1st/12/22

unit V

State space Analysis

① **State:-**
It is the group of variables which summarize the history of the given system inorder to predict the future values.

② **State Variable:-** The smallest set of variables that determines the state of the system is known as state variables.

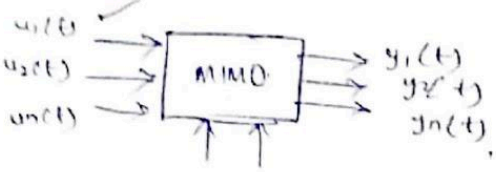
i.e. $x_1(t), x_2(t) \ldots\ldots x_n(t)$

③ **State Vector:-** It is the vector which contains state variables

$$[x_1(t)\ x_2(t) \ldots\ldots x_n(t)]$$

④ **State Models:-**

When we are Analyzing multi input and multioutput system with state space Analysis. the output Eqn $y(t)$ and state Eqn $\dot{x}(t)$ or $\frac{d}{dt}(x(t))$ Collectively to form state Model



State Eqn

$$\frac{d}{dt}(x_1(t)) = a_{11}x_1(t) + a_{12}x_2(t) + b_{11}u_1(t) + b_{12}u_2(t)$$

$$\dot{x}_1(t) = a_{11}x_1(t) + a_{12}x_2(t) + b_{11}u_1(t) + b_{12}u_2(t) \quad \text{---(i)}$$

$$\frac{d}{dt} x_2(t) = a_{21}x_1(t) + a_{22}x_2(t) + b_{21}u_1(t) + b_{22}u_2(t)$$

$$\dot{x}_2(t) = a_{21}x_1(t) + a_{22}x_2(t) + b_{21}u_1(t) + b_{22}u_2(t) \quad \text{—②}$$

Eqn ① & ② represent in Matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$\boxed{\dot{x} = Ax + Bu}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

O/p Eqns =

$$y_1(t) = c_{11}x_1(t) + c_{12}x_2(t) + D_{11}u_1(t) + D_{12}u_2(t)$$

$$y_2(t) = c_{21}x_1(t) + c_{22}x_2(t) + b_{21}u_1(t) + D_{22}u_2(t)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$\boxed{y = Cx + Du}$$

x1 -

5 State transition matrix := ($\phi(t)$)

It is a matrix which satisfies linear homogeneous state Eqn. then we can say state transition matrix.

$A' = A$

$\dot{x}_1 = Ax + Bu$ ... $u = 0$

$\dot{x}_1 = Ax$ { if linear when i/p is zero }

$$\phi(t) = e^{At} = \mathcal{L}^{-1}\left[SI - A\right]^{-1}$$

Properties of state transition Matrix := (STM)

$$\phi(t) = e^{At}$$

① $\phi(0) = e^{A(0)} = 1 = I$

$\phi(0) = I$

② $\phi(t) = e^{At} = \frac{1}{e^{At}}$

$\phi(-t) = \phi(t)^{-1}$

③ $\phi(t)^m = e^{At^m} = e^{mAt}$

$\phi(t)^m = \phi(tm)$

④ $\phi(t_2 - t_1) \cdot \phi(t_1 - t_0) = e^{A(t_2 - t_1)} \times e^{A(t_1 - t_0)}$

$= e^{A[t_2 - t_1 + t_1 - t_0]}$

$= e^{A[t_2 - t_0]}$

$= \phi(t_2 - t_0)$

(5) $\phi[t_1+t_2] = e^{A(t_1+t_2)}$

$= e^{At_1} \times e^{At_2}$

$\phi(t_1+t_2) = \phi(t_1) \times \phi(t_2)$ ✓

Derivation of Transfer function from state model

$\dot{x} = Ax + Bu \longrightarrow (1)$

$y = Cx + Du \longrightarrow (2)$

Apply L.T on b.s to Eqn (1)

$\mathcal{L}\left\{\frac{d}{dt}(x)\right\} = \mathcal{L}\{Ax+Bu\}$

$sX(s) = AX(s) + BU(s)$

$IsX(s) - AX(s) = BU(s)$

$X(s)[sI-A] = BU(s)$

$X(s) = [sI-A]^{-1}BU(s)$

Apply L.T on b.s to Eqn (2)

