3.8 Combinatorics

Combinatorics - Methods of Counting

Ex 1 How many different 4 digit PINs are there?

Ex 2 a) How many different 10 digit phone numbers are there with no restriction?
b) Like a) but your number cannot begin with 0 or 1

These are examples of the fundamental counting principle (FCP).

FCP - when counting with sequential choices, multiply the results at each choice.

Ex 3 How many ways can 10 students line up?

Ex 4 a) How many ways can you reorder the letters of the word COMPUTER?
b) As a) but you must keep the vowels together.

Factorials get used alot in combinatorics so here's the definition:

$$n! = n \cdot (n-1) \cdot (n-2) \cdot ... \cdot 3 \cdot 2 \cdot 1$$

Other properties of Factorials

$$n! = n \cdot (n-1)! \quad 0! = 1$$

Ex 3 and 4 are examples of Permutations

Permutation - ${}_nP_r$ denotes the number of ways of creating an ordered list of r elements taken from a pool of n elements. ${}_nP_r = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-r+1)$ or

$${}_nP_r = \frac{n!}{(n-r)!}$$

Ex 5 There are 4 people who can be selected to create a committee. How many ways can we create this committee of:

a) 3 people
b) 2 people
c) Any number of people

Combination - $\binom{n}{r}$ denotes the number of ways of creating an unordered list (or subset) of r elements taken from a pool of n elements

$$\binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

PASCALS TRIANGLE

Ex 5 When $\left(1+\frac{x}{2}\right)^n$ is expanded in ascending powers of x, the coefficient of x^3 is 70.

(a) Find the value of n

(b) Hence, find the coefficient of x^2

Ex 6 There is a pool of 22 people (9 men, 13 women) to form a 12 person jury for an upcoming case. How many ways can the jury be formed if:

a) There are 6 men and 6 women on the jury?

b) There are only men or only women?

c) There must be at least one man on the jury?

HL book: p.191#1-5,7-10

Day 2 Repeated elements

Day 3 Another day of these but from question bank