# Tower of Hanoi POW

**Question:**
If the monks moved one disc per second, how long would it take them to complete *n* discs?

**Problem Statement:**
For this problem, we had to find a formula that was able to express the smallest amount of time needed to move *n* number of discs using three pegs.

**Process:**

1. Played the game multiple times in order to find the least number of moves possible for each level, where it adds one more disk per level.
2. Found that the least number of moves possible for three disks was seven moves.
3. Found that the least number of moves possible for four disks was fifteen moves.
4. Looked at the number of moves per disk in order to reach the least number of moves in those levels.
5. Found that, in each level, the largest block moves once, the second-largest moves twice, the third-largest moves four times, and the fourth-largest moves eight times. We found a pattern in the moves per disk, where the number of moves added from one level to the next was double that of the previous number added (see table below).

| Level | Number of Disks (n) | Least Number of Moves (y) | Number of Moves Added |
|---|---|---|---|
| 1 | 3 | 7 | |
| 2 | 4 | 15 | + 8 |
| 3 | 5 | 31 | + 16 |
| 4 | 6 | 63 | + 32 |
| 5 | 7 | 127 | + 64 |

6. Using this pattern, we then worked to find an equation to match our results. We found this equation after hearing the work done by other groups. The equation that the other groups found that match our own results is: $2^n - 1 = y$.
7. In terms of the question, knowing that each move takes one second, we can assume that each number of moves for each level would then also represent the least amount of time it would take to complete that level. For example, for three disks, it takes seven moves, therefore, it takes seven seconds to complete that level.

**Solution:**

If the monks moved 1 disk per second, it would take them $2^n - 1$ seconds to move *n* discs.

**Reflection:**

- How confident are you that your solution is correct?

  I am confident that the solution is correct, as it remains consistent throughout the n, y table above and the equation was discovered by multiple groups working on this problem.

- What was the most challenging part of solving this POW?

  The most challenging part of solving this POW, for me, was finding the least number of moves that can be played on each level of the game. It turned out to be time-consuming and difficult to know whether or not it was truly the least amount without realizing the pattern. Had we not been prompted by Ande or peers to keep trying to find a smaller amount of moves, I feel that we would have remained with our current numbers and had false information to use, therefore, being unsuccessful in solving the POW.

- Was your process justified?

  I know that my answer is correct if we look at the discs separately. If we have two discs, there would be three moves needed to complete the level, two to move the top disc, one to move the bottom disc. It holds true for all levels.