Including Firm Adaptation to Risk

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Including Firm Adaptation to Risk

Now what about recourse (simplspr.gms)

What is recourse?

Make a Decision now for example investment in capital goods

Then make a decision later – but must adjust in face of prior decision cannot entirely undo it so we are stuck with earlier level of capital goods investment

Suppose we have the following decision

Today we can invest in a machine which costs $3

During the machine life we use it under differing price capacity and yield events that are uncertain

Two projected futures exist
At the time we use the machines we know the conditions

Two states of nature can occur

	Price	Yield with invest	Yield without invest	Unit that can be produced	Probability
Son 1	4	1.2	1.1	2	0.3
Son 2	6	1.9	0.9	2.2	0.7

Including Firm Adaptation to Risk

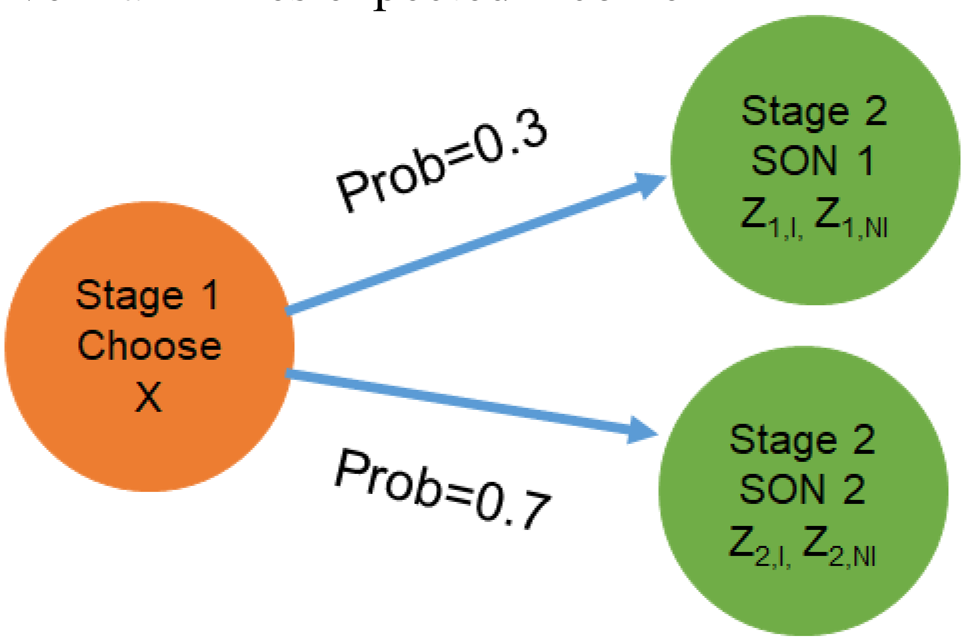
Now what about recourse (simplspr.gms)

Problem will have 2 stages

Stage 1 Investment stage when we choose whether to buy a machine for which we define a single variable X
Stage 2 Operation stage when we use the machine and know prices, capacity and yield which results in variable to operate with (I) or without (NI) the investment under each state of nature (the 4 variables Z below)

Max	$-3X$	$+0.3*4($	$1.2 *Z_{1,I}$	$+1.1 * Z_{1,NI})$	$+0.7*6($	$1.9 * Z_{2,I}$	$+0.9 * Z_{2,NI})$	
s.t.	$-2X$		$+ Z_{1,I}$					≤ 0
			$+ Z_{1,I}$	$+ Z_{1,NI}$				≤ 2
	$-2.2X$					$+ Z_{2,I}$		≤ 0
						$+ Z_{2,I}$	$+ Z_{2,NI}$	≤ 2.2

Objective maximizes expected income



Including Firm Adaptation to Risk

Now what about recourse (simplspr.gms)

$$\text{Max} \quad -3X + 0.3*4(1.2 * Z_{1,I} + 1.1 * Z_{1,NI}) + 0.7*6(1.9 * Z_{2,I} + 0.9 * Z_{2,NI})$$

$$\begin{array}{lllllll}
\text{s.t.} & -2X & + Z_{1,I} & & & & \leq 0 \\
 & & + Z_{1,I} & + Z_{1,NI} & & & \leq 2 \\
 & -2.2X & & & + Z_{2,I} & & \leq 0 \\
 & & & & + Z_{2,I} & + Z_{2,NI} & \leq 2.2
\end{array}$$

