NAME:_____

Precalculus ACP
Unit 5 Packet

- 5.1 Six Trig Functions – All Angles
- 5.2 Building the Unit Circle – Similar Triangles
- 5.3 More Unit Circle
- 5.4 Trigonometry of Special Angles
- 5.5 Solving Trig Equations
- 5.6 Mixed Practice
- 5.7 Review
- 5.8 Assessment # 5 on _____

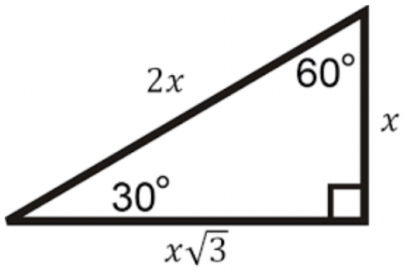
Unit 5 Learning Goals:

1. **Angle Fundamentals**
    a. Determine what quadrant an angle is in
    b. Determine coterminal angles given an angle in radians and/or degrees
    c. Determine a reference angle given an angle in radians and/or degrees
    d. Convert an angle's measure from radians to degrees or from degrees to radians.
2. **Trigonometric Expressions**
    a. understand how the values on the unit circle are defined.
    b. determine the values of the six trigonometric functions of a given angle using special right triangles or a table of points on the unit circle
    c. determine the values of the remaining five trigonometric functions of an angle given one of the functions and the quadrant in which the angle lies.
    d. Given the sign(+/-) of two non-reciprocal trigonometric functions of the same angle determine the quadrant of the angle. Then find the remaining four trigonometric functions.
    e. Identify where the secant, cosecant, tangent, and cotangent functions are undefined.
3. **Trigonometric Equations**
    a. solve trigonometric equations given a table of points on the unit circle or for special right triangle angles on a given domain (0°-360° or $0 \; to \; 2\pi$)

5.1: 6 Trig Functions – All Angles

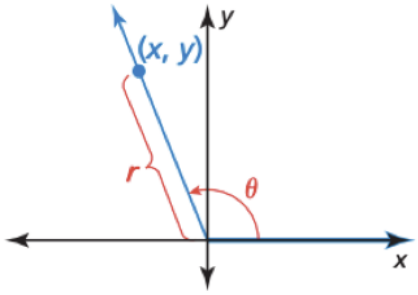
Warm Up: Evaluate each expression. Simplify as much as possible.

1. $sec30° =$
2. $sin30° =$
3. $cot60° =$

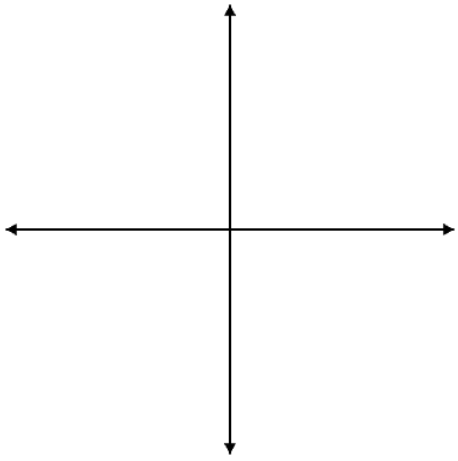


Goal: Find the trig ratios of ANY angle in standard position.

-Where is the right triangle in the diagram to the right?

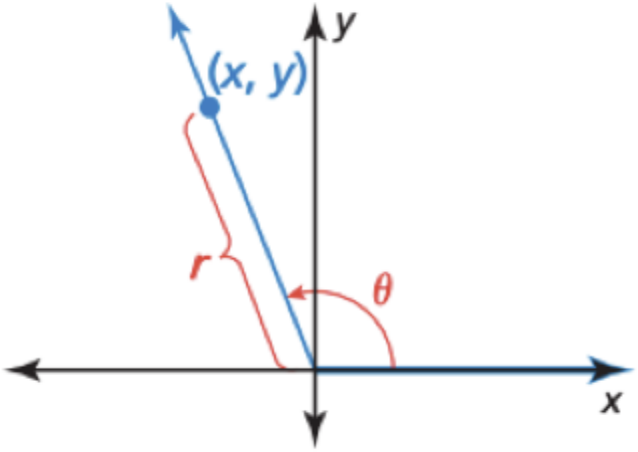


Example 1: Find the sine, cosine, and tangent of the angle 135°. The point $(-6, 6)$ is on the terminal side of this angle.



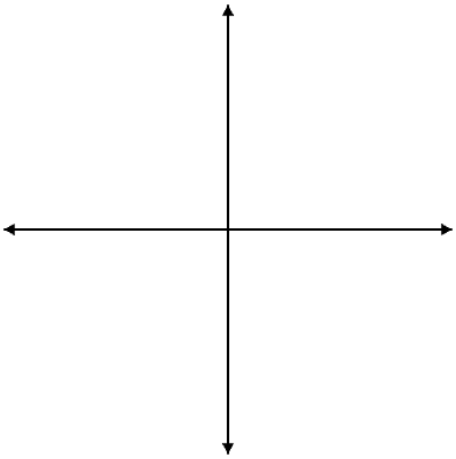
How would the triangle we draw for 135° be similar or different from the triangle for 45°?

Let's build this in general terms:



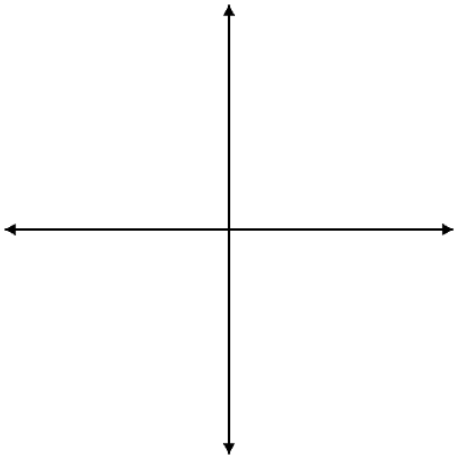
$sin\theta =$	$csc\theta =$
$cos\theta =$	$sec\theta =$
$tan\theta =$	$cot\theta =$

Example 2: The point $(8, \ -6)$ is on the terminal side of an angle. Find the value of the six trig functions of that angle.