$y = Cx + Du$

$Y(s) = CX(s) + DU(s)$

$Y(s) = C[[sI-A]^{-1}BU(s) + DU(s)]$

$Y(s) = C[C[sI-A]^{-1}B + D]U(s)$

$\frac{Y(s)}{U(s)}$

$\frac{Y(s)}{U(s)} = C[sI-A]^{-1}B + D$ ✓

Pblm

The state Eqns of the time linear invariant system is given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

find Transfer function

$$A = \begin{bmatrix} 0 & 5 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$D = [0]$$

$$\frac{Y(s)}{U(s)} = C\left[sI - A\right]^{-1} B + D$$

$$\frac{Y(s)}{U(s)} = C\left[sI - A\right]^{-1} B$$

$$\left[sI - A\right] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 5 \\ -1 & -2 \end{bmatrix}$$

$$\left[sI - A\right] = \begin{bmatrix} s & -5 \\ 1 & s+2 \end{bmatrix}$$

$$\left[sI - A\right]^{-1} = \frac{\text{Adj}\left[sI - A\right]}{|sI - A|}$$

$$|sI - A| = s(s+2) - (1 \times -5$$

$$|sI - A| = s^2 + 2s + 5$$

Scanned with O

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \text{Adj } A \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{Adj}\,[sI-A] = \begin{bmatrix} s+2 & -(-5) \\ -1 & s \end{bmatrix} = \begin{bmatrix} s+2 & 5 \\ -1 & s \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{\text{Adj}\,[sI-A]}{|sI-A|} = \frac{\begin{bmatrix} s+2 & 5 \\ -1 & s \end{bmatrix}}{s^2+2s+5}$$

$$\frac{Y(s)}{U(s)} = C\,[sI-A]^{-1}B$$

$$= \underset{1\times 2}{[1\ 1]} \begin{bmatrix} s+2 & 5 \\ -1 & s \end{bmatrix}_{2\times 2} \underset{2\times 1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$= \frac{1}{s^2+2s+5} [1\ 1] \begin{bmatrix} s+2+5 \\ -1+s \end{bmatrix}$$

$$= \frac{[1\ 1]}{s^2+2s+5} \begin{bmatrix} -s+7 \\ s-1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+5} [2s+6]$$

$$\frac{Y(s)}{U(s)} = \frac{2s+6}{s^2+2s+5}$$

Obtain the state Model for the system is

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 10y = 5u(t)$$

Here y is the Output and u is the input Variables.

Scanned with O

Let $x_1(t) = y(t)$

$\dot{x}_1(t) = x_2(t)$

$\dot{x}_2(t) = x_3(t)$

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 10y(t) = 5u(t)$$

Apply L.T

$$s^2Y(s) + 5sY(s) + 10Y(s) = 5U(s)$$

$$Y(s)[s^2+5s+10] = 5U(s)$$

$$\frac{Y(s)}{U(s)} \Rightarrow \frac{5}{s^2+5s+10}$$

$$Y(s) = 5x_1(t)$$

$$U(s) = s^2+5s+10$$

$$U(s) = \dot{x}_2(t) + 5\dot{x}_1(t) + 10x_1(t)$$

$$U(s) = \dot{x}_2(t) + 5x_2(t) + 10x_1(t)$$

$$\dot{x}_2(t) = -10x_1(t) - 5x_2(t) + u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10 & -5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

output eqn

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 5 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t)$$

3) "A linear time invariant system is $D^3y + 3D^2y + 3Dy + y = u$ where $D = \frac{d}{dt}$ and y = output u = input.

1) find state model (or) write state eqn

2) find state transition Matrix (STM):-

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = u$$

Apply L.T on b.s.

$$s^3Y(s) + 3s^2Y(s) + 3sY(s) + Y(s) = U(s)$$

$$Y(s)\left[s^3 + 3s^2 + 3s + 1\right] = U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 3s^2 + 3s + 1}$$

$$Y(s) = 1$$

$$U(s) = s^3 + 3s^2 + 3s + 1$$

let

$$y_1(t) = x_1(t)$$

$$s = \dot{x}_1(t) = x_2(t) \quad \text{—①}$$

$$s^2 = \dot{x}_2(t) = x_3(t) \quad \text{—②}$$

$$s^3 = \dot{x}_3(t) = x_4(t)$$

$$U(s) = s^3 + 3s^2 + 3s + 1$$

$$u(t) = \dot{x}_3(t) + 3\dot{x}_2(t) + 3\dot{x}_1(t) + 1x_1(t)$$

$$u(s) = \dot{x}_3(t) + 3\dot{x}_3(t) + 3x_2(t) + x_1(t)$$

$$\dot{x}_3(t) = -x_1(t) - 3x_2(t) - 3x_3(t) + u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

