Instructions

The exam will begin at 7pm on Tuesday, October 29 in room MPHY 203.

The exam will consist of 20 multiple choice questions. You will have 60 minutes to complete the exam. A formula sheet with the basic fundamental equations and important constants will be provided on the exam. An example of this sheet is given at the end of this document. Additionally, you may bring a single, one sided sheet (standard US letter size or smaller) with your own notes or helpful formulas written on it.(The amount of notes which can be brought will be increased by half a page per exam, i.e. half page for exam 1, full page for exam 2, one and a half for exam 3, 2 pages for the final exam) These sheets will be inspected before the exam starts. You will be required to show your student ID on submission.

Note: In the event that you are unable to take the exam at the scheduled time, or an external event beyond your control interferes with your ability to take the examination, you ***MUST*** notify me as soon as possible so that alternate arrangements can be made (within reason).

Recommendations for studying

- Chapters 3 - 6
- Lectures 6 - 10
- Homework 3 - 6
- A calculator is highly recommended (especially one with trigonometric functions)

Preparation for Exam

Concepts to know

* This is an overview of items to know, it is not necessarily comprehensive and not all items here may appear on the exam. Use it as a general guideline on what to study. *

- Various universal and common constants (h, c, e^-, m_e, m_p, etc.)
  - Unit conversions
- Discovery of x-ray and electron
  - Millikan drop experiment and determining the charge of an electron
- Rydberg Equation: $\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right)$ $\quad R_H = 1.096776 \times 10^7 \text{ m}^{-1}$
- Blackbody radiation
  - Concept of Wien's Displacement Law and Stephan-Boltzmann law
    - $\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$
    - $R(T) = \int_0^\infty \mathcal{I}(\lambda, T) d\lambda = \epsilon \sigma T^4$
  - Ultraviolet Catastrophe and what it meant for classical physics
  - Planck's Radiation Law: $\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$
- Planck's Postulates:
  - The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nhf$, where n is an integer, f is the frequency of the radiation, and h is called Planck's constant ($h = 6.6261 \times 10^{-34}$ J·s).
  - The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by: $\Delta E = hf$
- Concept and equations of the photo-electric effect.
  - $E = hf$
  - $\frac{1}{2} m v_{\max}^2 = eV_0 = hf - \phi$
- Compton Effect

- $$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

- The various atomic models, their strengths, their weaknesses, and what lead to their conception.
- Bohr radius: $a_0 = 0.529$Å
- How to calculate changes in electron states
- The Correspondence Principle
- Electron shells and Mosely Plot
- Braggs Law and Bragg Planes
  - $n\lambda = 2d \sin\theta$ (n = integer)
- De Broglie waves
  - $\lambda = \frac{h}{p}$
- Properties of Wave motion
  - Phase and Group Velocity
    - Phase Velocity: $v_{ph} = \frac{\lambda}{T} = \frac{\omega}{k}$
    - Group Velocity: $u_{gr} = \frac{dE}{dp} = \frac{pc^2}{E}$
- Uncertainty Principle
  - Position-Momentum: $\Delta p_x \, \Delta x \geq \frac{\hbar}{2}$
  - Energy-Time: $\Delta E \, \Delta t \geq \frac{\hbar}{2}$
- The Copenhagen Interpretation
  - The uncertainty principle of Heisenberg
  - The complementarity principle of Bohr

   - The statistical interpretation of Born, based on probabilities determined by the wave function
- All aspects of a particle-in-a-box
   - Energy of n-th level: $E_n = n^2 \frac{\hbar^2 \pi^2}{2ml^2}$
- General form of the Schrodinger Wave Equation
   - $\Psi(x,t) = Ae^{i(kx-\omega t)} = A[\cos(kx-\omega t) + i\sin(kx-\omega t)]$
- What is Normalization and why it is needed
   - How to Normalize an equation.
   - Normalizations of common functions (exponentials, sines)
- Boundary Conditions for Wave Equations
   - In order to avoid infinite probabilities, the wave function must be finite everywhere.
   - In order to avoid multiple values of the probability, the wave function must be single valued.
   - For finite potentials, the wave function and its derivative must be continuous. This is required because the second-order derivative term in the wave equation must be single valued. (There are exceptions to this rule when V is infinite.)
   - In order to normalize the wave functions, they must approach zero as x approaches infinity.

Preparation for Exam

Thermodynamics and Kinetic Theory:
$\Delta U = Q + W,\ pV = nRT,\ \bar{K} = \frac{3}{2}k_B T = \frac{1}{2}mv_{\rm rms}^2,\ U = \frac{3}{2}Nk_B T,\ k_B = R/N_A$

Lorentz transformation along x-direction

$$x' = \gamma(x - vt)\ ,\quad t' = \gamma(t - \frac{v}{c^2}x)\ ,\quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\ ,\quad u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}\ ,\quad u'_y = \frac{u_y}{\gamma(1 - \frac{u_x v}{c^2})}$$

Time dilation and length contraction:
$\Delta t' = \gamma \Delta t\ ,\quad L' = L/\gamma\quad (\Delta t,\ L$: "proper" time, length)

