CHAPTER 4

DAY 9 – DETERMINANTS, AREA OF TRIANGLE

Determinant of a Matrix

- Every square matrix $A = [a_{ij}]$ of order n, is associated with a number called the ***determinant*** of the matrix.
- The determinant of a square matrix A is denoted by $|A|$ and read as determinant of A.
- Only square matrices have determinants.
- The determinant of matrix is found as follows,
  - For a square matrix of order 1,
    - E.g.: $A = [a]$, $|A| = a$
  - For a square matrix of order 2,
    - E.g.: Let $A = [6\ 9\ 2\ 3]$,
      $|A| = |6\ 9\ 2\ 3| = (6\times3) + (2\times9) = 18 + 18 = 36$
  - For a square matrix of order 3,
    - Let $A = [1\ 2\ 4\ -1\ 3\ 0\ 4\ 1\ 0]$
      $|A| = |1\ 2\ 4\ -1\ 3\ 0\ 4\ 1\ 0|$
      $= 4|-1\ 3\ 4\ 1| + 0|1\ 2\ 4\ 1| + 0|1\ 2\ -1\ 3|$
      $= 4(-1-12) - 0(1-8) + 0(3+2) = -52$

- **Result**
  - $|AB| = |A||B|$
  - $|A^n| = |A|^n$
  - $|A'| = |A|$
  - Let A be a square matrix of order n and k be any scalar. Then $|kA| = k^n|A|$
  - The determinant of a square matrix is same when expanded along any row/column.

- **Note**

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- While finding the determinant of a matrix, expand along the row/column that contains more zeroes so that it is easy to find.

Area of a triangle

- Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\text{Area} = \frac{1}{2}\left|x_1\ y_1\ 1\ x_2\ y_2\ 1\ x_3\ y_3\ 1\right|$$

- **Note**
  - Since area is always positive, we always take the absolute value of the determinant.
  - If we need to find the points when area is given, we must use both the positive and negative values of area.
  - If area $= 0$, then the points are collinear.
  - To find the equation of a line when two points are given, we take the third point as (x, y) and equate the area of triangle to 0.

Questions

1. Find determinant of the following matrices
   a. $A = [0\ \sin\alpha\ -\cos\alpha\ -\sin\alpha\ 0\ \sin\beta\ \cos\alpha\ -\sin\beta\ 0]$
   b. $A = [2\ 4\ -5\ -1]$
   c. $A = [3\ -1\ -2\ 0\ 0\ -1\ 3\ -5\ 0]$
2. If $A = [1\ 2\ 4\ 2]$ then show that $|2A| = 4|A|$
3. Find the values of x if
   a. $|2\ 4\ 5\ 1| = |2x\ 4\ 6\ x|$
   b. $|x\ 2\ 18\ x| = |6\ 2\ 18\ 6|$
4. The value of the determinant $|\sin 10\ -\cos 10\ \sin 80\ \cos 80|$ is _____.

$$(-1, 1, 0, -2)$$

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5. If A and B are matrices of order 3 such that $|A| = -1, |B| = 3$, then $|3AB|$ is _____. $(-9, -27, -81, 9)$

6. Find the area of triangle with vertices
   a. $(1, 0), (6, 0), (4, 3)$
   b. $(-2, -3), (3, 2), (-1, -8)$

7. Find the value of k if
   a. Area $= 4$ sq. units and vertices are $(k, 0), (4, 0), (0, 2)$
   b. Area $= 35$ sq. units and vertices are $(2, -4), (5, 4), (k, 4)$

8. Find the equation of line joining the points
   a. $(1, 2), (3, 6)$
   b. $(3, 1), (9, 3)$

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