(b) $STM = \phi(t) = e^{At} = L^{-1}[sI - A]^{-1}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}_{3\times 3}$$

$$sI - A = s\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 3 & s+3 \end{bmatrix}$$

$$\det |sI - A| = s[s(s+3) - (3(-1))] - (-1)[0 - (-1)]$$

$$= s[s^2 + 3s + 3] + 1$$

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$|sI - A| = s^3 + 3s^2 + 3s + 1$$

$$|sI - A| = (s+1)^3$$

$$[sI - A]^{-1} = \frac{Adj[sI - A]}{|sI - A|}$$

$$\begin{bmatrix} + & - & + \\ s & -1 & 0 \\ - & + & - \\ 0 & s & -1 \\ + & - & + \\ 1 & 3 & s+3 \end{bmatrix}$$

$$Adj[sI - A] = [\text{cofactors of } [sI - A]]^T$$

Cofactors of $[sI - A]$ =

$$\begin{bmatrix} +\begin{vmatrix} s & -1 \\ 3 & s+3 \end{vmatrix} & -\begin{vmatrix} 0 & -1 \\ 1 & s+3 \end{vmatrix} & +\begin{vmatrix} 0 & s \\ 1 & 3 \end{vmatrix} \\ -\begin{vmatrix} -1 & 0 \\ 3 & s+3 \end{vmatrix} & +\begin{vmatrix} s & 0 \\ 1 & s+3 \end{vmatrix} & -\begin{vmatrix} s & -1 \\ 1 & 3 \end{vmatrix} \\ +\begin{vmatrix} -1 & 0 \\ s & -1 \end{vmatrix} & -\begin{vmatrix} s & 0 \\ 0 & -1 \end{vmatrix} & +\begin{vmatrix} s & -1 \\ 0 & s \end{vmatrix} \end{bmatrix}$$

$$\begin{bmatrix} s^2+3s+3 & -1 & -s \\ s+3 & s^2+3s & -(3s+1) \\ 1 & s & s^2 \end{bmatrix}^T$$

$$\text{Adj}\,[sI - A] = \begin{bmatrix} s^2+3s+3 & s+3 & 1 \\ -1 & s^2+3s & s \\ -s & -(3s+1) & s^2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\text{Adj}\,(sI - A)}{|sI - A|}$$

$$= \frac{\begin{bmatrix} s^2+3s+3 & s+3 & 1 \\ -1 & s^2+3s & s \\ -s & -(3s+1) & s^2 \end{bmatrix}}{(s+1)^3}$$

$$e^{At} = \mathcal{L}^{-1}\left\{ \begin{bmatrix} \frac{s^2+3s+3}{(s+1)^3} & \frac{s+3}{(s+1)^3} & \frac{1}{(s+1)^3} \\ \frac{-1}{(s+1)^3} & \frac{s^2-3s}{(s+1)^3} & \frac{s}{(s+1)^3} \\ \frac{-s}{(s+1)^3} & \frac{-[3s+1]}{(s+1)^3} & \frac{s^2}{(s+1)^3} \end{bmatrix} \right\}$$

above terms converted into partial fraction

$$\frac{s^2+3s+3}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$\frac{s^2+3s+3}{(s+1)^3} = \frac{A(s+1)^2 + B(s+1) + C(1)}{(s+1)^3}$$

$$s^2+3s+3 = A[s^2+2s+1] + Bs + B + C$$

$$s^2+3s+3 = As^2 + 2As + A + Bs + B + C$$

Compare Coefficients

$A = 1$	$3 = 2A + B$	$A + B + C = 3$
	$3 = 2(1) + B$	$1 + 1 + C = 3$
	$B = 3 - 2$	$2 + C = 3$
	$\boxed{B = 1}$	$C = 3 - 2$
		$C = 1$

$$\frac{s^2+3s+3}{(s+1)^3} = \frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}$$

2nd term

$$\frac{s+3}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$s+1 = s^2A + s(2A+B) + (A+B+C)$$

$A=0$ | $1 = 2A+B$, $B=1$ | $A+B+C=1$, $0+1+C=1$, $C=0$

$$\frac{s+3}{(s+1)^3} = \frac{1}{(s+1)^2} + \frac{2}{(s+1)^3}$$

3rd term

$$\frac{1}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$1 = s^2A + s(2A+B) + (A+B+C)$$

$A=0$ | $2A+B=0$, $2(0)+B=0$, $B=0$ | $A+B+C=1$, $0+0+C=1$, $C=1$

5th term

$$\frac{s^2+3s}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$s^2+3s = s^2A + s(2A+B) + A+B+C.$$

$A=1$ | $3 = 2A+B$, $3=2+B$, $B=3-2$, $B=1$ | $A+B+C=0$, $1+1+C$, $2+C=0$, $C=-2$.