Note one decision variable (X) in first stage, 2 for each event at second stage (Z). Thus shows operation under 2 mutually exclusive second stages. ie at the same time we cannot have 2 prices, yields and capacities

When we solve we get

Solution obj=18.44 X=1 $Z_{1,I}=2$ $Z_{2,I}=2.2$

Note the X tells how to invest now, the Z's tell how to use later

Including Firm Adaptation to Risk

Stochastic Programming Model with Resources (SPR)

(simplspr.gms)

$$\text{Max} \quad -3X \quad +0.3*4(\quad 1.2 * Z_{1,I} \quad +1.1 * Z_{1,NI}) \quad +0.7*6(\quad 1.9 * Z_{2,I} \quad +0.9 * Z_{2,NI})$$

$$
\begin{array}{lllllll}
\text{s.t.} & -2X & +Z_{1,I} & & & & \leq 0 \\
 & & +Z_{1,I} & +Z_{1,NI} & & & \leq 2 \\
 & -2.2X & & & +Z_{2,I} & & \leq 0 \\
 & & & & +Z_{2,I} & +Z_{2,NI} & \leq 2.2
\end{array}
$$

```
SET STATE          STATES OF NATURE     /Son1  , Son2/
    item  /price,yieldwith,yieldwithout,capacity,PROBability/;

table data(item,STATE)     Stochastic data
                      Son1         Son2
price                  4             6
yieldwith             1.2           1.9
yieldwithout          1.1           0.9
capacity                2           2.2
PROBability            0.3          0.7   ;
set invest(item) /yieldwith,yieldwithout/;
 POSITIVE VARIABLES
          BuyMachine        first stage variable
          Use(state,invest) second stage variables;
 VARIABLES
          PROFIT                   TOTALPROFIT
  EQUATIONS
          OBJT                     OBJECTIVE FUNCTION ( PROFIT )
          linkcapacity(state)      New invest capacity AVAILABLE
          totcapacity(state)       Total Capacity AVAILABLE;
  OBJT..  PROFIT =E=        -3*BuyMachine
        +SUM(STATE,data("PROBability",STATE)*data("price",STATE)
         *sum(invest,data(invest,state)*Use(state,invest))) ;
  linkcapacity(state)..     -data("capacity",STATE)*BuyMachine
                            + Use(STATE,"yieldwith") =l= 0;

  totcapacity(STATE)..   sum(invest,Use(STATE,invest))=L=
                          data("capacity",STATE);
 MODEL BASICSPR /ALL/;
```

Including Firm Adaptation to Risk - Risk with Recourse (simplspr.gms)

$$\text{Max} \quad -3X \quad +0.3*4(\quad 1.2 * Z_{1,I} \quad +1.1 * Z_{1,NI}) \quad +0.7*6(\quad 1.9 * Z_{2,I} \quad +0.9 * Z_{2,NI})$$

$$\begin{array}{llllll} \text{s.t.} & -2X & +Z_{1,I} & & & \leq 0 \\ & & +Z_{1,I} & +Z_{1,NI} & & \leq 2 \\ & -2.2X & & & +Z_{2,I} & \leq 0 \\ & & & & +Z_{2,I} \quad +Z_{2,NI} & \leq 2.2 \end{array}$$

Solution

```
---- EQU linkcapacity  New invest capacity AVAILABLE
        LOWER      SLACK      UPPER     MARGINAL
Son1     -INF        .          .          .
Son2     -INF        .          .         1.364

---- EQU totcapacity  Total Capacity AVAILABLE
        LOWER      SLACK      UPPER     MARGINAL
Son1     -INF        .         2.000     1.440
Son2     -INF        .         2.200     6.616

  BuyMachine  first stage variable
                     LOWER    LEVEL    UPPER     MARGINAL
---- VAR BuyMachine   .       1.000    +INF         .

---- VAR Use  second stage variables
                   LOWER    LEVEL    UPPER     MARGINAL
Son1.yieldwith       .      2.000    +INF        .
Son1.yieldwithout    .        .      +INF      -0.120
Son2.yieldwith       .      2.200    +INF        .
Son2.yieldwithout    .        .      +INF      -2.836

                   LOWER      LEVEL      UPPER    MARGINAL
---- VAR PROFIT    -INF      17.436      +INF         .
```

