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## Learning Outcomes

- Write the terms of a sequence defined by an explicit formula
- Write the formula for the  $n$ th term of a sequence
- Find the terms of an alternating sequence

Launch Desmos Calculator

One way to describe an ordered list of numbers is as a **sequence**. A sequence is a function whose domain is a subset of the counting numbers. The sequence established by the number of hits on the website is

$$\{2, 4, 8, 16, 32, \dots\}$$

The **ellipsis** (...) indicates that the sequence continues indefinitely. Each number in the sequence is called a **term**. The first five terms of this sequence are 2, 4, 8, 16, and 32.

Listing all of the terms for a sequence can be cumbersome. For example, finding the number of hits on the website at the end of the month would require listing out as many as 31 terms. A more efficient way to determine a specific term is by writing a formula to define the sequence.

One type of formula is an **explicit formula**, which defines the terms of a sequence using their position in the sequence. Explicit formulas are helpful if we want to find a specific term of a sequence without finding all of the previous terms. We can use the formula to find the  **$n$ th term of the sequence**, where  $n$  is any positive number. In our example, each number in the sequence is double the previous number, so we can use powers of 2 to write a formula for the  $n$ th term.

$$\begin{array}{ccccccc} \{2, & 4, & 8, & 16, & 32, & \dots, & ?, \dots\} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \\ \{2^1, & 2^2, & 2^3, & 2^4, & 2^5, & \dots, & 2^n, \dots\} \end{array}$$

The first term of the sequence is  $2^1=2$ , the second term is  $2^2=4$ , the third term is  $2^3=8$ , and so on. The

$n^{\text{th}}$  term of the sequence can be found by raising 2 to the  $n^{\text{th}}$  power. An explicit formula for a sequence is named by a lower case letter  $a, b, c, \dots$ , with the subscript  $n$ . The explicit formula for this sequence is

$$a_n = 2^n$$

Now that we have a formula for the  $n^{\text{th}}$  term of the sequence, we can answer the question posed at the beginning of this section. We were asked to find the number of hits at the end of the month, which we will take to be 31 days. To find the number of hits on the last day of the month, we need to find the 31<sup>st</sup> term of the sequence. We will substitute 31 for  $n$  in the formula.

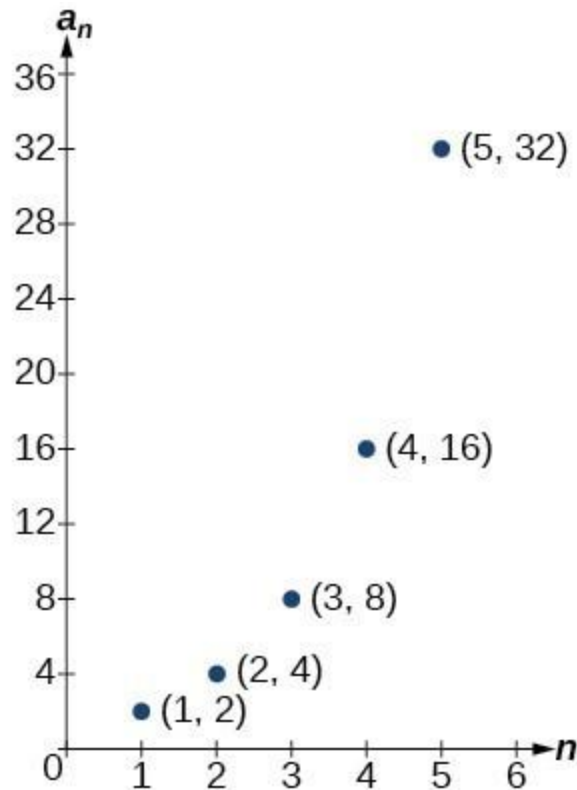
$$a_{31} = 2^{31} \quad \&= \text{2,147,483,648}$$

If the doubling trend continues, the company will get 2,147,483,648 hits on the last day of the month. That is over 2.1 billion hits! The huge number is probably a little unrealistic because it does not take consumer interest and competition into account. It does, however, give the company a starting point from which to consider business decisions.

Another way to represent the sequence is by using a table. The first five terms of the sequence and the  $n^{\text{th}}$  term of the sequence are shown in the table.

$n$	1	2	3	4	5	$n$
$n^{\text{th}}$ term of the sequence, $a_n$	2	4	8	16	32	$2^n$

Graphing provides a visual representation of the sequence as a set of distinct points. We can see from the graph below that the number of hits is rising at an exponential rate. This particular sequence forms an exponential function.



