

College Algebra Corequisite Instructor Guide

Module 10: Rational and Radical Functions

Table of Contents

Table of Contents	1
Learning Outcomes	2
Summary of Module	2
Module Resources	4
Cheat Sheet	4
Worksheets/Handouts	4
Activity One: Theme Park Design Challenge	5
Evidence-Based Teaching Practice	5
Background	5
Instructions	6
Discussion Prompts	7
Reflection	8
Online Variation	8
Assignments	9
Analyzing Asymptotic Behavior of Rational Functions Discussion	9
Rational and Radical Functions Writing Task	9

Learning Outcomes

[Detailed Course Learning Outcome Spreadsheet is linked here.](#)

Topic	Student Learning Goals
Rational Functions	• Use arrow notation to describe how rational functions behave at different points • Apply rational functions to solve real-world problems • Find out which values are allowed for rational functions and understand what limits these values set • Spot where rational functions shoot up to infinity (vertical asymptotes) and explore why these points are important • Find where rational functions level off at infinity (horizontal asymptotes) and see how these affect the graph • Draw graphs of rational functions carefully, including places where the graph breaks or levels out, and where it's not defined
Inverses and Radical Functions	• Learn how to find the inverse (or "reverse") of a polynomial function when it's possible. • Figure out how to limit the domain of a polynomial function so you can find its inverse. • Use radical functions to solve real-world problems.
Variations	• Solve direct variation problems • Solve inverse variation problems • Solve problems involving joint variation

Summary of Module

Background You'll Need

The assumed prerequisite skills:

- Figure out which values make a rational expression impossible to calculate (like dividing by zero)
- Find the x-intercept and y-intercept of graphs.
- Discover or calculate the inverse of a function

Rational Functions

In these sections, students explore rational functions beginning with vertical and horizontal asymptotes and using arrow notation to describe function behavior. They analyze domain restrictions, removable discontinuities, and examine how different degrees between numerator and denominator affect end behavior. Through various examples, students practice identifying key features of rational functions, including asymptotes and intercepts, and use these to sketch accurate graphs. The section concludes with students learning to work backwards from graphs to write rational functions by recognizing how graphical features correspond to algebraic structure.

Inverses and Radical Functions

In this section, students learn to recognize and understand radical functions as inverses of polynomial functions. They begin by exploring the core concepts through a smartphone brightness application, then work with square roots, cube roots, and higher-order roots to understand their behavior. Students analyze domains and ranges of radical functions, practice finding inverse functions by restricting domains when needed, and solve equations containing radicals. They get hands-on practice with switching between polynomial and radical forms, graphing these functions, and identifying key features like restricted domains. The section culminates with students applying these skills to real-world problems involving surface area, volume, and other physical applications where radical functions naturally occur.

Variations

In this section, students explore variation relationships through practical scenarios and worked examples. They start by examining direct variation using a commission-based sales model, where they create tables, analyze relationships, and solve for unknowns. Moving to inverse variation, students work with real-world applications like ocean temperatures and travel times, practicing how to find constants and solve related equations. The section wraps up with joint variation problems where students handle multiple variables simultaneously.

Module Resources

Cheat Sheet



Rational and Radical Functions : Cheat Sheet

Worksheets/Handouts



[Theme Park Design Challenge Worksheet](#)



[Theme Park Design Challenge Worksheet Answer Key](#)



[Analyzing Asymptotic Behavior of Rational Functions Discussion](#)



[Rational and Radical Functions Writing Task](#)

Activity One: Theme Park Design Challenge

Evidence-Based Teaching Practice



Success Skills

Educators help students develop a sense of self-efficacy about being a college student by engaging them in a professional engineering design process where they apply mathematical concepts to solve real-world problems and present their solutions.



Prior Knowledge Assessment

Educators evaluate what students know at the beginning of the learning unit by connecting to students' personal theme park experiences and reviewing key mathematical concepts before applying them to design challenges.



Contextualization

Educators help students make sense of theoretical material by demonstrating how it applies to relevant "real world" situations by using theme park design to illustrate applications of rational functions, radical functions, and variation relationships.

Background

In this module, students have developed strong foundational skills in working with rational functions, including analyzing asymptotes, domain restrictions, and end behavior based on degree relationships between numerator and denominator. They've explored radical functions as inverses of polynomials, mastering concepts of restricted domains and ranges, and studied variation relationships, understanding how quantities can vary directly, inversely, or jointly with each other. This activity challenges students to apply these mathematical concepts in an engaging theme park design project where they will use rational functions to model ride paths, radical functions to analyze structural requirements, and variation relationships to optimize ride capacity. Through this real-world application, students will see how these different function types work together in engineering design while strengthening their understanding of key concepts through active problem-solving.

Instructions

Time Estimate: 60-75 minutes

1. Conversation starter

Ask students about their experiences at theme parks. What makes rides thrilling? Have they ever wondered how engineers design rides to be both exciting and safe?

2. Review

Briefly review the key properties of rational functions (asymptotes, behavior), radical functions (domains, inverses), and variation relationships.

3. Split the class into groups of 3-4 students

Each group will need graph paper, calculators, and the design challenge worksheet.