Including Firm Adaptation to Risk

$$\begin{array}{llll}
Max & \sum_k p_k Y_k & & \\
s.t. & -Y_k + \sum_l c_j X_j + \sum_n e_{nk} Z_{nk} & = 0 & \text{for all } k \\
& \sum_j a_{ij} X_j & \leq b_i & \text{for all } i \\
& -\sum_j d_{mjk} X_j + \sum_n f_{mnk} Z_{nk} & \leq 0 & \text{for all } m, k \\
& \sum_n g_{wnk} Z_{nk} & \leq s_{wk} & \text{for all } w, k \\
& Y_k & \gtreqless 0 & \text{for all } k \\
& X_j, \quad Z_{nk} & \geq 0 & \text{for all } j, n, k
\end{array}$$

Max	E									
	-E	$+0.3Y_1$	$+0.7Y_2$							$= 0$
		$-Y_1$		$-3X$	$+4*(1.2 * Z_{1,I}$	$+1.1 *Z_{1,NI})$				$= 0$
s.t.				$-2X$	$+ Z_{1,I}$					≤ 0
					$+ Z_{1,I}$	$+ Z_{1,NI}$				≤ 2
			$-Y_2$	$-3X$				$+6(1.9 * Z_{2,I}$	$+0.9 *Z_{2,NI})$	$= 0$
				$-2.2X$				$+ Z_{2,I}$		≤ 0
								$+ Z_{2,I}$	$+ Z_{2,NI}$	≤ 2.2

Including Firm Adaptation to Risk - Add Risk Aversion(spraver.gms)

Back to Unified model

Max	E								$-\ \emptyset(\sum_i P_i(Y_i - E)^2)^{0.5}$	
	E	$+0.3Y_1$	$+0.7Y_2$							$= 0$
		$-Y_1$		$-3X$	$+4*(1.2 * Z_{1,I}$	$+1.1 *Z_{1,NI})$				$= 0$
s.t.				$-2X$	$+ Z_{1,I}$					≤ 0
					$+ Z_{1,I}$	$+ Z_{1,NI}$				≤ 2
			$-Y_2$	$-3X$			$+6(1.9 * Z_{2,I}$	$+0.9 *Z_{2,NI})$		$= 0$
				$-2.2X$			$+ Z_{2,I}$			≤ 0
							$+ Z_{2,I}$	$+ Z_{2,NI}$		≤ 2.2

```
objt.. riskobj=e=avgincome  -rap*(sum(state, data("probability",state)
                                  *sqr(income(state)-avgincome))**0.5);
avgincomeacct..     sum(state,data("probability",state)*income(state))=e=avgincome;
incomeacct(state)..  income(state)=E=  -3*BuyMachine+
   data("price",STATE)*sum(invest,data(invest,state)*Use(state,invest)) ;
linkcapacity(state).. -data("capacity",STATE)*BuyMachine + Use(STATE,"yieldwith")
                     =l= 0;
totcapacity(STATE)..   sum(invest,Use(STATE,invest))=L= data("capacity",STATE);
```

Including Firm Adaptation to Risk

Multiple Stages(sellspr.gms)

Suppose we can sell now, in 6 months and in one year and between now and then we get to observe the prices. If we sell now we get the certain price of $2, 6 months from now either 2.2 or 2.15, one year from now 2.01 or 2.44 with probability conditional on six month price