Scanned with O

$$\frac{s^2+3s+1}{(s+1)^3} = \frac{1}{s+1} + \frac{1}{(s+1)^2} - \frac{2}{(s+1)^3}$$

6th term

$$\frac{s}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$s = s^2A + s(2A+B) + (A+B+C)$$

$A=0$ | $1 = 2A+B$, $B=1$ | $A+B+C=0$, $0+1+C$, $C=-1$

$$\frac{s}{(s+1)^3} = \frac{1}{(s+1)^2} - \frac{1}{(s+1)^3}$$

7th term

$$-s = s^2A + s(2A+B) + (A+B+C)$$

$A=0$ | $-1 = 2A+B$, $B=-1$ | $A+B+C=0$, $0+1+C$, $+1 = -C$, $C=1$

$$\frac{-s}{(s+1)^3} = \frac{-1}{(s+1)^2} + \frac{1}{(s+1)^3}$$

8th term

$$\frac{-(3s+1)}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$-3s-1 = s^2A + s(2A+B) + (A+B+C)$$

$A=0$ | $-3 = 2A+B$ | $-1 = A+B+C$, $-1 = -3+C$

$$\frac{-(3s+1)}{(s+1)^3} = \frac{-3}{(s+1)^2} + \frac{2}{(s+1)^3}$$

9th term

$$\frac{s^2}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

$$s^2 = s^2A + s(2A+B) + (A+B+C)$$

$A=1$ | $2A+B=0$, $2+B$, $B=-2$ | $A+B+C=0$, $1-2+C=0$, $-1+C$, $+1=+C \therefore C=1$

$$\frac{s^2}{(s+1)^3} = \frac{1}{(s+1)} - \frac{2}{(s+1)^2} + \frac{1}{(s+1)^3}$$

$e^{At} = L^{-1}$ [

$\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}$; $\frac{1}{(s+1)^2} + \frac{2}{(s+1)^3}$; $\frac{1}{(s+1)^3}$

$\frac{-1}{(s+1)^2}$; $\frac{1}{s+1} + \frac{1}{(s+1)^2} - \frac{2}{(s+1)^3}$; $\frac{1}{(s+1)^2} - \frac{1}{(s+1$

$\frac{-1}{(s+1)^2} + \frac{1}{(s+1)^3}$; $\frac{-3}{(s+1)^2} + \frac{2}{(s+1)^3}$; $\frac{1}{s+1} - \frac{2}{(s+1)^2} + \frac{1}{(s+1$

]

$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$; $L^{-1}\left[\frac{1}{(s+a)^2}\right] = te^{-at}$

$L^{-1}\left[\frac{1}{(s+a)^3}\right] = \frac{t^2}{2} e^{-at}$

$\mathcal{L}^{-1}\left\{\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}\right\} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3}\right\}$

1st term

$e^{-t} + te^{-t} + \frac{t^2}{2}e^{-t} \Rightarrow e^{-t}\left(1 + t + \frac{t^2}{2}\right)$

2nd term

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2} + \frac{2}{(s+1)^3}\right\} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3}\right\}$

$= te^{-t} + t\frac{t^2}{2}e^{-t} \Rightarrow e^{-t}\left(1 + t^2\right) \Rightarrow e^{-t}(1+t)$

3rd term

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^3}\right\} \Rightarrow \frac{t^2}{2}e^{-t}$

4rth term :=

$\mathcal{L}^{-1}\left\{\frac{-1}{(s+1)^3}\right\} = -\left[\frac{t^2}{2}e^{-t}\right]$

5thterm :=

$\mathcal{L}^{-1}\left\{\frac{1}{s+1} + \frac{1}{(s+1)^2} - \frac{2}{(s+1)^3}\right]$

$e^{-t} + te^{-t} - e^{t}\frac{t^2}{2}e^{-t} \Rightarrow e^{-t}\left(1 + t + t^2\right)$

$\Rightarrow e^{-t}\left(t^2 + t + 1\right)$

6th term :=

$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2} - \frac{1}{(s+1)^3}\right\}$

$te^{-t} - \frac{t^2}{2}e^{-t} \Rightarrow e^{-t}\left[t - \frac{t^2}{2}\right]$

$= \frac{e^{-t}}{2}\left(2t - t^2\right)$.