Relativistic Doppler effect : $f' = f\sqrt{\frac{1+v/c}{1-v/c}}\ ,\quad c = f\lambda = f'\lambda'$

Relativistic 3-momentum and energy

$$\vec{p} = \gamma m\vec{u}\ ,\ E = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}\ ,\ K = E - mc^2\ ,\ \vec{u} = \frac{\vec{p}}{E}c^2\ ,\ M_{\rm bound} = m_1 + m_2 + E_B$$

Total energy and momentum conservation: $E_{\rm in} = E_{\rm out}\ ,\quad \vec{P}_{\rm in} = \vec{P}_{\rm out}$

Thomson experiment: $\frac{e}{m} = \frac{V\tan\Theta}{B^2 dl}$

Blackbody: $I = P/A = \sigma T^4\ ,\ \lambda_{\rm max}T = const\ \ (const = 2.9\cdot 10^{-3} mK)\ ,$

$$u(f,T) = 8\pi h\frac{f^3}{c^3}\frac{1}{\exp(hf/k_BT) - 1}\ ,\ E_\gamma = hf\ ,\ c = f\lambda$$

Photoeffect: $K_{\rm max} = |eV_s|\ ,\ K_{\rm max} = hf - \phi\ ,\ K = \frac{1}{2}m_e v_e^2$

Compton effect: $\lambda' - \lambda = \lambda_e(1 - \cos\Theta)$

Rutherford scattering: $b = \frac{Z_1Z_2ke^2}{2K_1\tan(\theta/2)}\ ,\ f = nt\,\pi b^2(\theta)\ ,\ \frac{dN}{dA} \equiv N(\theta) = \frac{N_1 n_2 t}{16R^2\sin^4(\theta/2)}\left(\frac{ke^2Z_1Z_2}{K_1}\right)^2\ ,$

Bohr Model: $rm_ev_e = n\hbar\ ,\ E_e = K_e + U_e = \frac{1}{2}U_e\ ,\ U_e = -\frac{ke^2}{r}\ ,\ r_n = n^2a_0\ ,\ a_0 = \frac{\hbar^2}{km_ee^2}\ ,$

$$E_n = -\frac{1}{n^2}\frac{ke^2}{2a_0} = -\frac{E_1}{n^2},\ E_\gamma = E_1\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right),\ \frac{1}{\lambda} = R\left|\frac{1}{n_i^2} - \frac{1}{n_f^2}\right|,\ \mu_e = \frac{m_e}{1 + m_e/m_p}$$

X-Ray spectra: $E_\gamma = (Z-1)^2E_1(1/n_i^2 - 1/n_f^2)$

de Broglie wavelength: $\lambda = \frac{h}{p}$

Phase and group velocity: $v_{\rm ph} = \frac{\omega}{k} = c\sqrt{1 + \left(\frac{mc}{p}\right)^2}\ ,\quad v_{\rm gr} = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{c^2}{v_{\rm ph}}$

Quantum mechanical probability density: $P(x,t) = |\psi(x,t)|^2$

Uncertainty principle: $\Delta x\ \Delta p \gtrsim \frac{\hbar}{2}\ ,\quad \Delta t\ \Delta E \gtrsim \frac{\hbar}{2}$

Potential energy levels:

infinite square well: $E_n = n^2 \frac{\hbar^2\pi^2}{2mL^2}$,

finite square well (small n): $E_n \simeq n^2 \frac{\hbar^2\pi^2}{2m(L+2\delta)^2} - U_0$ with $\delta \simeq \frac{\hbar}{\sqrt{2mU_0}}$

simple harmonic oscillator: $E_n = (n + \frac{1}{2})\hbar\omega$ with $\omega^2 = \frac{k}{m}$

Constants and Conversions

$c = 3 \cdot 10^8\, m/s$, $k_B = 8.62 \cdot 10^{-5}\, eV/K$, $R = 8.31 J/K \cdot mol$, $\sigma = 5.67 \cdot 10^{-8}\, W/(m^2K^4)$,
$e = 1.6 \cdot 10^{-19}C$, $1\, eV = 1.6 \cdot 10^{-19}J$, $1\, MeV = 10^6\, eV$, $1\, GeV = 10^9\, eV$,
$1u = 1.66 \cdot 10^{-27}\, kg = 931.5\, MeV/c^2$, $m_p = 938.3 MeV/c^2$, $m_n = 939.6 MeV/c^2$, $m_e = 511\, keV/c^2$,
$1\, fm = 10^{-15}m$, $1\, nm = 10^{-9}m$,
$h = 6.63 \cdot 10^{-34}Js = 4.14 \cdot 10^{-15}\, eVs$, $\hbar = h/(2\pi)$, $\lambda_e = h/(m_e c) = 0.00243\, nm$,
$hc = 1.24 \cdot 10^{-6}eVm = 1240\, eVnm$, $\hbar c = 197\, eVnm = 197\, MeVfm$
$a_0 = 0.0529\, nm$, $E_1 = ke^2/(2a_0) = 13.6\, eV$, $R = 1.1 \cdot 10^7 m^{-1} = 1/(91.2\, nm)$, $ke^2 = 1.44\, eVnm$