Patterns in Higher Order Derivatives

In this module, you learned rules for taking derivatives of many types of functions. Oftentimes, these rules lead to some interesting patterns when taking higher order derivatives.

Partner up with a classmate as instructed by your teacher and discuss the following questions.

1. Is the derivative of a polynomial function also a polynomial function? If not, provide a counterexample. If so, what basic differentiation rules lead you to this conclusion?

2. Consider a polynomial function $p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ where $n \geq 0$ is a whole number. Find a general formula for the $(n + 1)^{\text{th}}$ derivative of $p(x)$. (**Hint:** Try taking the 3rd derivative of a quadratic function, the 4th derivative of a cubic function, the 5th derivative of a degree 4 polynomial function, etc. and see if you can find a pattern.)

3. You saw in this module that $\frac{d^{4n}}{dx^{4n}}(\sin x) = \sin x$ and $\frac{d^{4n+1}}{dx^{4n+1}}(\sin x) = \cos x$ for $n \geq 1$. Continue this pattern to find $\frac{d^{4n+2}}{dx^{4n+2}}(\sin x)$ and $\frac{d^{4n+3}}{dx^{4n+3}}(\sin x)$.

4. Now find similar formulas for the derivatives of $\cos x$. How do the derivatives of $\sin x$ compare to the derivatives of $\cos x$?

5. You saw in this module that the natural exponential function is its own derivative. In other words, the derivative of $E(x) = e^x$ is $E'(x) = e^x$. Find a general formula for the n^{th} derivative $E^{(n)}(x)$ of the natural exponential function.

6. Can you think of any other functions that have this same property where $f'(x) = f(x)$?