

Calculus 2

Differentiation Quiz Review

Name: _____

Monday Quiz

- Multiple-choice questions
- Mixed differentiation problems
- Implicit differentiation
- Find & Correct the Error
- A trigonometric derivative derivation

Important: You will be asked to derive the derivative of $\tan x$.

Start with $\tan x = \sin x / \cos x$ and use the Quotient Rule. Do not simply state the derivative from memory.

The derivative rules shown in this review packet are for study and review only. No derivative formula chart will be provided on the quiz. You are expected to know the derivative rules, including inverse trigonometric derivatives, from memory.

A. Derivative Reference

Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Constant Multiple / Sum and Difference

$$\frac{d}{dx}[cu] = c \frac{du}{dx} \quad / \quad \frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Trigonometric Derivatives

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) =$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

B. Derive the Missing Rule

Starting with $\tan u = \sin u / \cos u$, use the Quotient Rule to derive the derivative of $\tan u$.

C. Inverse Trig Reference

Review these formulas carefully. They will not be provided on the quiz.

$$\frac{d}{dx} (\sin^{-1}u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx} (\cos^{-1}u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{d}{dx} (\tan^{-1}u) = \frac{1}{1+u^2} \frac{du}{dx}$$

D. Choosing an Efficient Approach

There may be more than one correct way to differentiate a function. Looking at the structure of the expression can help you choose an efficient approach and reduce algebraic errors.

Ask yourself: What structure do I see? Are any functions nested inside other functions?

Worked Example 1

$$y = (x^3 + 2) \sqrt{5x - 1}$$

This expression involves a product, and the second factor also requires the Chain Rule.

$$y' = 3x^2 \sqrt{5x - 1} + (x^3 + 2) \left(\frac{1}{2\sqrt{5x - 1}} \right) (5)$$

$$y' = 3x^2 \sqrt{5x - 1} + 5(x^3 + 2) / (2\sqrt{5x - 1})$$

Worked Example 2

$$y = \sin(2x) / (x^2 + 4)$$

The expression is written as a quotient. The numerator also requires the Chain Rule.

$$y' = [(x^2 + 4)(2 \cos(2x)) - \sin(2x)(2x)] / (x^2 + 4)^2$$

E. Mixed Differentiation

Differentiate. Show enough work to make your reasoning clear.

1. $f(x) = 5x^4 - 3x^{3/2} + 7x - 2$

2. $y = (x^2 + 4) \sin x$

3. $y = (3x^2 - 1) / (x + 2)$

4. $y = (2x^3 - 5x + 1)^4$

5. $y = \cos(3x^2 + 1)$

6. $y = x^3(2x - 1)^5$

7. $y = \tan(3x^2 + 1)$

8. $y = \sin^{-1}(2x)$

9. $y = x^2 \tan^{-1}x$

F. Implicit Differentiation

Find dy/dx .

10. $x^2 + y^2 = 36$

11. $x^2 + xy + y^2 = 12$

12. $x^2 + \tan y = 5$

G. Multiple Choice — Understanding Differentiation

13. If $y = (3x^2 + 1)^5$, which expression correctly represents y' ?

- A. $5(3x^2 + 1)^4$
- B. $30x(3x^2 + 1)^4$
- C. $6x(3x^2 + 1)^5$
- D. $30x(3x^2 + 1)^5$

14. If $y = y(x)$, then $d/dx (y^3) =$

- A. $3y^2$
- B. $3y^2(dy/dx)$
- C. $y^2(dy/dx)$
- D. $3y(dy/dx)$

H. Identify, Then Differentiate

15. $y = x^2 \cos(3x)$

Which rules are useful? Select all that apply.

Power Rule Product Rule Quotient Rule Chain Rule

Then find y' .

I. Find & Correct the Error

16. A student differentiates

$$y = (x^2 + 1) \sin x$$

as follows:

$$y' = 2x \cos x$$

Identify the error and find the correct derivative.

17. Given

$$x^2 + xy + y^2 = 9$$

a student writes

$$2x + x(dy/dx) + 2y = 0$$

Find the error(s) and write the correct derivative equation.