$sin\theta =$	$csc\theta =$
$cos\theta =$	$sec\theta =$
$tan\theta =$	$cot\theta =$

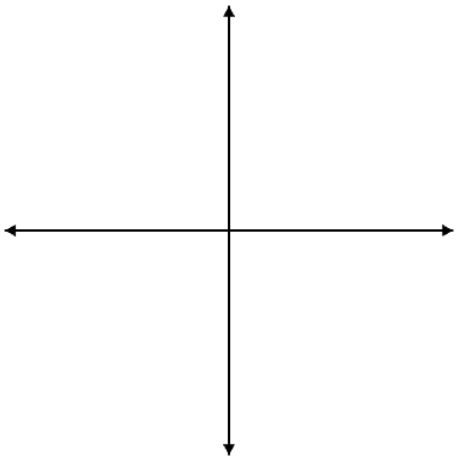
Example 3: Find the 6 trig ratios for $-\frac{3\pi}{4}$



$sin\theta =$	$csc\theta =$
$cos\theta =$	$sec\theta =$
$tan\theta =$	$cot\theta =$

Group Discussion Question:

1. In what quadrant(s) will $sin\theta$ be positive?
2. In what quadrant(s) will $cos\theta$ be positive?
3. In what quadrant(s) will $tan\theta$ be positive?



4. What about the reciprocal functions? Secant? Cosecant? Cotangent?

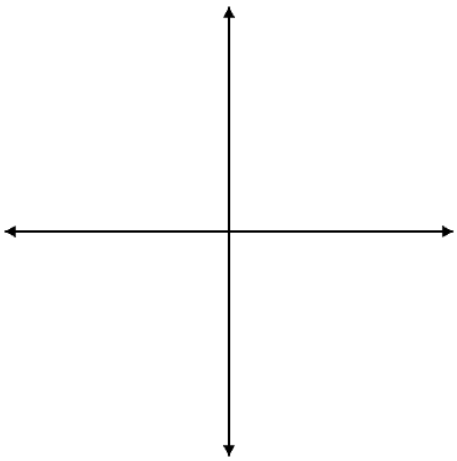
Example 4: If $cos\theta = -\frac{4}{7}$ and $tan\vartheta > 0$. Find $sin\theta$ and $cos\theta$.

5.1 Classwork:

Point P is on the terminal side of the angle θ. Evaluate the following trigonometric functions for θ.

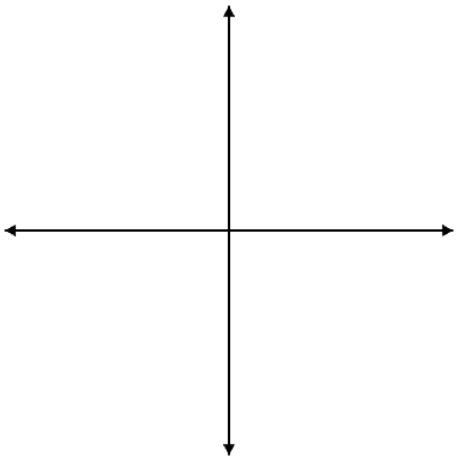
1. P(-1,-6)

$sin\theta =$
$sec\theta =$
$tan\theta =$

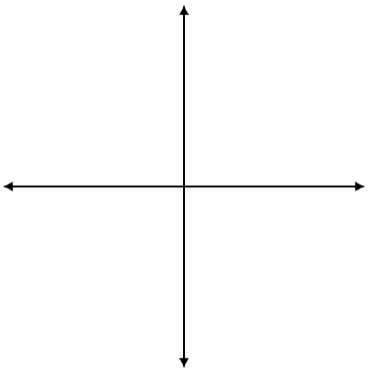


2. P(6, -8)

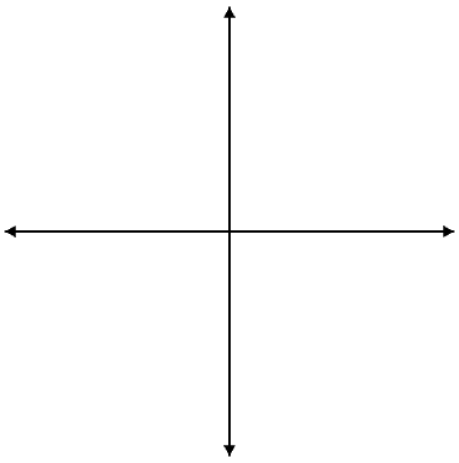
$sin\theta =$
$sec\theta =$
$tan\theta =$



3. Find $cos\theta$ and $cot\theta$ if $sin\theta = \frac{1}{4}$ and θ is in the second quadrant.



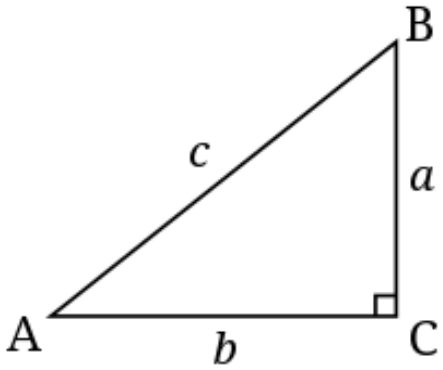
4. Find $sin\theta$ and $cos\theta$ if $cot\theta = \frac{3}{7}$ and θ is in the third quadrant.