7th term =

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}\right\} \Rightarrow te^{-t} - \frac{t^2}{2}e^{-t}$$

$$e^{-t}\left[-1 + \frac{t^2}{2}\right]$$

$$= \frac{e^{-t}}{2}(-2 + t^2)$$

8th term =

$$\mathcal{L}^{-1}\left\{\frac{-3}{(s+1)^2} + \frac{2}{(s+1)^3}\right\}$$

$$-3te^{-t} + \frac{t^2}{2}e^{-t}$$

$$-3te^{-t} + t^2e^{-t}$$

$$e^{-t}\left[-3t + t^2\right] \Rightarrow e^{-t}\left[t^2 - 3t\right]$$

9th term :=

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{2}{(s+1)^2} + \frac{1}{(s+1)^3}\right\}$$

$$e^{-t} - 2te^{-t} + \frac{t^2}{2}e^{-t}$$

$$e^{-t}\left[1 - 2t + \frac{t^2}{2}\right]$$

$$\frac{e^{-t}}{2}\left[2 - 2t + t^2\right] \Rightarrow \frac{e^{-t}}{2}\left[t^2 - 2t + 2\right]$$ (9)

$$e^{-At} = \begin{bmatrix} \frac{e^{-t}}{2}(t^2+2t+2) & ; & e^{-t}(t^2+t) & ; & \frac{t^2}{2}e^{-t} \\ -\frac{t^2}{2}e^{-t} & ; & e^{-t}(-t^2+t+1) & ; & \frac{e^{-t}}{2}(2+t-t^2) \\ \frac{e^{-t}}{2}[\quad & ; & \frac{e^{-t}}{2}(2+t-t^2) & ; & e^{-t}[t^2-3t] \end{bmatrix}$$ (8)

(4) Given $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

Find STM & find the Total Response for unit Response step.

When $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$SI - A \Rightarrow S\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$SI - A \Rightarrow S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$\det |SI - A| = S(S+3) - (-1)(2)$$

$$\det |SI - A| = S^2 + 3S + 2$$

$$SI - A = S^2 + 3S + 2$$

$$[SI - A]^{-1} = \frac{Adj\,[SI - A]}{|SI - A|}$$

$$Adj \text{ of } |SI - A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} S+3 & -(-1) \\ -2 & S \end{bmatrix} \Rightarrow \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

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$$\text{Adj}\,|SI-A| = \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$[SI-A]^{-1} = \frac{\text{Adj}\,(SI-A)}{|SI-A|}$$

$$[SI-A]^{-1} = \frac{\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}}{S^2+3S+2}$$

$$[SI-A]^{-1} = \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{1}{S^2+3S+2} \\ \frac{-2}{S^2+3S+2} & \frac{S}{S^2+3S+2} \end{bmatrix}\right\}$$

1st term :-

$$\frac{S+3}{S^2+3S+2} = \frac{A}{S+1} + \frac{B}{S+2} \quad -(1)$$

$$\frac{S+3}{S^2+3S+2} = \frac{A(S+2)+B(S+1)}{(S^2+3S+2)}$$

$$S+3 = A(S+2)+B(S+1) \quad -(2)$$

put $S=-1$ in (2)

$$-1+3 = A(-1+2)+B(-1+1)$$

$$2 = A+0$$

$$\boxed{A=2}$$

$$S+3 = A(S+2)+B(S+1)$$

put $s=-2$ in (2)

$$-2+3 = A(-2+2)+B(-2+1)$$

$$1 = 0-B$$

$$\boxed{B=-1}$$

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Sub A and B values.