Lastly, we can write this particular sequence as

$$\{2, 4, 8, 16, 32, \dots, 2^n, \dots\}$$

A sequence that continues indefinitely is called an **infinite sequence**. The domain of an infinite sequence is the set of counting numbers. If we consider only the first 10 terms of the sequence, we could write

$$\{2, 4, 8, 16, 32, \dots, 1024\}$$

This sequence is called a **finite sequence** because it does not continue indefinitely.

#### A General Note: Sequence

A **sequence** is a function whose domain is the set of positive integers. A **finite sequence** is a sequence whose domain consists of only the first  $n$  positive integers. The numbers in a sequence are called **terms**. The variable  $a_n$  with a number subscript is used to represent the terms in a sequence and to indicate the position of the term in the sequence.

$$[a]_1, [a]_2, [a]_3, \dots, [a]_n, \dots$$

We call  $[a]_1$  the first term of the sequence,  $[a]_2$  the second term of the sequence,  $[a]_3$  the third term of the sequence, and so on. The term  $[a]_n$  is called the  **$n$ th term of the sequence**, or the general term of the sequence. An **explicit formula** defines the  $n$ th term of a sequence using the position of the term. A sequence that continues indefinitely is an **infinite sequence**.

Q & A

### Does a sequence always have to begin with $[a]_1$ ?

*No. In certain problems, it may be useful to define the initial term as  $[a]_0$  instead of  $[a]_1$ . In these problems, the domain of the function includes 0.*

How To: Given an explicit formula, write the first  $n$  terms of a sequence.

1. Substitute each value of  $n$  into the formula. Begin with  $n=1$  to find the first term,  $[a]_1$ .
2. To find the second term,  $[a]_2$ , use  $n=2$ .
3. Continue in the same manner until you have identified all  $n$  terms.

Example: Writing the Terms of a Sequence Defined by an Explicit Formula

Write the first five terms of the sequence defined by the explicit formula  $[a]_n = -3n + 8$ .

Show Solution

Substitute  $n=1$  into the formula. Repeat with values 2 through 5 for  $n$ .

$$\begin{aligned} n=1 \quad \&\& \{a\}_1 = -3\left(1\right) + 8 = 5 \quad \&\& n=2 \quad \&\& \{a\}_2 = -3\left(2\right) + 8 = 2 \quad \&\& n=3 \\ \&\& \{a\}_3 = -3\left(3\right) + 8 = -1 \quad \&\& n=4 \quad \&\& \{a\}_4 = -3\left(4\right) + 8 = -4 \quad \&\& n=5 \quad \&\& \\ \&\& \{a\}_5 = -3\left(5\right) + 8 = -7 \quad \&\& \end{aligned}$$

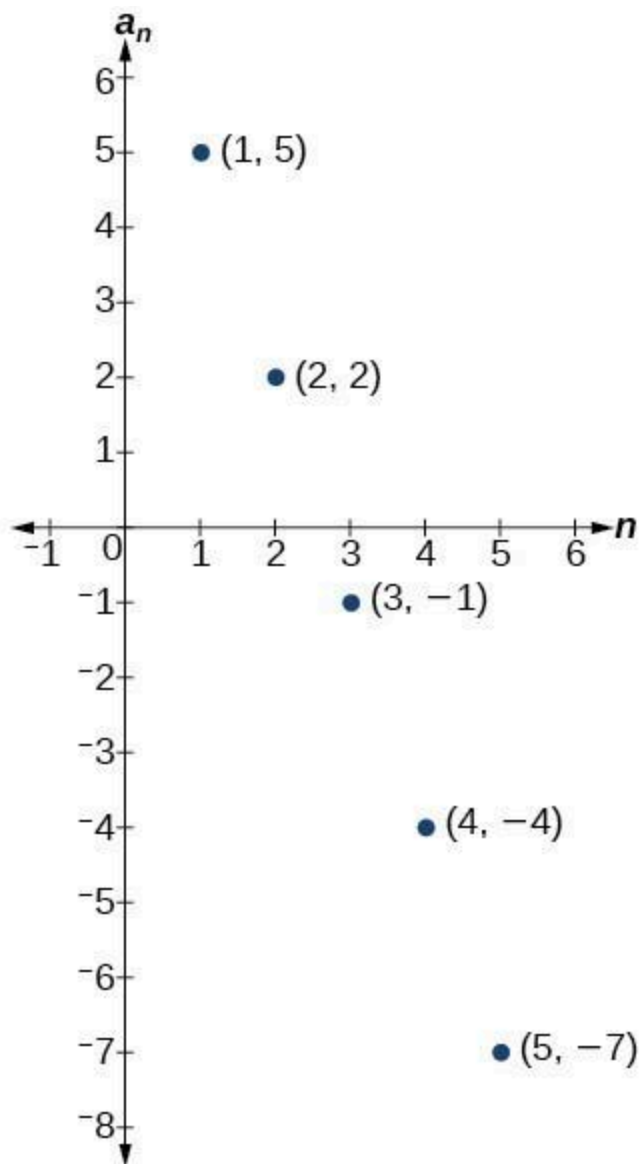
The first five terms are  $\{5, 2, -1, -4, -7\}$ .