	Period 1			Period 2				Period 3									
								Period 2 State 1		Period 2 State 1		Period 2 State 2		Period 2 State 2			
	Average Ending Net Worth			State 1		State 2		State A		State B		State A		State B			
		Sell	Keep	Sell	Keep	Sell	Keep	Sell	End Worth	Sell	End Worth	Sell	End Worth	Sell	End Worth		
Objective	1															max	
Starting Stock		1	1													≤	100
Avg End Worth	1								-0.21		-0.49		-0.27		-0.03	=	0
Stock Kept pd 1 to 2 s1			-1	1	1											≤	0
Stock Kept pd 1 to 2 s2			-1			1	1									≤	0
Stock Kept pd 2 to 3 s1-sA					-1			1								≤	0
Ending Worth s1-sA		2		2.2				2.01	-1							=	0
Stock Kept pd 2 to 3 s1-sB					-1					1						≤	0
Ending Worth s1-sB		2		2.2						2.44	-1					=	0
Stock Kept pd 2 to 3 s2-sA							-1					1				≤	0
Ending Worth s2-sA		2				2.15						2.01	-1			=	0
Stock Kept pd 2 to 3 s2-sB							-1							1		≤	0

Ending Worth s2-sB	2	2.15	2.44	-1	=	0

Including Firm Adaptation to Risk - Multiple Stages(sellspr.gms)

```
*   SECTION A    SET DEFINITION
SET          PERIODS  TIME PERIODS  /T1,T2,T3/
             STATE2   STATES OF NATURE FOR PERIOD2  /S21,S22/
             STATE3   STATES OF NATURE FOR PERIOD3  /S31,S32/          ;
*   SECTION B    DATA DEFINITION
 SCALAR      INVENTORY     STOCK ON HAND     /100/
             PRICE1        PRICE IN PERIOD1  /2.00/
  PARAMETER PRICE2(STATE2)     PRICE AT STATE NATURE 2 /S21 2.20 , S22 2.15/
             PROB2(STATE2)     PROBABILITY OF STATE OF NATURE PERIOD 2
                                                    /S21 .7 , S22 .3/
              PRICE3(STATE3) PRICE AT STATE OF NATURE 3
                    /      S31    2.01  , S32  2.44 /
   TABLE      PROB3(STATE2,STATE3) PROBABILITY OF STATES IN PD 3
*                                 CONDITIONAL ON STATE RESULTING IN PD 2
                         S31    S32
                  S21     .3     .7
                  S22     .9     .1
    POSITIVE VARIABLES
             SELL1                       SALES IN PERIOD 1
             SELL2(STATE2)               SALES IN PERIOD 2 BY STATE
             SELL3(STATE2,STATE3)        SALES IN PERIOD 3 BY STATE IN PD 2 & 3
             KEEP1                       STOCK KEPT ON HAND FROM PERIOD 1 TO 2
             KEEP2(STATE2)               STOCK KEPT ON HAND FROM PD 2 TO 3
    VARIABLES
           AVGWORTH                      TOTAL ENDING NET WORTH
           ENDWORTH(STATE2,STATE3)       INCOME BY STATE OF NATURE;
    EQUATIONS
           OBJT                        OBJECTIVE FUNCTION
           BALANCE1                    INITIAL STOCK AVAILABLE
           KEPT12(STATE2)              STOCK KEPT FROM PERIOD 1 INTO 2
           KEPT23(STATE2,STATE3)       STOCK KEPT FROM PERIOD 2 INTO 3
           WORTHBAL(STATE2,STATE3)     WORTH BALANCE BY STATE OF NATURE;
  OBJT.. AVGWORTH =e=
      SUM((STATE2,STATE3),
           PROB2(STATE2)*PROB3(STATE2,STATE3)*ENDWORTH(STATE2,STATE3));
  WORTHBAL(STATE2,STATE3)..
     PRICE1*SELL1 + PRICE2(STATE2)*SELL2(STATE2)
     + PRICE3(STATE3)*SELL3(STATE2,STATE3) =E=  ENDWORTH(STATE2,STATE3);
  BALANCE1..     SELL1 + KEEP1 =L=  INVENTORY;

  KEPT12(STATE2)..   -KEEP1 + SELL2(STATE2) + KEEP2(STATE2) =L= 0;

  KEPT23(STATE2,STATE3)..   -KEEP2(STATE2) + SELL3(STATE2,STATE3) =L= 0;
  MODEL STOCSTOCK /ALL/;
  SOLVE STOCSTOCK USING LP MAXIMIZING AVGWORTH;
```

