Poker is built on a foundation of mathematics. Two of the most fundamental equations in poker are minimum defence frequency, and pot odds. These two equations intercept at the Golden Ratio.
Graph: https://www.desmos.com/calculator/cwylof8qdu

Definitions:

b = bet
p = pot
x = bet / pot
Minimum Defense Frequency: MDF = pot / (pot + bet)
Pot Odds(as a percentage): bet / (pot + bet + bet)
φ = Golden Ratio = 1.61803…

Start by reducing MDF and Pot Odds to a single variable; the bet-to-pot ratio 'x'. Expressing functions in terms of x:

$$MDF = \frac{pot}{pot + bet} \rightarrow \frac{p}{p+b} \rightarrow \frac{p}{p+(px)} \rightarrow \frac{1}{x+1}$$

$$Pot\ Odds\ (\%) = \frac{bet}{pot + bet + bet} \rightarrow \frac{b}{p+2b} \rightarrow \frac{px}{p+2(px)} \rightarrow \frac{x}{2x+1}$$

Now we set MDF equal to Pot Odds:

$$\frac{1}{x+1} = \frac{x}{2x+1} \rightarrow$$

$$2x + 1 = x(x+1) \rightarrow$$

$$x^2 - x - 1 = 0$$

The roots of this polynomial will give us interception points. By the Quadratic Formula:

$$x = \frac{1 \pm \sqrt{5}}{2} = (\varphi, \frac{-1}{\varphi})$$

Since both the bet and the pot cannot be negative, x must always be positive. Therefore we take the positive root only, x = φ

$$When\ \frac{bet}{pot} = \varphi:$$

$$MDF = Pot\ Odds = 2 - \varphi \approx 0.382...$$

All this to say, when you overbet the pot ~161.8%, your opponent's pot odds and minimum defense frequency are equal, ~38.2%.

Part 2 - Just having fun

Villain fires a bluff at Hero on the turn for 1/φ of the pot. Their bluff only needs to work 2-φ of the time to be profitable. Hero is getting (1+φ):1 on a call and should continue with at least 1/φ of their range to remain unexploitable. Hero calls.

The river blanks. Villain overbets for φ times the pot. Their bluff needs to work 1/φ of the time in order for the bluff to be profitable. Hero is getting φ:1 on a call and should continue with at least (2-φ) of their range to remain unexploitable. Hero needs (2-φ) equity to continue. Hero calls and takes it down with a pair of suited aces.

Tournament payouts are apparently based on the golden ratio.

Part 3: The Golden Ratio φ and poker: GTO Solver edition

(This is the last one guys, I promise. Probably.)

What happens when we make a GTO Solver bet the Golden Ratio? I've been harassing r/poker with this topic lately, so I decided to let Piosolver test my thesis.

Board cards: 22223

Player 1 range: 50% winning hands, 50% losing hands
Player 2 has a bluff catcher, can only beat a bluff.

Player 1 can check or overbet the Golden ratio (φ) times the pot. (161.8%)

Piosolver Simulation Results:

Player 1's betting range consists of 0.618 (1/φ) bluffs for every value hand, which means it bluffs 61.8% (1/φ) of the time.

Player 1's betting range has 61.8% (1/φ) equity.

Player 2 will fold 61.8% (1/φ) of the time to a bet.
Player 2's equity, pot odds, and MDF are all 38.2% (2-φ) after a bet.

--

Explanation:

It is very strange to see all these variables lining up. This only happens when Pot Odds equal Minimum Defense Frequency, which only occurs when you bet according to the Golden Ratio. (See part 1). I think this test proves that what I predicted in part 2 is true.

--

Part 1
https://www.reddit.com/r/poker/comments/dxij9p/the_golden_ratio_is_hiding_in_poker/

Part 2
https://www.reddit.com/r/poker/comments/dxxy41/bluffing_with_the_golden_ratio_φ/

Proofs and notes:
https://docs.google.com/document/d/1IFDJSyws2q5dQ9sRMQtq4A0RZLGu4CyVD-9wZHz3Gdg/edit

-----

Same experiment as above.