5 .1 Homework Work Page

5.2 Building the Unit Circle: Similar Triangles

Directions: Use the given side lengths to find the trig ratios and the measure of angle A of right triangle ABC. Reduce your fractions for the trig ratios.



	Side a =	Side b =	Side c =	sin(A) =	cos(A)=	tan(A)=
Triangle 1	a = 4	b = 3	c = 5			
Triangle 2	a = 12	b = 9	c = 15			
Triangle 3	a = 2	b = $\frac{3}{2}$	c = $\frac{5}{2}$			
Triangle 4	a = $\frac{4}{5}$	b = $\frac{3}{5}$	c = 1			

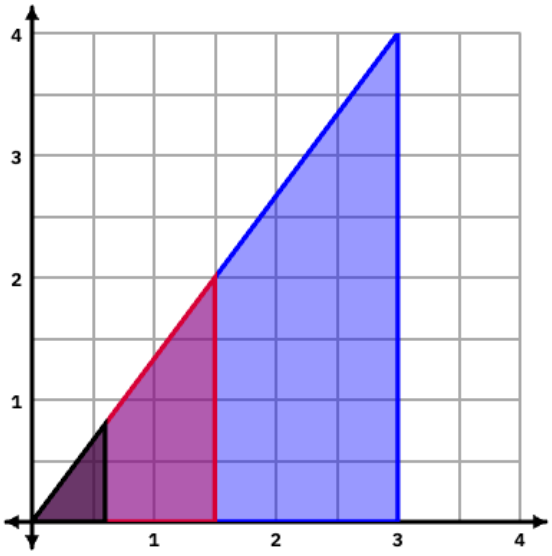
	Side a =	Side b =	Side c =	sin(B) =	cos(B)=	tan(B)=
Triangle 5	a = 3	b = $3\sqrt{3}$	c = 6			
Triangle 6	a = 5	b = $5\sqrt{3}$	c = 10			
Triangle 7	a = $\frac{3}{2}$	b = $\frac{3\sqrt{3}}{2}$	c = 3			
Triangle 8	a = $\frac{1}{2}$	b = $\frac{\sqrt{3}}{2}$	c = 1			

2. What is the relationship between triangles 1-4 ? What about the triangles 5-8 ? How do you know?

3. What do you notice about the $sin\theta$, $cos\theta$, and $tan\theta$ for each set of four triangles? What about the triangles make this happen?

Notes: Together

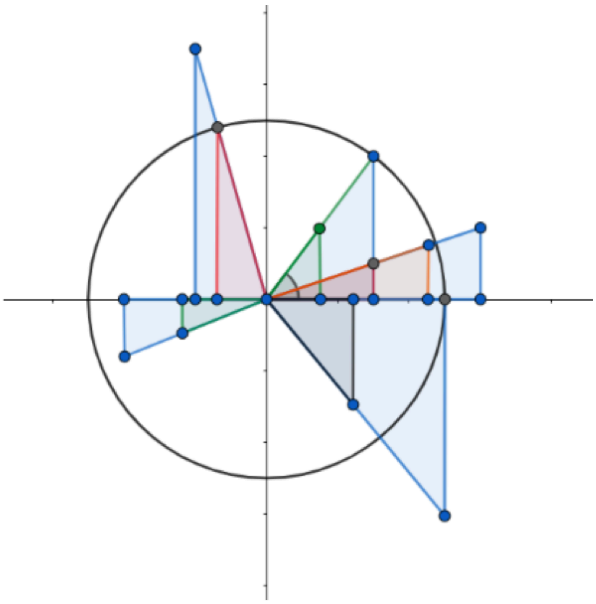
For ANY set of _____ triangles:



So to standardize our thinking about similar triangles and angles let's set the angle of interest, theta θ, at the origin of a coordinate plane....

	Side a =	Side b =	Side c =
Triangle 1	a = 4	b = 3	c = 5
Triangle 2	a = 2	b = $\frac{3}{2}$	c = $\frac{5}{2}$
Triangle 3	a = $\frac{4}{5}$	b = $\frac{3}{5}$	c = 1

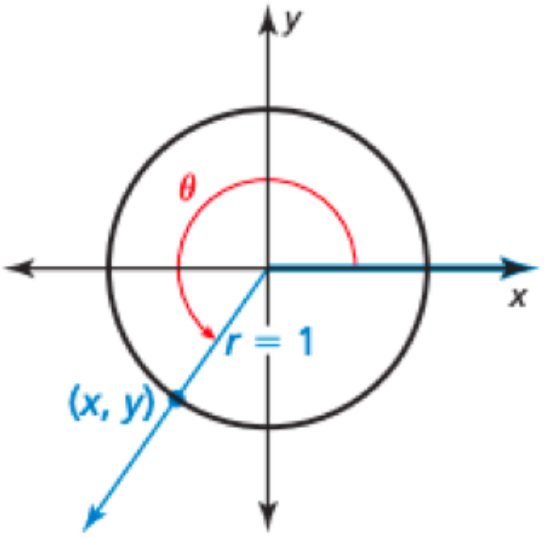
We want to create a way to represent EVERY "family" of similar triangles on the same coordinate plane.
This is challenging because we have triangles in all four quadrants and similar triangles can come in many different shapes and sizes



We can do a few things however....

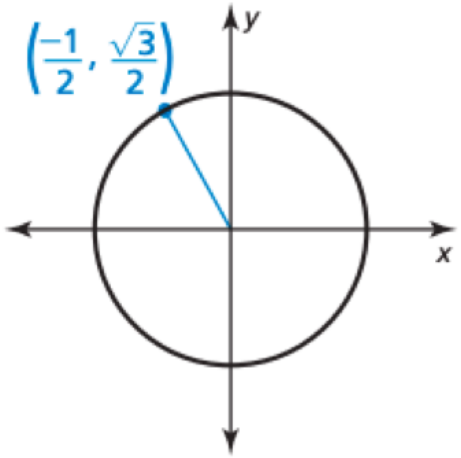
1. Let the reference angle of each angle be the distance from the terminal side of the angle (the hypotenuse of the triangle) to the x-axis.
2. Pick ONE triangle to represent each family.
3. Standardize the size of the "representative" triangle for each family of similar triangles by making them all have the same hypotenuse.