4. Present the Challenge

Introduce the theme park design challenge: teams must design a new roller coaster attraction, considering three key aspects: Ride path design (rational functions), Support structure requirements (radical functions), Capacity planning (variation relationships)

5. Design Phase (25 minutes)

Teams work on their designs, using specific functions to model different aspects:

- Create a rational function for a section of the ride path
- Use radical functions to determine support beam requirements
- Apply variation principles to optimize rider capacity and wait times

6. Analysis Phase (15 minutes)

Groups analyze their designs for safety and efficiency:

- Check domain restrictions and asymptotes for physical feasibility
- Verify structural support calculations
- Optimize capacity based on variation relationships

7. Presentation Phase (15 minutes)

Each team presents their design, explaining:

- Their chosen functions and why they selected them
- How they addressed safety considerations
- Their capacity optimization strategy

8. Final Discussion (10 minutes)

Class discussion about different approaches and insights gained.

Discussion Prompts

- **How did domain restrictions on your rational and radical functions influence your design decisions? What real-world factors do these restrictions represent?**

Misconception: Students often view domain restrictions as purely mathematical rules to be memorized rather than meaningful constraints. For instance, when working with a rational function like $f(x) = (x^2 + 4)/(x - 3)$ for a spiral track element, students typically just write " $x \neq 3$ " without considering that this represents a physical boundary the track cannot cross. Similarly, with radical functions like $h = \sqrt{L - x}$ for height calculations, students might treat the domain restriction $x \leq L$ as just a mathematical rule, missing that it represents the physical impossibility of negative heights or supports extending beyond their base. The discussion should help students recognize that these "mathematical rules" directly translate to safety barriers, structural limitations, and physical constraints that could mean the difference between a safe ride and a dangerous one.

- **Why might theme park engineers prefer to use rational functions over polynomials for modeling certain ride elements? What advantages do they offer?**

Goal: This discussion should lead students to discover the unique capabilities of rational functions in engineering design contexts. While polynomials might seem simpler, they can't effectively model situations with limiting behaviors - a crucial consideration in ride design.

- **In optimizing ride capacity, how did you determine whether to use direct, inverse, or joint variation? What real-world factors influenced this decision?**

Sample Answer: Theme park optimization requires understanding multiple variation relationships working together. Wait time (T) shows inverse variation with the number of cars (n) - doubling cars halves wait time, following $T = k/n$. However, wait time has direct variation with the number of riders (r), as $T = kr$. Combining these gives $T = kr/n$. Operating costs introduce joint variation with both cars and track length ($C = kmL$), creating a complex optimization problem. For example, reducing wait time by 50% by adding cars increases operational costs proportionally, potentially making the improvement financially unfeasible. Engineers must balance these competing variations to find optimal solutions that consider both guest experience and operational efficiency.

Reflection

After the activity, we recommend that students complete exit cards with the following reflection questions:

- Which aspect of the design process (path design, structural support, or capacity planning) did you find most challenging and why?
- How did your understanding of asymptotes change when applying them to a physical design problem?
- What surprised you most about the relationship between mathematical functions and real-world design constraints?
- How did your team decide which type of function to use for different aspects of the design?
- What strategies did you develop for checking whether your mathematical solutions were physically realistic?
- If you were to redesign your attraction, what would you do differently and why?
- How has this activity changed your perspective on the practical applications of these functions?
- What questions do you still have about applying mathematical models to real-world engineering problems?

Online Variation

To adapt this activity for an online environment, use breakout rooms for team collaboration and digital whiteboards for design work. Teams can use shared documents to work on calculations together and create digital presentations of their designs. The class discussion can take place in the main room with screen sharing for design presentations. Use online polling for peer feedback on designs and discussion board posts for teams to share their work and reasoning between synchronous sessions.

Assignments

Analyzing Asymptotic Behavior of Rational Functions Discussion

In this discussion, students will explore the unique behaviors of rational functions, particularly focusing on asymptotic behavior and key graphical features. By selecting one of four provided rational functions, students will conduct a comprehensive analysis identifying domains, vertical and horizontal asymptotes, removable discontinuities, and intercepts. They will create properly labeled graphs showing all key features and explain the behavior of their function near asymptotes using arrow notation. Students will also compare the asymptotic behavior of rational functions to polynomial functions they've previously studied. After posting their analysis, they'll engage with classmates by comparing and contrasting asymptotic behaviors, asking thoughtful questions, and discussing real-world applications of these mathematical concepts.

[Analyzing Asymptotic Behavior of Rational Functions Discussion](#)

We ask that you make your own copy to edit and adjust to fit the needs of your classroom

Rational and Radical Functions Writing Task: Modeling Relationships in Physical Systems

In this writing task, students will apply rational and radical functions to model relationships in physical systems through a comprehensive wind turbine farm scenario. The assignment is divided into four interconnected parts where students will: develop equations modeling power generation using direct variation principles; analyze a rational function representing turbine efficiency including finding maximum points and asymptotes; determine inverse functions to relate power output to wind speed; and apply joint variation concepts to calculate optimal turbine spacing. The assignment culminates with students reflecting on how rational and radical functions provide unique insights for modeling physical systems.

[Rational and Radical Functions Writing Task](#)

We ask that you make your own copy to edit and adjust to fit the needs of your classroom