## Analysis of the Solution

The sequence values can be listed in a table. A table is a convenient way to input the function into a graphing utility.

$n$	1	2	3	4	5
$\{a\}_n$	5	2	-1	-4	-7

A graph can be made from this table of values. From the graph in Figure 2, we can see that this sequence represents a linear function, but notice the graph is not continuous because the domain is over the positive integers only.



Try It

Write the first five terms of the sequence defined by the **explicit formula**  $a_n = 5n - 4$ .

Show Solution

The first five terms are  $\{1, 6, 11, 16, 21\}$



[See this interactive in the course material.](#)

## Investigating Explicit Formulas

We've learned that sequences are functions whose domain is over the positive integers. This is true for other types of functions, including some **piecewise functions**. Recall that a piecewise function is a function defined by multiple subsections. A different formula might represent each individual subsection.

**How To:** Given an explicit formula for a piecewise function, write the first  $n$  terms of a sequence

1. Identify the formula to which  $n=1$  applies.
2. To find the first term,  $a_1$ , use  $n=1$  in the appropriate formula.
3. Identify the formula to which  $n=2$  applies.
4. To find the second term,  $a_2$ , use  $n=2$  in the appropriate formula.
5. Continue in the same manner until you have identified all  $n$  terms.

**Example:** Writing the Terms of a Sequence Defined by a Piecewise Explicit Formula

Write the first six terms of the sequence.

$$a_n = \begin{cases} n^2 & \text{if } n \text{ is not divisible by 3} \\ \frac{n}{3} & \text{if } n \text{ is divisible by 3} \end{cases}$$

Show Solution

Substitute  $n=1, n=2$ , and so on in the appropriate formula. Use  $n^2$  when  $n$  is not a multiple of 3. Use  $\frac{n}{3}$  when  $n$  is a multiple of 3.