The Unit Circle:



Example 1: θ is an angle that has the point $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on its terminal side.

1. Is the point $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on the unit circle? How can you tell?

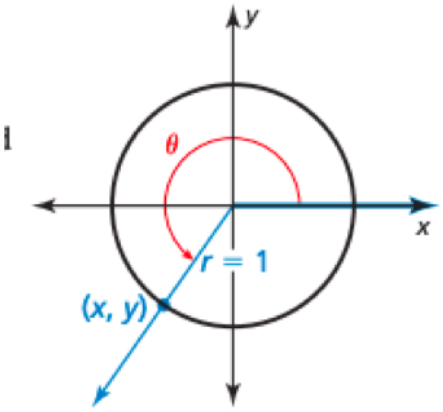


2. Find the sine, cosine, and tangent of θ .

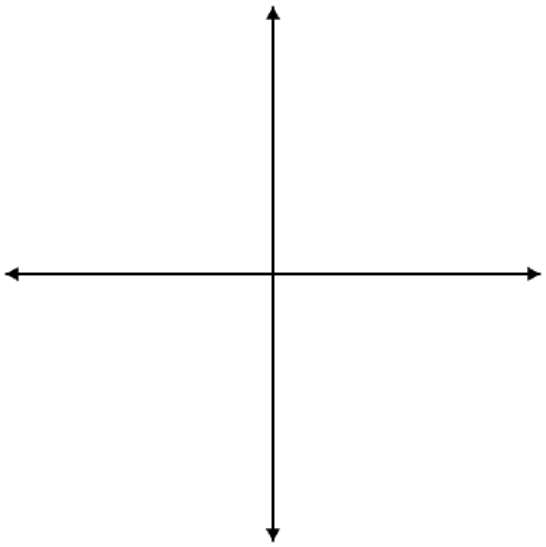
3. What is the value of θ? (Find the reference angle first!)

Generalization:

For trig functions:	For trig functions on the unit circle:
$\sin\theta =$	$\sin\theta =$
$\cos\theta =$	$\cos\theta =$
$\tan\theta =$	$\tan\theta =$



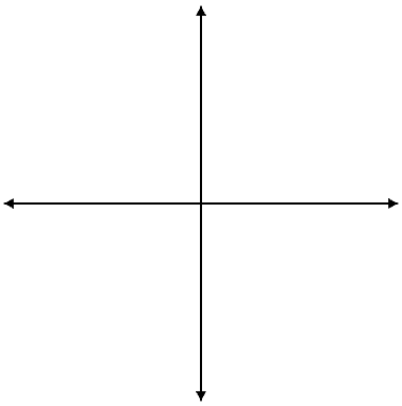
Example 2: Find the $cos\frac{5\pi}{6}$, $sin\frac{5\pi}{6}$, and $tan\frac{5\pi}{6}$.



Steps:

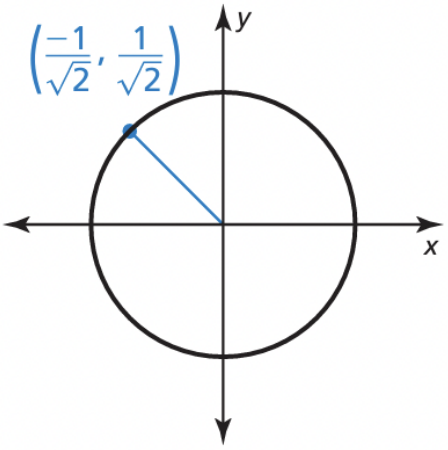
1. Draw your angle & find reference angle.
2. Label the sides of the triangle according to reference angle and quadrant.
3. Evaluate the trig expression!

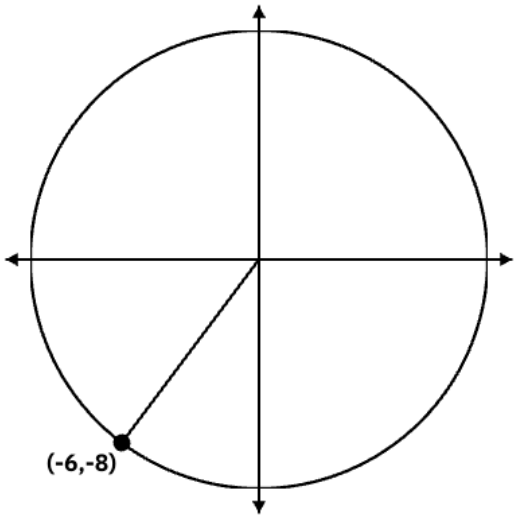
Example 3: Find the $csc(-405°)$.