When you bet φ, while holding a bluff:value ratio of φ:1,

Your (IP) betting range equity, total betting frequency, Break Even frequency, and EV are all φ^(-1).
Your bluff frequency and equity (before betting) are φ^(-2).
Your opponent's (OOP) MDF, EV, calling frequency, and calling equity are all φ^(-2).

IP will over-realize his equity by φ
OOP Will under-realize his equity by φ^(-1)

Hey guys, I think there's more to this Golden Ratio thing.

For those who don't know, the golden ratio φ, is an irrational constant found throughout nature, like pi or e. φ = 1.618...

Imagine a river where you (IP) are either holding the nuts or air, and your opponent (OOP) only has bluff catchers. Stack-to-pot ratio of φ. Your range has a bluff:value ratio of φ:1. Piosolver has the option IP to check or go all-in for 161.8% (φ).

RESULTS:

IP equity realization: 161.8% = φ
OOP Equity realization: 61.8% = φ^-1

IP Equity: 38.2% = φ^-2

OOP Equity: 61.8% = φ^-1

Break-Even%: 61.8% = φ^-1
Minimum Defence Frequency: 38.2% φ^-2

IP Betting Frequency: 61.8% = φ^-1
IP Checking Frequency: 38.2% = φ^-2

IP betting range Equity: 61.8% = φ^-1
IP bluff frequency: 38.2% = φ^-2

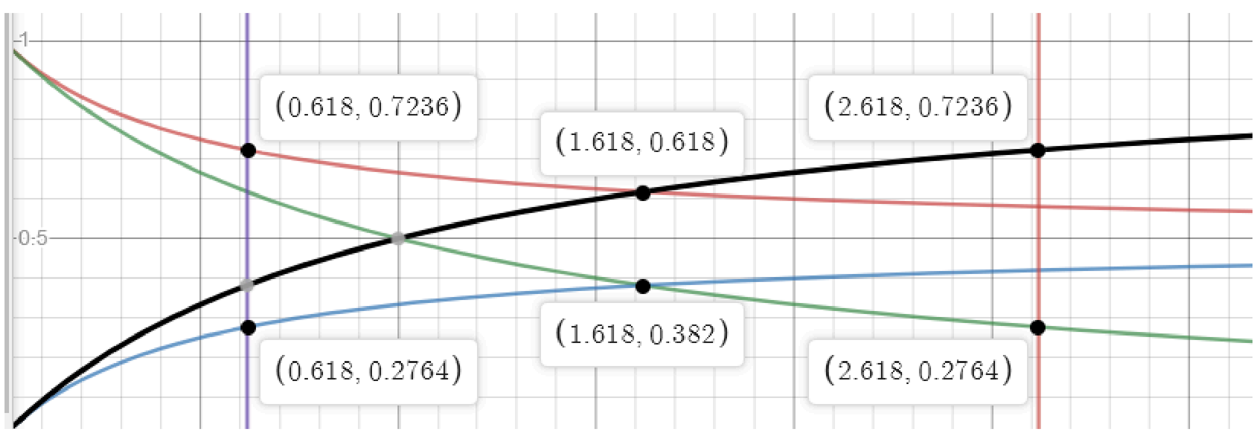
OOP Folding Frequency: 61.8% = φ^-1
OOP Calling Frequency: 38.2% = φ^-2
OOP Equity after IP bet: 38.2% = φ^-2

IP Expected Value(%): 61.8% = φ^-1
OOP Expected Value(%): 38.2% = φ^-2

The Golden Ratio appears to be a fundamental constant hidden in the maths of GTO poker.

Piosolver Simulation:

https://drive.google.com/open?id=1k8Y_ZPp1Xm0psdvBb2U7Psu1zEmIWh4Y



Red Curve: IP Minimum bet equity
Green Curve: OOP Minimum defence frequency
Black Curve: Break Even%
Blue Curve: OOP Minimum required equity to call (AKA Pot Odds)

All above are functions of x, where x= bet/pot

Black/red intersection: (phi, phi^-1). BreakEven% =Min bet equity.

Green:Blue: (phi, phi^-2): MDF = Pot Odds