

Effective Mathematics Teaching Practice 1: Establish mathematics goals to focus learning

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

What are teachers doing?

- ☐ Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons or throughout a unit.
- ☐ Identifying how the goals fit within a mathematics learning progression.
- ☐ Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.
- ☐ Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions.

What are students doing?

- ☐ Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?).
- ☐ Using learning goals to stay focused on their progress in improving their understanding of mathematics content and practices.
- ☐ Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going.
- ☐ Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

Shift 1: From *stating-a-standard* toward *communicating expectations for learning*.

From	Level 1	Level 2	Level 3	Toward
Teacher shares broad performance goals and/or those provided in standards or curriculum documents	Teacher... a. does not establish goals or does not clearly define goals for mathematical understanding. b. fails to explain how the mathematical goals contribute to enduring understandings. c. fails to use goals to guide decision making. d. does not use concrete and semi-concrete (representational) activities or does not make any connections between activities and the mathematical goals. e. does not use a concrete--semi-concrete (representational)--abstract learning progression to meet these goals.	Teacher inconsistently ... a. establishes clear and detailed goals that indicate the mathematics students are learning b. explains how the mathematical goals contribute to enduring understandings. c. uses these goals to guide decision making. d. connects concrete and semi-concrete (representational) activities to the conceptual understanding of the mathematical goals. e. uses a concrete--semi-concrete (representational)--abstract learning progression to meet these goals.	Teacher consistently ... a. establishes clear and detailed goals that indicate the mathematics students are learning. b. explains how the mathematical goals contribute to enduring understandings. c. uses these goals to adjust instruction. d. connects concrete and semi-concrete (representational) activities to the conceptual understanding of the mathematical goals. e. uses a concrete--semi-concrete (representational)--abstract learning progression to meet these goals.	Teacher creates lesson-specific learning goals and communicates these goals at critical times within the lesson to ensure students understand the lesson's purpose and what is expected of them.

Effective Mathematics Teaching Practice 2. Implement tasks that promote reasoning and problem solving

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

What are teachers doing?

- ☐ Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.
- ☐ Selecting tasks that provide multiple entry points through the use of varied tools and representations.
- ☐ Posing tasks on a regular basis that require a high level of cognitive demand.
- ☐ Supporting students in exploring tasks without taking over student thinking.
- ☐ Encouraging students to use varied approaches and strategies to make sense of and solve tasks.

What are students doing?

- ☐ Persevering in exploring and reasoning through tasks.
- ☐ Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.
- ☐ Using tools and representations as needed to support their thinking and problem solving.
- ☐ Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Shift 2: From routine tasks toward reasoning tasks.

From	Level 1	Level 2	Level 3	Toward
Teacher uses tasks involving recall of previously learned facts, rules or definitions and provides students with specific strategies to follow.	Teacher... a. does not provide the opportunity for students to explore and solve problems, or provides opportunities that do not build on and extend their current mathematical understanding. b. does not select tasks that provide multiple entry points. c. poses tasks that require a low level of cognitive demand. d. does not provide opportunities for students to discuss tasks or often takes over student thinking. e. does not select tasks that allow students to make sense of and solve using varied approaches and strategies or provides one specific strategy or approach for students to use to solve tasks.	Teacher inconsistently ... a. provides opportunities for students to explore and solve problems that attempt to build on and extend their current mathematical understanding. b. selects tasks that provide multiple entry points. c. poses tasks that require a high level of cognitive demand. d. provides opportunities for students to discuss tasks or sometimes take over student thinking. e. selects tasks that allow students to make sense of and solve using those approaches and strategies presented in class.	Teacher consistently ... a. provides opportunities for students to explore and solve problems that build on and extend their current mathematical understanding. b. selects tasks that provide multiple entry points. c. poses tasks that require a high level of cognitive demand. d. provides opportunities for students to discuss tasks without taking over student thinking. e. selects tasks that allow students to make sense of and solve using varied approaches and strategies.	Teacher uses tasks that lend themselves to multiple representations, strategies or pathways encouraging student explanation (how) and justification (why/when) of solution strategies.