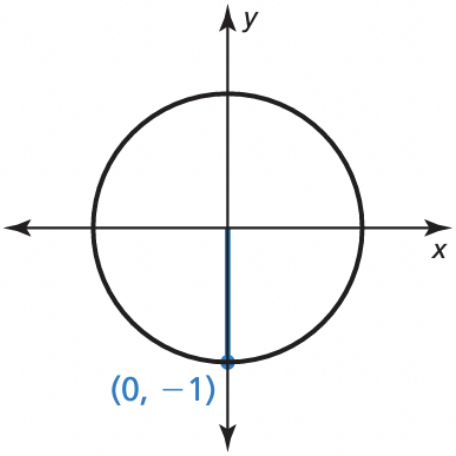


5.2 Classwork:

1. Are the following points on the unit circle? Show work to support your answer.







2. Find the $cos(210°)$.

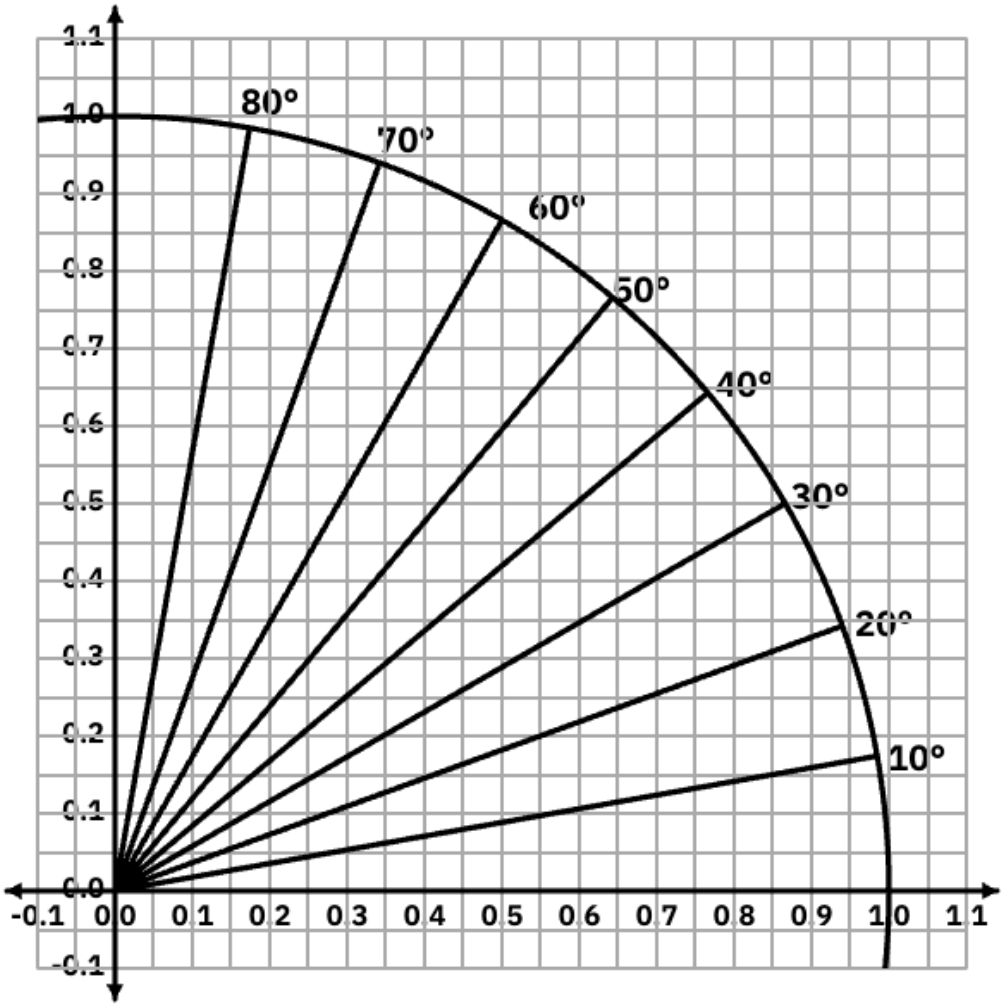
3. $cot\left(-\frac{\pi}{3}\right)$

5.2 Homework Work Page

5.3 More Unit Circle

1. The circle below has a radius of one unit. Each line represents 0.05 units. The multiples of 10° are listed along the circumference of the circle. Estimate the coordinates of the points on the circle.

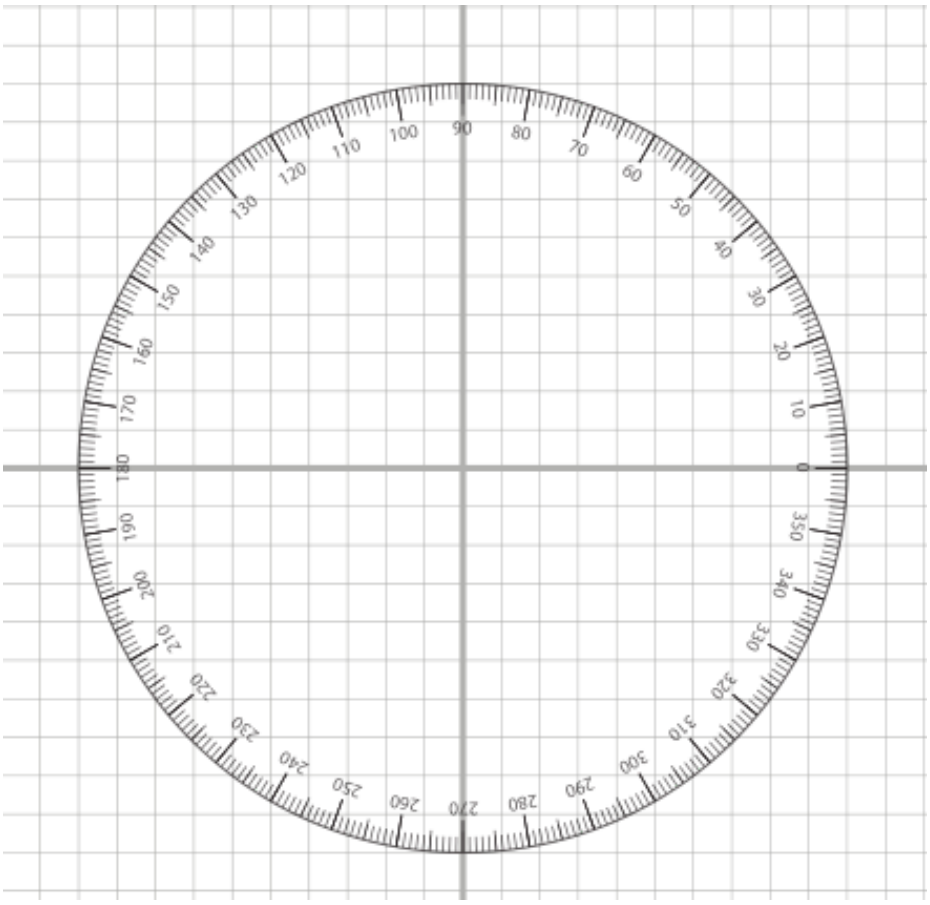
Angle	x-value	y-value
0°		
10°		
20°		
30°		
40°		
50°		
60°		
70°		
80°		
90°		