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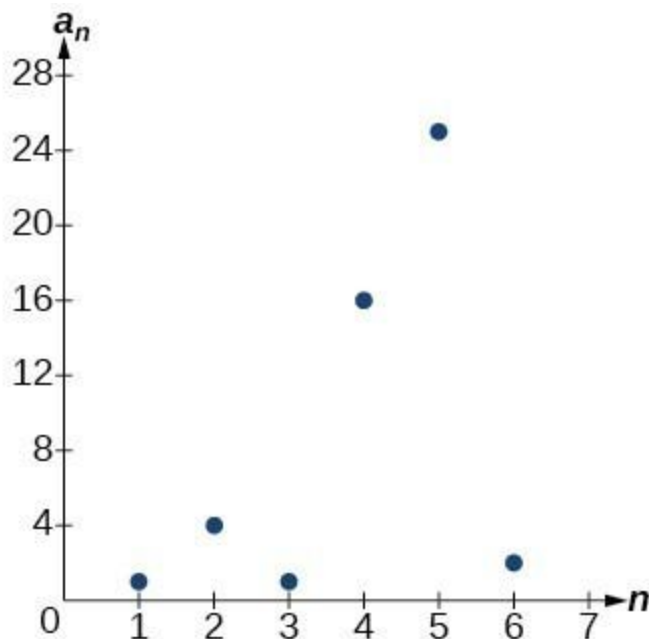
\begin{align}
&a_1=1^2=1 \quad \& \text{1 is not a multiple of 3. Use } n^2. \\
&a_2=2^2=4 \quad \& \text{2 is not a multiple of 3. Use } n^2. \\
&a_3=\frac{3}{3}=1 \quad \& \text{3 is a multiple of 3. Use } \frac{n}{3}. \\
&a_4=4^2=16 \quad \& \text{4 is not a multiple of 3. Use } n^2. \\
&a_5=5^2=25 \quad \& \text{5 is not a multiple of 3. Use } n^2. \\
&a_6=\frac{6}{3}=2 \quad \& \text{6 is a multiple of 3. Use } \frac{n}{3}.
\end{align}

```

The first six terms are  $\{1, 4, 1, 16, 25, 2\}$ .

## Analysis of the Solution

Every third point on the graph shown in Figure 5 stands out from the two nearby points. This occurs because the sequence was defined by a piecewise function.



Try It



Write the first six terms of the sequence.

$$a_n = \begin{cases} 2n^3 & \text{if } n \text{ is odd} \\ \frac{5n}{2} & \text{if } n \text{ is even} \end{cases}$$

Show Solution

The first six terms are  $\{2, 5, 54, 10, 250, 15\}$ .

## Finding an Explicit Formula

Thus far, we have been given the explicit formula and asked to find a number of terms of the sequence. Sometimes, the explicit formula for the  $n$ th term of a sequence is not given. Instead, we are given several terms from the sequence. When this happens, we can work in reverse to find an explicit formula from the first few terms of a sequence. The key to finding an explicit formula is to look for a pattern in the terms. Keep in mind that the pattern may involve alternating terms, formulas for numerators, formulas for denominators, exponents, or bases.

How To: Given the first few terms of a sequence, find an explicit formula for the sequence.

1. Look for a pattern among the terms.
2. If the terms are fractions, look for a separate pattern among the numerators and denominators.
3. Look for a pattern among the signs of the terms.
4. Write a formula for  $a_n$  in terms of  $n$ . Test your formula for  $n=1, n=2$ , and  $n=3$ .

Example: Writing an Explicit Formula for the  **$n$ th Term of a Sequence**

Write an explicit formula for the  $n$ th term of each sequence.

1.  $\left\{-\frac{2}{11}, \frac{3}{13}, -\frac{4}{15}, \frac{5}{17}, -\frac{6}{19}, \dots\right\}$
2.  $\left\{-\frac{2}{25}, -\frac{2}{125}, -\frac{2}{625}, -\frac{2}{3125}, -\frac{2}{15625}, \dots\right\}$

3.  $\left\{e^4, e^5, e^6, e^7, e^8, \dots\right\}$   
 Show Solution

Look for the pattern in each sequence.

- The terms alternate between positive and negative. We can use  $\left(-1\right)^n$  to make the terms alternate. The numerator can be represented by  $n+1$ . The denominator can be represented by  $2n+9$ .  

$$a_n = \frac{\left(-1\right)^n \left(n+1\right)}{2n+9}$$
- The terms are all negative.  

$$\begin{aligned} &\left\{-\frac{2}{25}, -\frac{2}{125}, -\frac{2}{625}, -\frac{2}{3125}, -\frac{2}{15625}, \dots\right\} \quad \& \quad \text{The numerator is 2.} \\ &\left\{-\frac{2}{5^2}, -\frac{2}{5^3}, -\frac{2}{5^4}, -\frac{2}{5^5}, -\frac{2}{5^6}, \dots, -\frac{2}{5^n}, \dots\right\} \quad \& \quad \text{The denominators are increasing powers of 5.} \end{aligned}$$
  
 So we know that the fraction is negative, the numerator is 2, and the denominator can be represented by  $5^{n+1}$ .

$$a_n = -\frac{2}{5^{n+1}}$$

- The terms are powers of  $e$ . For  $n=1$ , the first term is  $e^4$  so the exponent must be  $n+3$ .  

$$a_n = e^{n+3}$$

Try It

Write an explicit formula for the  $n$ th term of the sequence.

$$\{9; -81,729; -6,561; 59,049\}$$

Show Solution

$$a_n = \left(-1\right)^{n+1} 9^n$$



[See this interactive in the course material.](#)

Try It

Write an explicit formula for the  $a_n$  term of the sequence.

$$\left\{-\frac{3}{4}, -\frac{9}{8}, -\frac{27}{12}, -\frac{81}{16}, -\frac{243}{20}, \dots\right\}$$

Show Solution

$$a_n = -\frac{3^n}{4n}$$



[See this interactive in the course material.](#)

Try It

Write an explicit formula for the  $n$ th term of the sequence.

$$\left\{\frac{1}{e^2}, \frac{1}{e}, 1, e, e^2, \dots\right\}$$

Show Solution

$$a_n = e^{n-3}$$

## Investigating Alternating Sequences

Sometimes sequences have terms that alternate in sign. The steps to finding terms of the sequence are the same as if the signs did not alternate. Let's take a look at the following sequence.

$$\{2, -4, 6, -8, \dots\}$$

Notice the first term is greater than the second term, the second term is less than the third term, and the third term is greater than the fourth term. This trend continues forever. Do not rearrange the terms in numerical order to interpret the sequence.

How To: Given an explicit formula with alternating terms, write the first  $n$  terms of a sequence.

1. Substitute each value of  $n$  into the formula. Begin with  $n=1$  to find the first term,  $a_1$ . The sign of the term in the explicit formula is given by the  $(-1)^n$  if the first term is negative and  $(-1)^{n-1}$  if the first term is positive.
2. To find the second term,  $a_2$ , use  $n=2$ .
3. Continue in the same manner until you have identified all  $n$  terms.

Example: Writing the Terms of an Alternating Sequence Defined by an Explicit Formula

Write the first five terms of the sequence.

$$a_n = \frac{(-1)^n n^2}{n+1}$$

Show Solution

Substitute  $n=1$ ,  $n=2$ , and so on in the formula.