Including Firm Adaptation to Risk - Multiple Stages(sellspr.gms) Solution

```
                          LOWER      SLACK      UPPER    MARGINAL
---- EQU BALANCE1         -INF        .       100.000     2.263
---- EQU KEPT12  STOCK KEPT FROM PERIOD 1 INTO 2
       LOWER      SLACK      UPPER    MARGINAL
S21     -INF        .          .        1.618
S22     -INF        .          .        0.645
---- EQU KEPT23  STOCK KEPT FROM PERIOD 2 INTO 3
           LOWER      SLACK      UPPER    MARGINAL
S21.S31     -INF        .          .        0.422
S21.S32     -INF        .          .        1.196
S22.S31     -INF        .          .        0.543
S22.S32     -INF        .          .        0.073
---- EQU WORTHBAL  WORTH BALANCE BY STATE OF NATURE
           LOWER      SLACK      UPPER    MARGINAL
S21.S31      .          .          .       -0.210
S21.S32      .          .          .       -0.490
S22.S31      .          .          .       -0.270
S22.S32      .          .          .       -0.030
                          LOWER      LEVEL      UPPER    MARGINAL
---- VAR SELL1              .          .         +INF     -0.263
  SELL1  SALES IN PERIOD 1
---- VAR SELL2  SALES IN PERIOD 2 BY STATE
       LOWER      LEVEL      UPPER    MARGINAL
S21      .          .         +INF     -0.078
S22      .       100.000      +INF       .
---- VAR SELL3  SALES IN PERIOD 3 BY STATE IN PD 2 & 3
           LOWER      LEVEL      UPPER    MARGINAL
S21.S31      .       100.000      +INF       .
S21.S32      .       100.000      +INF       .
S22.S31      .          .         +INF       .
S22.S32      .          .         +INF       .
                          LOWER      LEVEL      UPPER    MARGINAL
---- VAR KEEP1              .       100.000      +INF       .
  KEEP1  STOCK KEPT ON HAND FROM PERIOD 1 TO 2

---- VAR KEEP2  STOCK KEPT ON HAND FROM PD 2 TO 3
       LOWER      LEVEL      UPPER    MARGINAL
S21      .       100.000      +INF       .
S22      .          .         +INF     -0.029
                          LOWER      LEVEL      UPPER    MARGINAL
---- VAR AVGWORTH         -INF     226.270      +INF       .
  AVGWORTH  TOTAL ENDING NET WORTH
---- VAR ENDWORTH  INCOME BY STATE OF NATURE
           LOWER      LEVEL      UPPER    MARGINAL
S21.S31     -INF     201.000      +INF       .
S21.S32     -INF     244.000      +INF       .
S22.S31     -INF     215.000      +INF       .
S22.S32     -INF     215.000      +INF       .
```

Red and blue are adaptation

$$
\begin{array}{lll}
Max & \overline{Y} - \psi \sum_k p_k \left(d_k^+ + d_k^-\right)^2 & \\
s.t. & -\overline{Y} + \sum_k p_k Y_k & = 0 \\
 & -\overline{Y} + Y_k - d_k^+ + d_k^- & = 0 \quad for\ all\ k \\
 & + \sum_j c_j X_j - Y_k + \sum_n e_{nk} Z_{nk} & = 0 \quad for\ all\ k \\
 & \sum_j a_{ij} X_j & \leq b_i \quad for\ all\ i \\
 & -\sum_j d_{mjk} X_j + \sum_n f_{mnk} Z_{nk} & \leq 0 \quad for\ all\ m, k \\
 & \sum_n g_{wnk} Z_{nk} & \leq s_{wk} \quad for\ all\ w, k \\
 & \overline{Y},\ Y_k & \overset{<}{\underset{>}{-}}\ 0 \quad for\ all\ k \\
 & X_j,\ d_k^+,\ d_k^-,\ Z_{nk} & \geq 0 \quad for\ all\ j, n, k
\end{array}
$$

