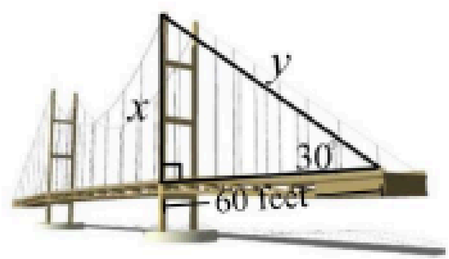
Practice

Find x and y.

1.

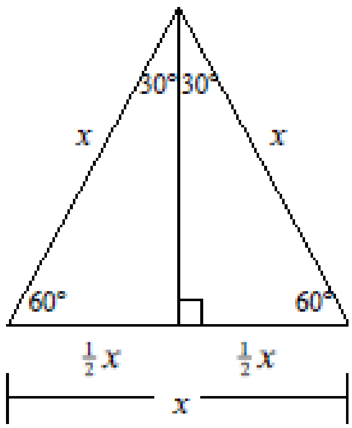


2.



3.

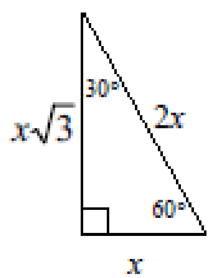
Consider an equilateral triangle. All angles measure 60°. If an altitude is drawn from any vertex, two 30°-60°-90° triangles are created.



$$a^2 + b^2 = c^2$$
$$\left(\tfrac{1}{2}x\right)^2 + b^2 = x^2$$
$$\tfrac{1}{4}x^2 + b^2 = x^2$$
$$b^2 = \tfrac{3}{4}x^2$$
$$b = \tfrac{\sqrt{3}}{2}x$$

The ratio of the measures of the sides is $\frac{1}{2} : \frac{\sqrt{3}}{2} : 1$ or $1 : \sqrt{3} : 2$.

For a 30°-60°-90° triangle, the hypotenuse is twice the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

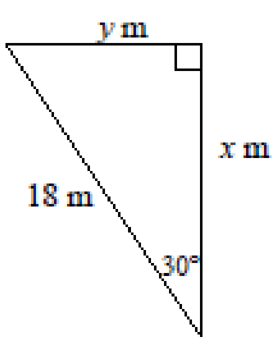
$a^2 + b^2 = c^2$ $x^2 + b^2 = (2x)^2$ $x^2 + b^2 = 4x^2$ $b^2 = 3x^2$ $\sqrt{b^2} = \sqrt{3x^2}$ $b = x\sqrt{3}$	

Example 2: Find *x* and *y*.

$18 = 2y$

$y = 9$

$x = 9\sqrt{3}$



1. **According to the theorem, the hypotenuse, 18, is twice the length of the short leg, *y*.**
2. **Solve for *y* (short leg).**
3. **The long leg, *x*, is the short leg times $\sqrt{3}$.**

4. The altitude of an equilateral triangle is 9 cm. Find the perimeter of the triangle.

5. The length of the diagonal of a square is $31\sqrt{2}$. Find the perimeter of the square.