In the graph below: The circle below has a radius of one unit. Each line represents 0.1 unit.

2. What will change about the values in your table for the second quadrant? What will remain the same?

Angle	X value	Y value
90°		
100°		
110°		
120°		
130°		
140°		
150°		
160°		
170°		



3. What do you think will happen to the x and y values in the third quadrant?

4. What do you think will happen to the x and y values in the fourth quadrant?

5. What patterns do you see?

Analysis Questions:

1.What is the x value on the unit circle when a ray intersects it at 110˚?

2. What is the $cos110°$?

3. Explain why questions 1 and 2 are really asking the same thing.

4. What is the y value on the unit circle when a ray intersects it at 150˚?

5. What is the $sin150°$? Why is this question the same as question # 4?

6. Evaluate each of the expressions below:

$sin250°$	$cos350°$	$tan130°$
$csc220°$	$sec190°$	$cot280°$

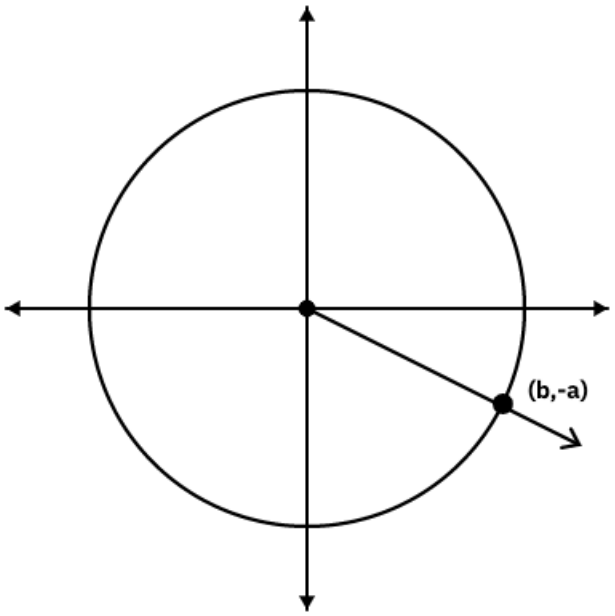
8. How are the $sin70°$ and the $sin110°$ related?

5.3 Homework Work Page

1. In the unit circle what is the

   $a.\ sin\,\theta$ $b.\ cos\,\theta$ $c.\ tan\theta$

2. A mystery angle θ is in the fourth quadrant intersects the unit circle $(b, -a)$. The angle with the same reference angle in quadrant 2, intersects the unit circle at what point?



3. Solve the following questions using the points on the unit circle given in the table below:

Angle (in °)	X Value	Y value
0	1	0
10	0.98	0.17
20	0.94	0.34
30	0.87	0.5
40	0.77	0.64
50	0.64	0.77
60	0.5	0.87
70	0.34	0.94
80	0.17	0.98
90	0	1

a. What is the point on the unit circle for 220°?

b. What is the sin 50°?

d. What is the cos -310°?

e. What is the point on the unit circle for 560°?

f. What is the point on the unit circle at $\frac{\pi}{6}$?

g. What is $sin\ \frac{19\pi}{18}$?

h. What is the value of $cos\frac{7\pi}{3}$?

5.4 Special Angles on Unit Circle:

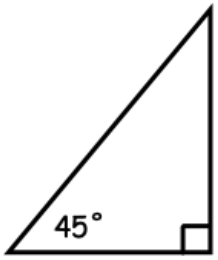
Warm Up: Which of the following expressions could you evaluate without a calculator or any additional information? Why ?

$$\tan\left(\frac{\pi}{4}\right) \qquad \sin 30^\circ \qquad \sec 50^\circ$$

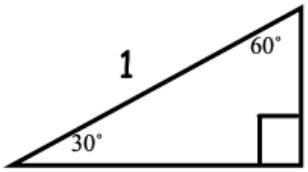
$$\cot 100^\circ \qquad \csc\left(\frac{\pi}{12}\right) \qquad \cos\left(\frac{6}{5}\right)$$

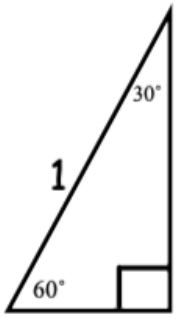
Evaluate the one(s) you can from above:

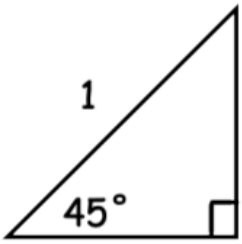
Notes: What were the relationships of our two special right triangles?