Table 14.19. Data on Uncertain Parameters in First SPR Example

Parameter	Value Under State of Nature 1	State of Nature 2
Probability	.6	.4
Corn Yield in bu	100	105
Wheat Yield in bu	40	38
Corn Harvest Rate hours per bu	.010	.015
Wheat Harvest Rate hours per bu	.030	.034
Corn Price per bu	3.25	2.00
Wheat Price per bu	5.00	6.00
Harvest Time hours	122	143

Table 14.20. Risk Free Formulation of First SPR Example

	Grow Corn	Grow Wheat	Income	Harvest Corn	Harvest Wheat	RHS	
Objective			1				
Land	1	1				$\leq$	100
Corn Yield Balance	$-yield_c$			1		$\leq$	0
Wheat Yield Balance		$-yield_w$			1	$\leq$	0
Harvest Hours				$+harvtime_c$	$+harvtime_w$	$\leq$	harvavail
Income	-100	-60	-1	$+price_c$	$+price_w$	=	0

Table 14.21. Formulation of First Stochastic Programming with Resources (SPR) Example

					State 1			State 2			
		Grow Corn	Grow Wht.	Inc. s1	Harv Corn s1	Harv Wht s1	Inc. s2	Harv Corn s2	Harv Wht s2		RHS
	Objective			.6			.4				max
	Land	1	1							≤	100
State 1	Corn s1	-100			1					≤	0
	Wheat s1		-40			1				≤	0
	Harvest Hours s1				.010	.030				≤	122
	Income s1	-100	-60	-1	3.25	5.00				=	0
State 2	Corn s2	-105						1		≤	0
	Wheat s2		-38						1	≤	0
	Harvest Hours s2							.015	.034	≤	143
	Income s2	-100	-60				-1	2.00	6.00	=	0

Table 14.22. Solution of First SPR Example

Equation	Slack	Shadow Price
Objective	16476	
Land	0	24.28
Corn s1	0	-1.95
Wheat s1	0	0.67
Harvest Hours s1	11.75	0
Income s1	0	-0.6
Corn s2	0	-3.00
Wheat s2	0	0.94
Harvest Hours s2	0	98.23
Income s2	0	-0.4

Variable	Solution Value	Marginal Cost
Grow Corn	48.8	0
Grow Wheat	51.2	0
Income S1	18144	0
Harvest Corn s1	4876	0
Harvest Wheat s1	2049	0
Income S2	13972	0
Harvest Corn s2	5120	0
Harvest Wheat s2	1947	0

Table 14.23. Second SPR Example Formulation (Partial Tableau)

	Corn	Soy	Wht	Avg Cost	Pos Prot Dev s1	Neg Prot Dev s1	Pos Eng Dev s1	Neg Eng Dev s1	Cost s1	Pos Cost Dev s1	Negs Cost Dev s1	Pos Prot Dev s2	Neg Prot Dev s2	Pos Eng Dev s2	Neg Eng Dev s2	Cost s2	Pos Cost Dev s2	Neg Cost Dev s2		
Objective				1						+	+						+	+		
Total Feed	1	1	1																=	1
Average Cost				1					-.25							-.25			=	0
Protein-s1	0.23	1.12	0.51		-1	1													=	0.6
Energy -s1	1.15	0.26	1.05				-1	1											=	0.9
Cost-s1	0.03	0.06	0.04		0.50	1.50	1.00	0.10	-1										=	0
Cost dev s1				-1					1	-1	1								=	0
Protein-s2	0.17	1.08	0.59									-1	1						=	0.6
Energy -s2	1.10	0.31	0.95											-1	1				=	0.9

Cost-s2	.03	.06	.04			0.50	1.50	1.00	0.10	-1			=	0
Cost dev s2				-1						1	-1	1	=	0