Mathematics Teaching Practice 3: Use and connect mathematical representations

Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

What are teachers doing?

- ☐ Selecting tasks that allow students to decide which representations to use in making sense of the problems.
- ☐ Allocating substantial instructional time for students to use, discuss and make connections among representations.
- ☐ Introducing forms of representations that can be useful to students.
- ☐ Asking students to make math drawings or use other visual supports to explain and justify their reasoning.
- ☐ Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.
- ☐ Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.

What are students doing?

- ☐ Using multiple forms of representations to make sense of and understand mathematics.
- ☐ Describing and justifying their mathematical understanding and reasoning with drawings, diagrams and other representations.
- ☐ Making choices about which forms of representations to use as tools for solving problems.
- ☐ Sketching diagrams to make sense of problem situations.
- ☐ Contextualizing mathematical ideas by connecting them to real-world situations.
- ☐ Considering the advantages or suitability of using various representations when solving problems.

Shift 3: From teachers about representations toward teaching through representations.

From	Level 1	Level 2	Level 3	Toward
Teacher shows students how to create a representation (e.g., a graph or picture).	Teacher... a. does not select tasks that allow students to choose representations (pictures, symbols, verbal, real-life situations, physical models) in making sense of problems. b. does not allocate sufficient instructional time for students to use, discuss, and make connections among representations. c. introduces and models a single representation that does not provide students with choices for making sense of problems, or introduces and models representations not aligned to the learning goal. d. does not ask students to use multiple representations to make connections, justify their reasoning, and deepen understanding of essential features and structure of mathematical concepts and procedures.	Teacher inconsistently ... a. selects tasks that allow students to choose representations (pictures, symbols, verbal, real-life situations, physical models) in making sense of problems. b. allocates instructional time for students to use, discuss, and make connections among representations. c. introduces and models a variety of representations that provide students with choices for making sense of problems. d. asks students to use multiple representations to make connections, justify their reasoning, and deepen understanding of essential features and structure of mathematical concepts and procedures.	Teacher consistently ... a. selects tasks that allow students to choose representations (pictures, symbols, verbal, real-life situations, physical models) in making sense of problems. b. allocates instructional time for students to use, discuss, and make connections among representations. c. introduces and models a variety of representations that provide students with choices for making sense of problems. d. asks students to use multiple representations to make connections, justify their reasoning, and deepen understanding of essential features and structure of mathematical concepts and procedures.	Teacher uses lesson goals to determine whether to highlight particular representations or to have students select a representation; in both cases, teacher provides opportunities for students to compare different representations and how they connect to key mathematical concepts.

Mathematics Teaching Practice 4: Facilitate meaningful mathematical discourse

Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

What are teachers doing?

- ☐ Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.
- ☐ Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.
- ☐ Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.
- ☐ Ensuring progress toward mathematical goals by making explicit connections among student approaches and reasoning.

What are students doing?

- ☐ Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.
- ☐ Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.
- ☐ Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.
- ☐ Identifying how different approaches to solving a task are the same and how they are different.

Shift 4: From *show-and-tell* toward *share-and-compare*.

From	Level 1	Level 2	Level 3	Toward
Teacher has students share their answers.	Teacher... a. does not engage students in discourse of mathematical ideas, reasoning, and approach. b. does not facilitate discourse among students by positioning them as authors of ideas, who explain and defend their approaches. c. does not make connections among student approaches and reasoning. d. does not select and sequence “evidence of student thinking” to highlight mathematical ideas and language for whole class analysis and discussion	Teacher inconsistently ... a. engages students in purposeful discourse of mathematical ideas, reasoning, and approaches, or regular discourse may not always be purposeful. b. facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches. c. makes connections among student approaches and reasoning or makes vague connections among student approaches and reasoning. d. selects and sequences “evidence of student thinking” to highlight mathematical ideas and language for whole class analysis and discussion	Teacher consistently ... a. engages students in purposeful discourse of mathematical ideas, reasoning, and approaches. b. facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches. c. makes connections among student approaches and reasoning clearly and explicitly. d. selects and sequences “evidence of student thinking” to highlight mathematical ideas and language for whole class analysis and discussion	Teacher creates a dynamic forum where students share, listen, honor and critique each other’s ideas to clarify and deepen mathematical understandings and language; teacher strategically invites participation in ways that facilitate mathematical connections.

