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**B. Sc. (Hons.) Math (Semester – 5<sup>th</sup>)**  
**DIFFERENTIAL GEOMETRY**  
**Subject Code: BMATS1-503**  
**Paper ID: 19131222**

**Time: 03 Hours**

**Maximum Marks: 60**

**Instruction for candidates:**

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

**Section – A**

**(2 marks each)**

Q1. Attempt the following:

- a. Discuss the nature of geodesic on a plane.
- b. Show that the involutes of a circular helix are plane curves.
- c. Prove that the tangent plane to the surface  $xyz = a^3$  and the co-ordinate planes bound a tetrahedron of constant volume.
- d. Calculate the fundamental magnitudes for the right helicoid  $x = u \cos v$ ,  $y = u \sin v$ ,  $z = cv$ .
- e. Prove that straight lines on a surface are the only asymptotic lines which are geodesics.
- f. Show that a necessary and sufficient condition that a curve be a straight line is that  $\kappa = 0$  at all points.
- g. Show that the principal normal to a curve is normal to the locus of centre of curvature at those points where the curvature is stationary.
- h. Find the length of the curve given as the intersection of the surfaces  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ,  $x = \cosh\left(\frac{z}{a}\right)$ .
- i. Find the radii of curvature and torsion of the helix  $x = a \cos u$ ,  $y = a \sin u$ ,  $z = au \tan \alpha$ .
- j. Consider the curve  $u = u(t)$ ,  $v = v(t)$ . Find the direction coefficients of the tangent to the curve.

**Section – B**

**(5 marks each)**

Q2. Prove that the curves of the family  $\frac{v^3}{u^2} = \text{constant}$ , are geodesics on a surface with metric  $v^2 du^2 - 2uv dudv + 2u^2 dv^2$  ;  $(u > 0, v > 0)$ .

Q3. Find the lines that have four point contact at  $(0, 0, 1)$  with the surface  $x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z = 1$ .

Q4. Discuss the geometrical interpretation of the second fundamental form.

Q5. If two curves have the same intrinsic equations then show that they are congruent.

Q6. Prove that for any curve  $[t', t'', t'''] = [r'', r''', r'''] = \kappa^3 (\kappa \tau' - \kappa' \tau) = \kappa^5 \frac{d}{ds} \left( \frac{\tau}{\kappa} \right)$ .

### Section – C

(10 marks each)

Q7. State and prove Gauss Bonnet theorem.

Q8. (a) Define the normal plane to a space curve at a point and find its equation. (7)

(b) Show that a real surface for which the equations  $E/L = F/M = G/N$  hold is either plane or spherical. (3)

Q9. If C represents a space curve and  $C_1$ , the locus of the centre of osculating sphere, then prove that the tangent, principal normal and binormal to  $C_1$  are parallel respectively to the binormal, principal normal and tangent to C at the corresponding points.