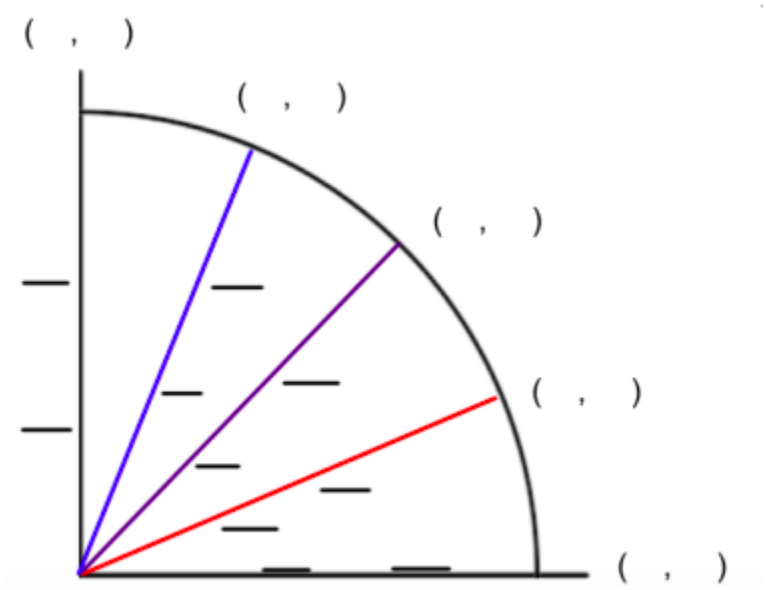
How can we apply this information to a triangle with hypotenuse 1?







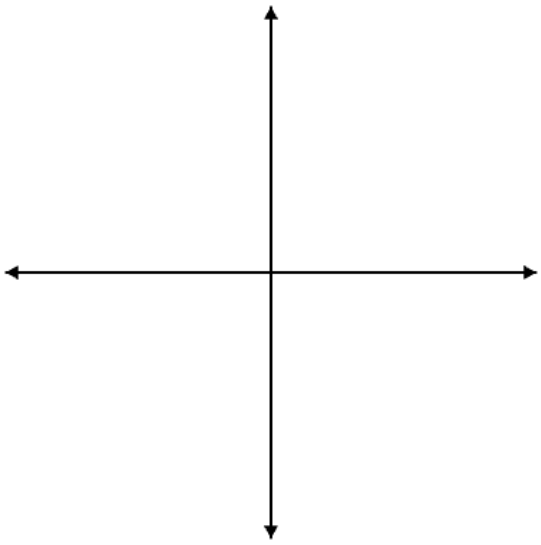
Special Angles in the First Quadrant of the Unit Circle:



Some precalculus teachers will have students complete this for the entire Unit circle, all four quadrants. Explain how you can use your knowledge of the first quadrant to evaluate an expression in any of the other three quadrants.

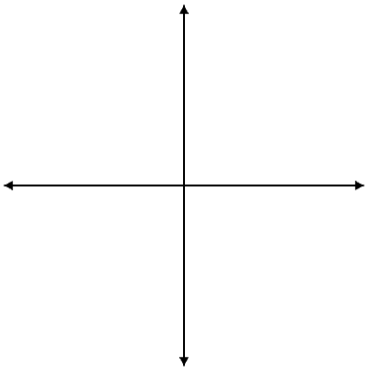
Quadrantal Angles: Angles whose terminal side lays on an x or y axis.

Example: Find the value of the six trigonometric functions for the angle: $-\frac{3\pi}{2}$



$sin\left(-\frac{3\pi}{2}\right) =$	$csc\left(-\frac{3\pi}{2}\right) =$
$cos\left(-\frac{3\pi}{2}\right) =$	$sec\left(-\frac{3\pi}{2}\right) =$
$tan\left(-\frac{3\pi}{2}\right) =$	$cot\left(-\frac{3\pi}{2}\right) =$

<u>Group Discussion Question:</u> Which of the 6 trig functions would be undefined and which would be zero if an angle had a terminal side that landed on the negative x-axis? (Hint: Start by thinking about the point on the unit circle that is on the negative x-axis.)



$sin\theta =$	$csc\theta =$
$cos\theta =$	$sec\theta =$
$tan\theta =$	$cot\theta =$

Example 2:

1. Evaluate the expressions below:

a. $sin\ \frac{5\pi}{4} =$

b. cos 120°=

c. $cot\ -\frac{7\pi}{3} =$

d. $cos\ \pi =$

Example 3: What angle, in degrees, would have a terminal ray that intersects the Unit Circle at the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$?

Practice:
Use the coordinate points given below (not your calculator for the following questions:

$$point\ at\ 30^\circ \left(\frac{\sqrt{3}}{2},\ \frac{1}{2}\right) \quad point\ at\ 45^\circ \left(\frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right) \quad point\ at\ 60 \left(\frac{1}{2}\ ,\frac{\sqrt{3}}{2}\right)$$

1. What are the exact values of:

sin 225°	$tan\ -\frac{3\pi}{4}$	$sin\ \frac{7\pi}{6}$
$cos\ \frac{5\pi}{3}$	$cot\ -\frac{7\pi}{3}$	$sec\ -\frac{5\pi}{6}$

2. What angle, in degrees, would have a terminal ray that intersects the Unit Circle at the point ....?

a. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ *b.* $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ *c.* $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ *d.* $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

3. What angle, in radians, would have a terminal ray that intersects the Unit Circle at the point...?

e. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ *f.* $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ *g.* $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ *h.* $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

5.4 Homework Work Page

5.5 Solving Trig Equations

Warm Up: What is/are the difference between the two equations below?

$$cos(0.77) = x \qquad\qquad cos(x) = 0.77$$

Which equation above can you solve if you have this table of the unit circle?

Angle (in °)	X Value	Y value
0	1	0
10	0.98	0.17
20	0.94	0.34
30	0.87	0.5
40	0.77	0.64
50	0.64	0.77
60	0.5	0.87
70	0.34	0.94
80	0.17	0.98
90	0	1

Example 1: Solve for all solutions over the interval $0° \leq \theta \leq 360°$

$csc\theta = \frac{100}{94}$

Example 2: What about change about our strategy if the equation was $csc\theta = -\frac{100}{94}$ instead?

