Department of Physics

University of Rajasthan

Computer Programming (Code: H02)

Assigned: 14 August 2019 *1st Home Assignment* Due: 31 August 2019

Note: This assignment is for the students who are having computer lab. Students can show their performance to the teacher in the lab. For lab you have to make a file by writing the answers of the following programs. Required formulas are given in the next page of this file. In parallel, students can solve problems given in THIS FILE (please click here).

1) Write a c-program which can calculate $\sin(x)$ ($\forall\ 0 \le x \le \pi$), $\cos(x)$ ($\forall\ 0 \le x \le \pi$), $\tan(x)$ ($\forall\ 0 \le x \le \pi$), $\log_e(1+x)$ ($\forall\ -1 \le x \le 1$), $\log_{10}(1+x)$ ($\forall\ -1 \le x \le 1$), $\log_e(x)$ ($\forall\ 0 \le x < \infty$), $\log_{10}(x)$ ($\forall\ 0 \le x < \infty$), $\exp(x)$ ($\forall\ x \in R$), $\sin^{-1}(x)$ ($\forall\ -1 \le x \le 1$), $\cos^{-1}(x)$ ($\forall\ -1 \le x \le 1$), $\tan^{-1}(x)$ ($\forall\ -\infty < x < \infty$), $\sinh(x)$ ($\forall\ x \in R$), $\cosh(x)$ ($\forall\ x \in R$), $\tanh(x)$ ($\forall\ x \in R$), $\sinh^{-1}(x)$ ($\forall\ x \in$ R), $\cosh^{-1}(x)$ ($\forall\ 1 \le x \le \infty$) and $\tanh^{-1}(x)$ ($\forall\ -1 < x < +1$) using their perfect expansion series. Compare them with actual value and print error for first 10 steps.

2) Make a simple calculator which can do binary operations addition, subtraction, multiplication and divide in any order. Expressions can have up to 100 terms. It should follow simple arithmetic precedence. Program should give output if equal sign (`'='`) entered at the end of expression. In other cases, program should prompt the error message while running. To check your program please check with these expressions:

Input	Output	Input	Output
`1+2`	`Expression is not complete.`	`4/2=`	`2`
`1+2=`	`3`	`1+2+3=`	`6`
`1-2=`	`-1`	`6/2+4/2=`	`5`
`1*2=`	`2`	`4/2*3+4-5/5*6-8+4/2-5=`	`-7`
`1g2-4=`	`Operation is not defined.`	`2+3=5-6`	`5`

3) Integrate a given function with
   a) Trapezoidal rule
   b) Simpson's ⅓ rule and
   c) Simpson's ⅜ rule.

   Do the process in all the following ways:
   i) for at least 1000 iterations and calculate the relative errors with different values of iterations.
   ii) Compare all processes with each other. Program should tell us which one gives the best results.
   iii) Each process should have error less than 0.001%.

4) Write a c-program which can calculate the value of π using area of a circle and the volume of a sphere (generate random numbers as needed). You can generate random numbers using rand() function.

Helpful things for above assignment:

Q1) Some expansions are:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \ldots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \ldots$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\tan(x) = \cfrac{x}{1-\cfrac{x^2}{3-\cfrac{x^2}{5-\cfrac{x^2}{7-\cfrac{x^2}{9-\ldots}}}}} = \cfrac{1}{\frac{1}{x}-\cfrac{1}{\frac{3}{x}-\cfrac{1}{\frac{5}{x}-\cfrac{1}{\frac{7}{x}-\cfrac{1}{\frac{9}{x}-\ldots}}}}}$$

$$\log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \ldots$$

$$\log_{10}(1 + x) = -\frac{\log_e(1+x)}{\log_e(1+a)}, \text{ Where } a = \text{-}0.9$$

For calculating $\log_e(x)$ and $\log_{10}(x)$, we can use above property for x = (1 - y) / (1 + y), where |y| < 1. So, we can write y = (x - 1)/(x+1).

