financialalgebra

6.6 Intro to Logarithms

Student Activity Packet

UNIT: INVESTING STRATEGIES & EXPONENTIAL FUNCTIONS

Name:

IN THIS LESSON, YOU WILL:

- Explain what the purpose of a logarithm is
- Convert between exponential and logarithmic functions
- Use the change of base formula to evaluate logarithms
- Solve exponential equations using logarithms
- Use the Rule of 72
- Determine the time required to meet investing goals

INTRO

EXPLORE: How Long Does Your Investment Take to Mature?

Delilah makes an investment of $10,000 at 7% annual interest and wants to wait until it reaches $15,500 before withdrawing her investment.

1. **Write an exponential equation that represents this situation where x represents years.**

2. **What problems do you run into trying to solve an equation like this?**

3. **For now, estimate the amount of time it will take to reach $15,500. (We'll revisit this later!)**

LEARN IT

VIDEO: Introduction to Logarithms

As you saw in the intro, we sometimes need to solve for an exponent but we haven't learned how to isolate a variable in the exponent part of an equation. That's where logarithms come in! Watch this introduction to logarithms. Then, answer the questions.

1. **Briefly summarize what logarithms are used for.**

PRACTICE IT

PRACTICE: Converting Logarithms
Complete each problem by converting to the specified form.

1. **Convert each exponential equation into logarithmic form.**

   a. $8^2 = 64$

   b. $10^0 = 1$

   c. $4^x = 128$

2. **Convert each logarithmic equation into exponential form.**

   a. $\log_2 32 = 5$

   b. $\log_7 49 = 2$

   c. $\log_5 x = 3$

LEARN IT

EXAMPLE
Before we can solve for an exponential equation, we need to convert it into logarithmic form.

1. **Review the completed example below.**

1 **Isolate:**
If there are any numbers on the same side of the equation as the exponent and base, move them to the other side

$$5 \cdot 2^3 + 6 = 46$$
$$5 \cdot 2^3 = 40$$
$$2^3 = 8$$

2 **Recall:**
Recall the connection between exponential equations and logarithmic equations and their meaning

Exponential: $b^x = c$
What number do you get when you raise b to the x power? The answer is c.

Logarithm: $\log_b c = x$
What number did you raise b to in order to get an answer of c? You raised b to the x power.

3 **Rewrite:**
Rewrite your example using the conversion above. Then explain what it means.

Exponential: $2^3 = 8$
What number do you get when you raise 2 to the 3 power? You get 8.

Logarithm: $\log_2 8 = 3$
What number did you raise 2 to in order to get an answer of 8? You raised 2 to the 3 power.

PRACTICE IT

PRACTICE: Converting to and Evaluating Logarithms
Complete each problem by converting to logarithmic form and, if necessary, solving for the variable.

1. **Isolate each base and exponent, then convert to logarithmic form.**
   a. $5^3 + 1 = 126$
   b. $3 \cdot 7^2 - 4 = 143$
   c. $4 \cdot 2^x + 1 = 33$

2. **Evaluate each logarithm**
   a. $\log_2 128$
   b. $\log_4 64$
   c. $\log_3 81$

3. **Convert each of the following to logarithmic form, then solve for the variable**
   a. $6^x = 36$
   b. $3^x - 7 = 74$
   c. $2 \cdot 4^x + 1 = 33$

LEARN IT

The Change of Base Formula
Some logarithms are easy to evaluate in your head, but there are some that aren't. Let's take a look at a new formula that will help you solve all logarithms.

Let's consider the following two logarithms to see why we need the change of base formula:

What is $\log_5 25$?

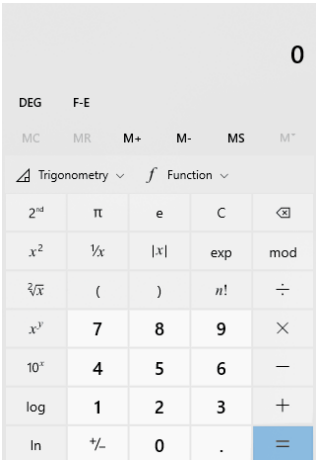
This one is easy to do in your head because this is asking the question: "What power did you raise 5 to to get an answer of 25?" Another way of thinking about this is the exponential form: $5^x = 25$. **We can easily get an answer of 2.**

What is $\log_5 30$?

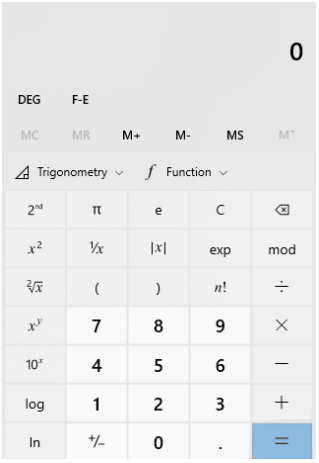
This logarithm is a little bit harder. The question here is "What power did you raise 5 to to get an answer of 30?" This is not obvious because it's a decimal. **We need the change of base formula for this example.**

The change of base formula lets us use the log button on our calculator to get an exact value for a logarithm.

$$\log_5 30 = \frac{\log 30}{\log 5}$$



This means that $\log_5 30 = 2.11328$. We can verify this by checking $5^{2.11238}$ to see if we get 30.



PRACTICE IT

PRACTICE: Evaluating Logarithms using the Change of Base Formula

1. **Evaluate each of the following using the change of base formula. Round to 3 decimal places.**
    a. $\log_4 35$

    b. $\log_8 50$

    c. $\log_{10} 1200$

Now that we know how to use logarithms to solve for an exponent, let's revisit Delilah's situation from the Intro. Recall that Delilah is investing $10,000 at 7% annual interest until her account reaches $15,500.

2. **Use the equation that you set up in the intro and solve it to calculate how long her money must be invested before it reaches its target amount.**

3. **Why might it be important for Delilah to know the exact amount of time it takes instead of an estimate?**

APPLY IT

Follow your teacher's directions to complete the Application Problems.

Teachers, you can find the Application problems linked in the Lesson Guide.