Source: National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*.

Mathematics Teaching Practice 5: Pose purposeful questions

Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

What are teachers doing?

- ☐ Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.
- ☐ Making certain to ask questions that go beyond gathering information to probing thinking and require explanation and justification.
- ☐ Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion.
- ☐ Allowing sufficient wait time so that students can formulate and offer responses.

What are students doing?

- ☐ Expecting to be asked to explain, clarify, and elaborate on their thinking.
- ☐ Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly.
- ☐ Reflecting on and justifying their reasoning, not simply providing answers.
- ☐ Listening to, commenting on, and questioning the contributions of their classmates.

Shift 5: From questions that seek expected answers toward questions that illuminate and deepen student understanding.

From	Level 1	Level 2	Level 3	Toward
Teacher poses closed and/or low-level questions, confirms correctness of responses, and provides little or no opportunity for students to explain their thinking.	Teacher... a. asks questions that take over or funnel student thinking. b. asks questions that simply gather information and do not probe thinking or require explanation and justification. c. does not ask questions that make the structure of mathematics more accessible for student learning.	Teacher inconsistently ... a. advances student understanding by asking questions that build on, but do not take over or funnel, student thinking. b. assesses thinking by asking questions that require explanation and justification. c. asks intentional questions that make the structure of mathematics more accessible for student learning.	Teacher consistently ... a. advances student understanding by asking questions that build on, but do not take over or funnel, student thinking. b. assesses thinking by asking questions that require explanation and justification. c. asks intentional questions that make mathematical ideas and relationships more visible for students.	Teacher poses questions that advance student thinking, deepen students' understanding, make the mathematics more visible, provide insights into student reasoning and promote meaningful reflection.

Mathematics Teaching Practice 6: Build procedural fluency from conceptual understanding

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

What are teachers doing?

- ☐ Providing students with opportunities to use their own reasoning strategies and methods for solving problems.
- ☐ Asking students to discuss and explain why the procedures they are using work to solve particular problems.
- ☐ Connecting student-generated strategies and methods to more efficient procedures as appropriate.
- ☐ Using visual models to support students' understanding of general methods.
- ☐ Providing students with opportunities for distributed practice of procedures.

What are students doing?

- ☐ Making sure that they understand and can explain the mathematical basis for the procedures that they are using.
- ☐ Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.
- ☐ Determining whether specific approaches generalize to a broad class of problems.
- ☐ Striving to use procedures appropriately and efficiently.

Shift 6: From teaching so that students replicate procedures toward teaching so that students select efficient strategies.

From	Level 1	Level 2	Level 3	Toward
Teacher approaches facts and procedures with the goal of speed and accuracy.	Teacher... a. does not provide students with opportunities to use their own reasoning strategies and methods for solving problems or provide a preferred strategy. b. does not ask students to discuss and explain why the procedures that they are using work. c. does not connect student-generated strategies and methods to more efficient procedures as appropriate. d. does not develop student understanding of the structure within procedural fluency, i.e., by using automaticity practice too soon. e. provides students with opportunities for rote practice of procedures that are not purposeful.	Teacher inconsistently ... a. provides students with opportunities to use their own reasoning strategies and methods for solving problems. b. asks students to discuss and explain why the procedures that they are using work. c. connects student-generated strategies and methods to more efficient procedures as appropriate. d. uses a concrete-semi-concrete (representational)--abstract sequence to develop some student understanding of the structure within procedural fluency. e. provides students with opportunities for practice of procedures that are brief, engaging, and purposeful.	Teacher consistently ... a. provides students with opportunities to use their own reasoning strategies and methods for solving problems. b. asks students to discuss and explain why the procedures that they are using work. c. connects student-generated strategies and methods to more efficient procedures as appropriate. d. uses an intentional concrete-semi-concrete (representational)-abstract sequence to develop student understanding of the structure within procedural fluency. e. provides students with regular opportunities for practice of procedures that are brief, engaging, and purposeful.	Teacher provides time for students to engage with mathematical problems, developing flexibility by encouraging student selection and use of efficient strategies; teacher provides opportunities for students to evaluate when a strategy is best suited for the problem at hand.