```

\begin{align}
&n=1 \quad \{a\}_1 = \frac{(-1)^1(2)^2}{1+1} = -\frac{1}{2} \\
&n=2 \quad \{a\}_2 = \frac{(-1)^2(2)^2}{2+1} = \frac{4}{3} \quad n=3 \quad \{a\}_3 = \frac{(-1)^3(3)^2}{3+1} = -\frac{9}{4} \\
&n=4 \quad \{a\}_4 = \frac{(-1)^4(4)^2}{4+1} = \frac{16}{5} \quad n=5 \quad \{a\}_5 = \frac{(-1)^5(5)^2}{5+1} = -\frac{25}{6}
\end{align}

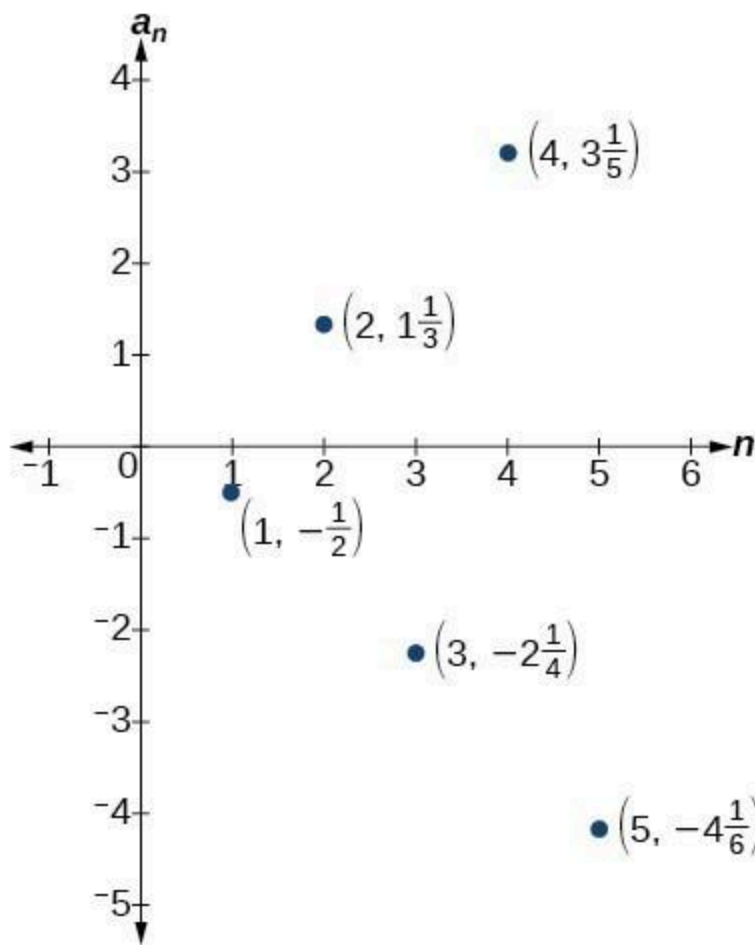
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The first five terms are

$-\frac{1}{2}, \frac{4}{3}, -\frac{9}{4}, \frac{16}{5}, -\frac{25}{6}$ .

## Analysis of the Solution

The graph of this function looks different from the ones we have seen previously in this section because the terms of the sequence alternate between positive and negative values.



Q & A

**In Example: Writing the Terms of an Alternating Sequence Defined by an Explicit Formula, does the  $(-1)$  to the power of  $n$  account for the oscillations of signs?**

*Yes, the power might be  $n, n+1, n-1$ , and so on, but any odd powers will result in a negative term, and any even power will result in a positive term.*

Try It

Write the first five terms of the sequence:

$$a_n = \frac{4n}{(-2)^n}$$

Show Solution

The first five terms are  $\left[-2, 2, -\frac{3}{2}, 1, -\frac{5}{8}\right]$ .



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