$$\frac{s+3}{s^2+3s+2} = \frac{2}{s+1} - \frac{1}{s+2}$$

$$\frac{s+3}{s^2+3s+2} = \frac{2(s+2) - (s+1)}{(s+1)(s+2)}$$

2nd term :=

$$\frac{1}{s^2+3s+2} = \frac{A}{s+1} + \frac{B}{s+2} \quad -(1)$$

$$1 = A(s+2) + B(s+1) \quad -(2)$$

put $s = -1$ in (2)

$$1 = A(-1+2) + B(-1+1)$$

$$1 = A(1) + 0$$

$$\boxed{A = 1}$$

$$1 = A(s+2) + B(s+1)$$

put $s = -2$

$$1 = A(-2+2) + B(-2+1)$$

$$1 = 0 - B$$

$$\boxed{B = -1}$$

$$\frac{1}{s^2+3s+2} = \frac{1}{s+1} - \frac{1}{s+2}$$

3rd term :=

$$\frac{-2}{s^2+3s+2} = \frac{A}{(s+1)} + \frac{B}{(s+2)} \quad -(1)$$

$$-2 = A(s+2) + B(s+1) \quad -(2)$$

put $s = -1$ in (2)

$$-2 = A(-1+2) + B(-1+1)$$

$$-2 = A \Rightarrow \boxed{A = -2}$$

$-2 = 1$

put $s = -2$

$-2 = A(-2+2) + B(-2+1)$

$+2 = 0 + B$

$\boxed{B = 2}$

Substituting B value

$$\frac{-2}{s^2+3s+2} = \frac{-2}{s+1} + \frac{2}{s+2}$$

4th term :-

$$\frac{s}{s^2+3s+2} = \frac{A}{s+1} + \frac{B2}{s+2} \quad -(1)$$

$$s = A(s+2) + B(s+1) \quad -(2)$$

put $s = -1$

$-1 = A(-1+2) + B(-1+1)$

$-1 = A$

$\boxed{A = -1}$

put $s = -2$

$-2 = -A(-2+2) + B(-2+1)$

$+2 = 0 + B(+1)$

$\boxed{B = 2.}$

$$\frac{s}{s^2+3s+2} = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$\mathcal{L}^{-1}\left[\begin{matrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{matrix}\right]$$

$$[sI-A]^{-1} = \begin{bmatrix} \mathcal{L}^{-1}\left\{\frac{2}{s+1} - \frac{1}{s+2}\right\} & \mathcal{L}^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right] \\ \mathcal{L}^{-1}\left[\frac{-2}{s+1} + \frac{2}{s+2}\right] & \mathcal{L}^{-1}\left[\frac{-1}{s+1} + \frac{2}{s+2}\right] \end{bmatrix}$$

$$[sI-A]^{-1} = \begin{bmatrix} 2e^{-t} + e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

1. Zero initial response (ZIR) $= e^{At}x(0)$
2. Zero state Response (ZSR) $= \mathcal{L}^{-1}\{\phi(s)BU(s)\}$

Total Response = ZIR + ZSR.

(1) $ZIR = e^{At}x(0)$

$$ZIR = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} + e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} - e^{-t} + 2e^{-2t} \end{bmatrix}$$

$2\times 2 \quad 2\times 1$

$$= \begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ -3e^{-t} + 4e^{-2t} \end{bmatrix}$$

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(2) $ZSR = \mathcal{L}^{-1}\{\phi(s) \cdot B \cdot U(s)\}$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad U(s) = \frac{1}{s}$$

$$\phi(s) = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$ZSR = \mathcal{L}^{-1}\left\{ \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s} \right\}$$

$$ZSR = \mathcal{L}^{-1}\left\{ \begin{bmatrix} \frac{1}{(s+1)(s+2)} \\ \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} \frac{1}{s} \end{bmatrix} \right\}$$

$$\mathcal{L}^{-1}\left\{ \begin{bmatrix} \frac{1}{s(s+1)(s+2)} \\ \frac{s}{s(s+1)(s+2)} \end{bmatrix} \right\}$$

$$ZSR = \mathcal{L}^{-1}\left\{ \begin{matrix} \frac{1}{s(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} \end{matrix} \right\}$$

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$$\frac{1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \quad \text{--- (1)}$$

$$\frac{1}{s(s+1)(s+2)} = \frac{A(s+1)+B(s+2)+C(s)}{s(s+1)(s+2)} \quad \text{--- (2)}$$