Mathematics Teaching Practice 7: Support productive struggle in learning mathematics

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

What are teachers doing?	What are students doing?
☐ Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. ☐ Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. ☐ Helping students realize that confusion and errors are a natural part of learning, but facilitating discussions on mistakes, misconceptions and struggles. ☐ Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	☐ Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. ☐ Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. ☐ Persevering in solving problems and realizing that it is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. ☐ Helping one another without telling their classmates what the answer is or how to solve the problem.

Shift 7: From *mathematics-made-easy* toward *mathematics-takes-time*.

From	Level 1	Level 2	Level 3	Toward
Teacher presents mathematics in small chunks so that students reach solutions quickly.	Teacher... a. does not give students time to struggle with tasks. b. does not ask questions that purposefully scaffold students' thinking or step in to do the work for them. c. contributes to students' fixed mindsets through a lack of discussions around mistakes, misconceptions, and struggles. d. does not provide specific feedback to support students with moving learning forward with mathematical ideas and/or persevering to solve problems.	Teacher inconsistently ... a. gives students time to struggle with tasks. b. asks questions that purposefully scaffold students' thinking without stepping in to do the work for them. c. helps students develop a growth mindset through facilitating discussions on mistakes, misconceptions, and struggles. d. provides specific feedback that supports students with moving learning forward to develop mathematical ideas and/or persevering to solve problems.	Teacher consistently ... a. gives students time to struggle with tasks. b. asks questions that purposefully scaffold students' thinking without stepping in to do the work for them. c. helps students develop a growth mindset by facilitating discussions on mistakes, misconceptions and struggles, and acknowledging students for their efforts (rather than an innate ability). d. provides specific feedback that supports students with moving learning forward to develop mathematical ideas and/or persevering to solve problems.	Teacher questions, encourages, provides time and explicitly states the value of grappling with mathematical tasks, making multiple attempts and learning from mistakes.

Source: National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*.

Mathematics Teaching Practice 8: Elicit and use evidence of student thinking

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

What are teachers doing?

- ☐ Identifying what counts as evidence of student progress toward mathematics learning goals.
- ☐ Eliciting and gathering evidence of student understanding at strategic points during instruction.
- ☐ Interpreting student thinking to assess mathematical understanding, reasoning, and methods.
- ☐ Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.
- ☐ Reflecting on evidence of student learning to inform the planning of next instructional steps.

What are students doing?

- ☐ Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse.
- ☐ Reflecting on mistakes and misconceptions to improve their mathematical understanding.
- ☐ Asking questions, responding to and giving suggestions to support the learning of classmates.
- ☐ Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.

Shift 8: From *looking at correct answers* toward *looking for students' thinking*.

From	Level 1	Level 2	Level 3	Toward
Teacher attends to whether an answer or procedure is (or is not) correct.	Teacher... a. does not gather evidence of student understanding during instructional time or may only gather evidence at the end of instructional time. b. does not use this evidence to adjust instruction to support and extend student learning. c. does not provide students opportunities to reflect on their work to capture understanding and misconceptions.	Teacher inconsistently ... a. elicits appropriate evidence of student understanding during instructional time. b. uses this evidence to adjust instruction to support and extend student learning. c. provides students opportunities to reflect on their work to capture understanding and misconceptions.	Teacher consistently ... a. elicits appropriate evidence of student understanding at strategic points during instructional time. b. uses this evidence to adjust instruction to support and extend student learning. c. provides students opportunities to reflect on their work to capture understanding and misconceptions.	Teacher identifies specific strategies or representations that are important to notice; strategically uses observations, student responses to questions and written work to determine what students understand; and uses these data to inform in-the-moment discourse and future lessons.