Table 14.24. Second SPR Example Risk Neutral Solution

	Slack	Shadow Price		Slack	Shadow Price
Objective	0.067		Corn Purchase	0.283	0
Total Feed	0	-0.14	Soybean Purchase	0.362	0
Average Cost	0.00	1.	Wheat Purchase	0.355	0
Protein-s1	0	0.125	Average Cost	0.067	0
Energy -s1	0	0.025	Pos Protein Dev s1	0.052	0
Cost-s1	0	252.66	Neg Protein Dev s1	0.	0.50
Cost dev s1	0	0.00	Pos Energy Dev s1	0.00	0
Protein-s2	0	0.125	Neg Energy Dev s1	0.108	0
Energy -s2	0	0.025	Cost - s1	0.081	0
Cost-s2	0	0.25	Pos Cost Dev - s1	0.014	0
Cost dev s2	0	0	Neg Cost Dev - s1	0.00	0
Protein-s3	0	-.366	Pos Protein Dev s2	0.049	0
Energy -s3	0	0.025	Neg Protein Dev s2	0.000	0.50
Cost-s3	0	0.25	Pos Energy Dev s2	0.	0.275
Cost dev s3	0	0	Neg Energy Dev s2	0.140	0
Protein-s4	0	.08	Cost - s2	0.083	0
Energy -s4	0	.025	Pos Cost Dev - s2	.016	0
Cost-s4	0	0.25	Neg Cost Dev - s2	0.00	0
Cost dev s4	0	0.00	Pos Protein Dev s3	0.	0.491
			Neg Protein Dev s3	0.	0.009
			Pos Energy Dev s3		0.275
			Neg Energy Dev s3	0.080	0
			Cost - s3	0.052	0
			Pos Cost Dev - s3	0.00	0
			Neg Cost Dev - s3	0.014	0
			Pos Protein Dev s4	0.	0.205
			Neg Protein Dev s4	0.	0.295
			Pos Energy Dev s4	0.	0.275
			Neg Energy Dev s4	0.067	0
			Cost - s4	0.051	0
			Pos Cost Dev - s4	0.	0
			Neg Cost Dev - s4	0.016	0

Table 14.25. SPR Second Example Problem Solution Under Varying Risk Aversion

RAP	0	0.1	0.2	0.3	0.4	0.500	0.600
Corn	0.283	0.249	0.245	0.244	0.288	0.296	0.297
Soybeans	0.362	0.330	0.327	0.326	0.340	0.342	0.342
Wheat	0.355	0.422	0.428	0.430	0.372	0.363	0.361
Avgcost	0.067	0.067	0.067	0.067	0.071	0.071	0.071
Cost s1	0.081	0.074	0.073	0.073	0.071	0.071	0.071
Cost s2	0.083	0.080	0.080	0.080	0.074	0.073	0.073
Cost s3	0.052	0.066	0.067	0.068	0.071	0.071	0.071
Cost s4	0.051	0.048	0.048	0.048	0.067	0.070	0.071
Std Error	0.015	0.012	0.012	0.012	0.002	0.001	0.001

RAP is the risk aversion parameter.

Table 14.26. Example Tableau for Third SPR Problem

		Average Ending Net Worth	Period 1		Period 2				Stage 3									
					State 1		State 2		Period 2 State 1				Period 2 State 2					
									Period 3 State A		Period 3 State B		Period 3 State A		Period 3 State B			
			Sell	Keep	Sell	Keep	Sell	Keep	Sell	End Worth	Sell	End Worth	Sell	End Worth	Sell	End Worth		
	Objective	1															max	
	Starting Stock		1	1													≤	100
	Avg End Worth	1								-0.42		-0.28		-0.21		-0.09	=	0
	Stock Kept pd 1 to 2 s1			-1	1	1											≤	0
	Stock Kept pd 1 to 2 s2			-1			1	1									≤	0
P2 S1	Stock Kept pd 2 to 3 s1-sA					-1			1								≤	0
	Ending Worth s1-sA		2.1412		2.332				2.18	-1							=	0
	Stock Kept pd 2 to 3 s1-sB					-1					1						≤	0
	Ending Worth s1-sB		2.1008		2.288						2.44	-1					=	0
P2 S2	Stock Kept pd 2 to 3 s2-sA							-1					1				≤	0
	Ending Worth s2-sA		2.1828				2.193						2.18	-1			=	0
	Stock Kept pd 2 to 3 s2-sB							-1							1		≤	0
	Ending Worth s2-sB		2.1012				2.111								2.44	-1	=	0