$$1 = As + A + Bs + 2B + Cs$$

$A = 1$ | $\times 1$

$A + 2B = 1$

$$\frac{1}{s(s+1)(s+2)} = \frac{A(s+1)(s+2)+B\,s(s+2)+C[s(s+1)]}{s(s+1)(s+2)}$$

$$1 = A(s^2+2s+s+2) + Bs^2+2Bs+Cs^2+Cs$$

$$1 = As^2+3As+2A+Bs^2+2Bs+Cs^2+Cs$$

$2A = 1$

$A = 1/2$

$A = 0.5$

$0 = A+B+C$

$0 = \frac{1}{2}+B+C$

$-\frac{1}{2} - C = B$

$\frac{-1-2C}{2} = B$

$3A + 2B + C = 0$

$3\left(\frac{1}{2}\right) + 2\left(\frac{-1-2C}{2}\right) + C$

$3\left(\frac{1}{2}\right) - 1 - 2C + C$

$\frac{3}{2} - 1 - C$

$\frac{3-2}{2} - C \Rightarrow \frac{1}{2} = C$

$-\frac{1}{2} - \frac{1}{2} = B$

$-\frac{2-2}{4} = B$

$-\frac{4}{4} = B$

$B = -1$

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$$ZSR = \mathcal{L}^{-1}\left\{ \begin{bmatrix} \frac{0.5}{s} & -\frac{1}{s+1} + \frac{0.5}{s+2} \\ \frac{1}{s+1} & -\frac{1}{s+2} \end{bmatrix} \right\}$$

$$ZSR = \begin{bmatrix} 0.5(1) - e^{-t} + 0.5e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

Total response = ZIR + ZSR

$$T.R = \begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ -3e^{-t} + 4e^{-2t} \end{bmatrix} + \begin{bmatrix} 0.5 - e^{-t} + 0.5e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} 3e^{-t} - 2e^{-2t} + 0.5 - e^{-t} + 0.5e^{-2t} \\ -3e^{-t} + 4e^{-2t} + e^{-t} - e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - 1.5e^{-2t} + 0.5 \\ -3e^{-t} + 3e^{-2t} + e^{-t} \end{bmatrix}$$

$$\text{Total response} = \begin{bmatrix} 2e^{-t} - 1.5e^{-2t} + 0.5 \\ -2e^{-t} + 3e^{-2t} \end{bmatrix}$$

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Controllability and Observability

Controllability :- A linear time invariant system is said to be completely state controllable if it is possible to transfer the system state from any initial state $x(0)$ to any other desired state (finite state) $x(t)$ in a specified time intervals, by a control vector that is output & Input Equation.

The concept of controllability and observability is introduced by kalman Hence, kalman test are used to find out whether the system is controllable and observable or not.

kalman test for Controllability :-

$$\dot{x} = Ax(t) + Bu(t)$$

$A = n \times n$, $B = n \times 1$

$$Q_c = \left[B : AB : A^2B : A^3B \cdots A^{n-1}B \right]$$

Where $B, AB, A^2B \ldots A^{n-1}B$ are columns of Matrix Q_c.

$\det |Q_c| \neq 0$.

the sys is Controllable

If

$\det |Q_c| = 0$ the sys is not controllable.

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Observability:-

A linear time invariant system is said to be Completely Observable if every state Eqn can be Completely $x(t)$ Can be Completely identified by Measurements of output $y(t)$. In a finite interval of time. If the system is not Completely observable means that few of state variables are not practically Measurable.

Kalman's test for Observability

$$\dot{x} = Ax(t)$$

$$y = Cx(t)$$

$A = n \times n$, $C = 1 \times n$

$$Q_0 = \left[C^T : A^T C^T : (A^T)^2 C^T : \ldots (A^T)^{n-1} C^T \right]$$

$|Q_0| = 0$

The system is not Observable.

$|Q_0| \neq 0$

The system is Observable.

Evaluate the Controllability and observability

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$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1\,0\,0]\, x + [0]\, u$$

Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}_{3\times 3}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $C = [1\,0\,0]$

1) Kalman's test for Controllability.

$n = 3$.

$$Q_C = [\, B : AB : A^2B \,]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}_{3\times 3} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 0 \\ -1 \\ -6 \end{bmatrix}$$

$$A^2B = A[AB] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -6 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0\times 0 + 1\times 1 + 0 \\ 0 + 0 - 6 \\ 0 - 11 + 36 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 25 \end{bmatrix}$$

$+ \quad - \quad +$

$\therefore 0 + 0 + 1(0 - 1)$

$-1 \neq 0.$

- the sys is Controllable.

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Q.6

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} C^T : A^T C^T : (A^T)^2 C^T \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(A^T)^2 C^T = A^T (A^T C^T) = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |Q_0| = 1$$

$\therefore |Q_0| \neq 0$

Then the system is completely observable.