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \ldots$$

$$\sin^{-1}(x) = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{33}{1152}x^9 + \ldots = \cfrac{x\sqrt{1-x^2}}{1-\cfrac{1.2x^2}{3-\cfrac{1.2x^2}{5-\cfrac{3.4x^2}{7-\cfrac{3.4x^2}{9-\cfrac{5.6x^2}{11-\ldots}}}}}}$$

$$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

$$\tan^{-1}(x) = \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \cfrac{x}{1+\cfrac{x^2}{3+\cfrac{4x^2}{5+\cfrac{9x^2}{7+\cfrac{16x^2}{9+\ldots}}}}}$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \frac{x^{11}}{11!} + \ldots = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \frac{x^{10}}{10!} + \ldots = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \cfrac{x}{1+\cfrac{x^2}{3+\cfrac{x^2}{5+\cfrac{x^2}{7+\cfrac{x^2}{9+\ldots}}}}}$$

$$\sinh^{-1}(x) = \log_e(x + \sqrt{1 + x^2})$$

$$\cosh^{-1}(x) = \log_e(x + \sqrt{x^2 - 1})$$

$$\tanh^{-1}(x) = \frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \frac{x^{11}}{11} + \ldots$$

Q2) You can use for or while loop. In that you have to put some conditions for equal sign and other signs.

Q3) Integration of a function $f(x)$ from $x = a$ to $x = b$ by Trapezoidal rule is:

$$\int_a^b f(x)\,dx = \frac{h}{2}[f(a) + 2 \times \{f(a + h) + f(a + 2 \times h) + f(a + 3 \times h) + \ldots + f(a + (n - 1) \times h\}$$

Where

$h = (b-a)/n;$

If we write $x_0 = a$ and $x_i = a + i{\times}h$ then we can write above formula as:

$$\int_a^b f(x)\,dx = \frac{h}{2}[f(x_0) + 2 \times \{f(x_1) + f(x_2) + f(x_3) + \ldots + f(x_{n-1})\} + f(x_n)]$$

Integration of a function $f(x)$ from $x = a$ to $x = b$ by Simpson ⅓ rule (n should be even):

$$\int_a^b f(x)\,dx = \frac{h}{3}[f(a) + 4 \times f(a + h) + 2 \times f(a + 2 \times h) + 4 \times f(a + 3 \times h) + 2 \times f(a + 4 \times h) -$$

$$\ldots + 2 \times f(a + (n - 2) \times h) + 4 \times f(a + (n - 1) \times h) + f(b)]$$

Where

$h = (b-a)/n;$

If we write $x_0 = a$ and $x_i = a + i{\times}h$ then we can write above formula as:

$$\int_a^b f(x)\,dx = \frac{h}{3}[f(x_0) + 4 \times \{f(x_1) + f(x_3) + f(x_5) + \ldots + f(x_{n-1})\} + 2 \times \{f(x_2) + f(x_4) + f(x_6) +$$

Integration of a function $f(x)$ from $x = a$ to $x = b$ by Simpson ⅜ rule (n should be divisible by 3):

$$\int_a^b f(x)\,dx = \frac{3h}{8}[f(a) + 3 \times f(a + h) + 3 \times f(a + 2 \times h) + 2 \times f(a + 3 \times h) + 3 \times f(a + 4 \times h)$$

$$+ 2 \times f(a + 6 \times h) + \ldots + 2 \times f(a + (n - 3) \times h) + 3 \times f(a + (n - 2) \times h) + 3 \times f(a + (n - 1) +$$

Where

$h = (b-a)/n;$

If we write $x_0 = a$ and $x_i = a + i{\times}h$ then we can write the above formula as:

$$\int_a^b f(x)\,dx = \frac{3h}{8}[f(x_0) + 3 \times \{f(x_1) + f(x_2) + f(x_4) + f(x_5) + f(x_7) + f(x_8) + \ldots + f(x_{n-2}) + f(x_{n-1})$$

$$+ 2 \times \{f(x_3) + f(x_6) + f(x_9) + \ldots + f(x_{n-6}) + f(x_{n-3})\} + f(x_n)]$$