Activity Guide - Public Key Bean Counting

Sending Secret Messages without agreeing on a secret key ahead of time
In this activity, cups filled with beans will represent information going back and forth between Alice and Bob. We do this activity to show you a simple version of something called Public Key Encryption so we can introduce you to the basic process of information exchange and to some of the terminology involved (which we’ll get to later).

This activity will show a technique for Alice and Bob to send secret messages to each other, without agreeing on a secret key ahead of time, and only by exchanging messages over public, insecure channels.

Background: A metaphor -- cup of beans as one-way function

- Imagine that putting some beans into a clear plastic cup and then putting a lid on the cup is an encryption function. Only the person who put the lid on is able to remove it.
- Everyone else can try to count the beans but they can’t take the lid off; they just have to stare into the cup (like trying to count the jelly beans in a jar at the carnival). This represents a computationally hard problem.
- In this activity, there is a wrinkle: a person can add beans to the cup by pushing them through the slot in the top of the lid. The result is that there will be more beans in the cup, but it’s still hard to count them by looking in from the outside.

Information Exchange Procedure

Materials

- A clear plastic cup
- A few handfuls of dried beans
- (optional) A lid with a slot in the top that would allow a bean to be pushed through. If you don’t have lids, you could use plastic wrap, or just use your powers of imagination.

Setup

- Decide who is playing Alice, Bob and Eve.
- Give all of the cups and lids to Alice to start.
- Alice and Bob should each have a handful of beans.

Eve:
Eve, you will direct all the action. You should read all the instructions of the procedure out loud to everyone, and Alice and Bob should follow along accordingly. (Alice and Bob can follow along on their sheets as well.) Eve reads….

Alice:

1. Alice, turn your back to Eve and Bob while you do this:
   - Put a random number of beans into a cup -- Remember this number (or write it down in a secret location).
   - Put the lid on the cup.

2. Then put the cup onto the table in front of Bob and Eve.
   NOTE: Eve and Bob can only guess how many beans are in the cup.
Bob:

1. Bob, take the cup off the table and turn your back to Alice and Eve while you do this:
   - Pick a secret number to send to Alice. Remember this number.
   - Count out that many beans and add them to the cup.

2. Then put the cup back onto the table.

NOTE: Even though she might be able to see the number of beans is different, Eve can only guess how many are in the cup.

Eve:

Quick question for Eve: Do you have any idea what secret number Bob is sending to Alice? Note: Unless Bob and Alice put so few beans into the cup that you can clearly see from the outside how many there were, your answer should be “No.” You might be able to make a guess, but you wouldn’t know for sure whether it was right. Okay...move on.

Alice:

Once more, turn your back to Bob and Eve while you do this:

- Take the cup off the table
- Remove the lid and dump out the beans.
- Count off the number of beans originally in the cup.

- What’s left is Bob’s secret number!

Recap:

- Alice and Bob did not have to agree on anything, or communicate ahead of time.
- Alice and Bob only exchanged information in public, right in front of Eve.
- Eve would have to be able to count the beans in the cup without opening it, both on the way over to Bob and on the way back to Alice, in order to determine what Bob was trying to send Alice.

Try it again?

- Change roles and try the procedure again to see how it works. Try to make it hard for Eve to guess the secret number. And, Eve, do try to guess.

- Here’s a fun wrinkle that makes it even more impossible for Eve: Use 3 cups!
  - Alice, put a random number of beans into 3 different cups (you need to remember how many total beans you used, or you could remember the 3 separate numbers).
  - Bob, for the number you wish to send, distribute the beans randomly into the 3 cups; it doesn’t matter how many go into each cup as long as the total is the number you want to send.
  - Alice, once you have the 3 cups back, either dump all the beans out and take away the total number of beans you originally put into the cups, or subtract the individual amounts from each cup. Either way, the beans left over are the ones Bob sent you.
What's the point of the cups and beans activity?

Public key cryptography is what makes secure transactions on the Internet possible. Obviously, computers don’t exchange information with beans in plastic cups; they use data (numbers mostly) and the methods of encryption use some math, which we will see in a later lesson. Here the number of beans represented data and the cups represented encrypted data. In order to see how the real thing works, we need to know some terms so we can talk about it accurately.

First, NOTICE:
- At no point did Bob or Alice agree on any secret password, number, or key.
- They only exchanged information in public.
- Bob can encrypt a secret message for Alice by using something that Alice puts out in public
- Eve could not tell what was going back forth without simply guessing either Alice or Bob’s private number.

Asymmetric Keys
The cups and beans represent asymmetric (pronounced “A-symmetric”) encryption because the procedure for encrypting a message (which Bob does) is different from the procedure for decrypting the message (which Alice does). Up to this point, the encryption schemes we’ve studied have been symmetric. This means that the key used to encrypt the message is the same key needed to decrypt the message.

Private Key
In the case of this activity, Alice’s secret number - the number of beans that she put into the cup originally is known as her private key. Only she knows it, and she never shares it with anyone.

Public Key
The sealed container sitting on the table represents Alice’s public key. In the real world a public key is something related to the private key, that can be safely shared in public, that another person can use to encrypt a message. In this case, the cup with the lid on top.

Encrypting (a message)
When Bob adds beans to the sealed cup, he is using a public key to encrypt a message. Since they get mixed in with the other beans (which are related to Alice’s private key), no one, not even Bob, knows how many total beans there are.

Decrypting (the message)
When Alice receives the cup back from Bob, she can decrypt the message by opening the lid and counting the beans. Since she knows how many beans she put in in the first place, she can subtract that number of beans and arrive at the number that Bob intended to send.

Public Key Cryptography
This entire form of exchange is called Public Key Cryptography. In this form of secure communication, every participant has both a public and a private key. When sending a message, the sender encrypts his message using the public key of the recipient.

The real math is actually not that complicated. It essentially uses multiplication and division instead of addition and subtraction. The next lesson shows how it works.