Table 14.27. Solution for Third SPR Example

Variable	Value	Reduced Cost	Variable	Slack	Shadow Price
Average Ending Net Worth	229.748	0	Objective	229.748	
Sell In Period 1	0	-0.162	Starting Stock	0	2.297
Keep From Period 1 to 2	100	0	Avg End Worth	0	1
Sell In Period 2 Under State 1	100	0	Stock Kept pd 1 to 2 s1	0	1.62
Keep From Period 2 to 3 Under State 1	0	-0.021	Stock Kept pd 1 to 2 s1	0	0.677
Sell In Period 2 Under State 2	0	-0.027	Stock Kept pd 2 to 3 s1-s1	0	0.916
Keep From Period 2 to 3 Under State 2	100	0	Ending Worth s1-s1	0	-0.42
Sell in Period 3 Under State 1 -- State A	0	0	Stock Kept pd 2 to 3 s1-s2	0	0.683
Ending Worth Under State 1 -- State A	233.2	0	Ending Worth s1-s2	0	-0.28
Sell In Period 3 Under State 1 -- State B	0	0	Stock Kept pd 2 to 3 s2-s1	0	0.458
Ending Worth Under State 1 -- State B	228.8	0	Ending Worth s2-s1	0	-0.21
Sell In Period 3 Under State 2 -- State A	100	0	Stock Kept pd 2 to 3 s2-s2	0	0.22
Ending Worth Under State 2 -- State A	218	0	Ending Worth s2-s2	0	-0.09
Sell In Period 3 Under State 2 -- State B	100	0			
Ending Worth Under State 2 -- State B	244	0			

14.1. E-V Model Efficient Frontier

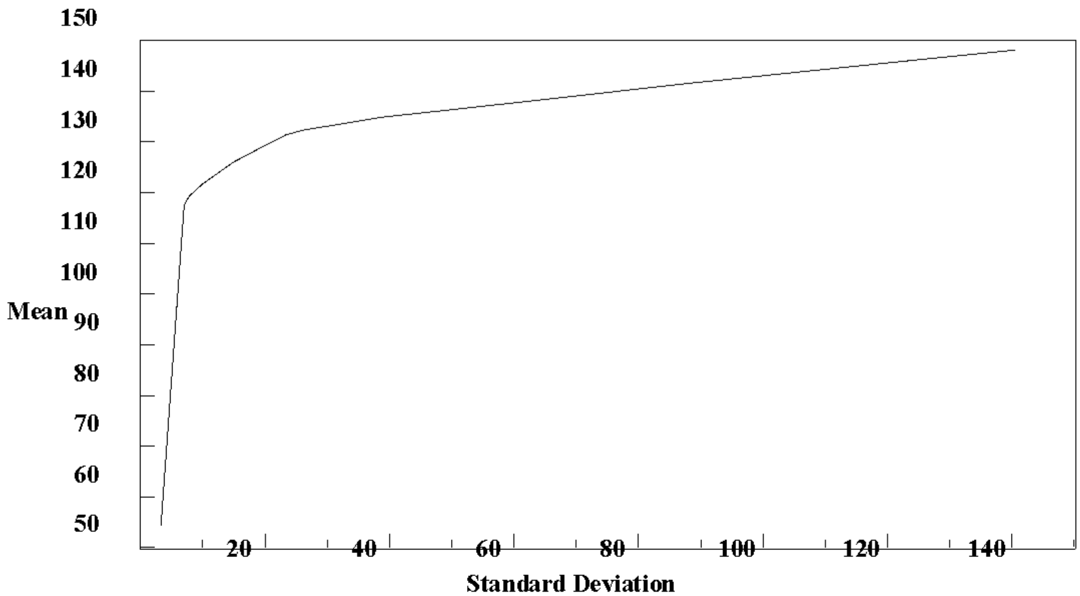


Figure 14.3: Decision Tree for Sequential Programming Example