Example 3: Solve for θ on the interval $0 < \theta < 2\pi$.

a. $sin\theta = 0$

b. $cos\theta = -\frac{\sqrt{2}}{2}$

c. $sinx = \frac{1}{2}$

What changes about problem a if I change the solving interval to $-2\pi < \theta < 2\pi$.

$sin\theta = 0$

Example 4: Solve for θ on the interval $0 < \theta < 2\pi$.

a. $sec\theta = -\frac{2\sqrt{3}}{3}$

b. $tan\theta = \sqrt{3}$

5.5 Classwork:

1. Solve the following equation with the table to the right over the interval $0 \leq \theta \leq 2\pi$

$$cot\theta = \frac{.94}{.34}$$

Angle Radians	X-Value	Y- Value
0	1	0
$\frac{\pi}{18}$	0.98	0.17
$\frac{\pi}{9}$	0.94	0.34
$\frac{\pi}{6}$	0.87	0.50
$\frac{2\pi}{9}$	0.77	0.64
$\frac{5\pi}{18}$	0.64	0.77
$\frac{\pi}{3}$	0.5	0.87
$\frac{7\pi}{18}$	0.34	0.94
$\frac{4\pi}{9}$	0.17	0.98
$\frac{\pi}{2}$	0	1

2. Create an equation, with the help of the table to the right, that has 2 solutions on the interval `$0 \leq \theta \leq 2\pi$.

3. Answer these questions fully for all angles in radians.– $2\pi \leq \theta \leq 2\pi$

   a. For what value (s) of θ, is $sin\theta = -\frac{\sqrt{3}}{2}$?

   b. For what value (s) of θ, is $cot\theta = \frac{\sqrt{3}}{3}$?

5.6 Homework:

Based on the group quiz from class today, please complete the following self-assessment. Then complete 5 questions. Choose your questions based on which topic(s) you need more practice with. Remember all assignments are helping you deepen your understanding of the unit circle!

	I can do this independently and explain my solution path(s) to my classmates and teacher.	I can do this independently.	I need more time. I need to see an example to help me.
Angle Fundamentals			
• Determine what quadrant an angle is in • Determine coterminal angles given an angle in radians and/or degrees • Determine a reference angle given an angle in radians and/or degrees • Convert an angle's measure from radians to degrees or from degrees to radians.			
Trigonometric Expressions			
• understand how the values on the unit circle are defined. • determine the values of the six trigonometric functions of a given angle using special right triangles, the unit circle, or a calculator. • determine the values of the remaining five trigonometric functions of an angle given one of the functions and the quadrant in which the angle lies. • Given the sign(+/-) of two non-reciprocal trigonometric functions of the same angle determine the quadrant of the angle. Then find the remaining four trigonometric functions. • Identify where the secant, cosecant, tangent, and cotangent functions are undefined.			
Trigonometric Equations			
• solve trigonometric equations on a given domain (0°-360° or $0 \; to \; 2\pi$)			

	Problems
Angle Fundamentals	1,5,7
Trig Expressions	1,2,3,4,5
Trig Equations	6,7,8,9

Evaluate the following expressions

1. $sin\frac{3\pi}{2} + tan\pi$

2. $3csc\frac{\pi}{3} + cot\frac{\pi}{4}$

3. Let $f(\theta) = sin\theta$ and $\theta = 60°$. Compute:

a. $f(2\theta)$ b. $[f(\theta)]^2$ c. $f(-\theta)$

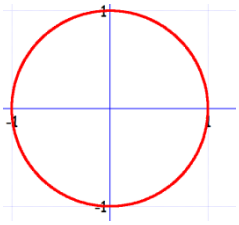
4. If the point (3,-7) is on the terminal side of an angle θ, Find $sec\theta$.

5. If $sec\theta = \frac{1}{5}$, is $csc\theta$ positive or negative. Explain how you know.

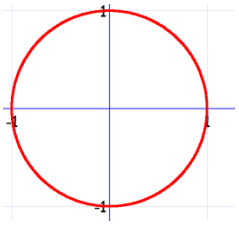
6. Is it possible for $sin\theta = 2$? Explain why or why not.

7. With the unit circles below, draw two terminal rays that indicates the two possible solutions to each equation in the interval $0 \leq \theta \leq 2\pi$.

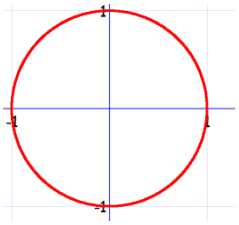
a. $sin\,\theta = 0.5$



b. $sin\,\theta = -0.7$



c. $cos\,\theta = 0.3$



8. Solve the $sec\theta = \frac{2\sqrt{2}}{2}$ equation: $sec\theta =$ on $0 \leq \theta \leq 2\pi$

9. Solve the equation: $tan\theta = -\sqrt{3}$ on $0 \leq \theta \leq 2\pi$

5.7 Review Warm Up:

1. How is the secant function defined on the unit circle? Explain how you know using your knowledge of right triangle trigonometry.

2. If $cos(54°) = 0.59$. Find $cos(234°)$

3. If $sec\theta < 0$, and $tan\theta = \frac{5}{9}$. Find the values of the other 5 trig functions of θ.

$sin\theta =$ $csc\theta =$

$cos\theta =$ $sec\theta =$

$tan\theta = \frac{5}{9}$ $cot\theta =$

4. Solve the equation $sec\theta = \sqrt{2}$ over the interval $0 \leq \theta \leq 2\pi$

5.7 Review :

1. Evaluate the following expressions using the table on the right.

a. $\sin \frac{5\pi}{18}$

b. $\csc 460^\circ$

c. $\cos \frac{17\pi}{9}$

d. $\sec 940^\circ$

e. $\tan 250^\circ$

f. $\cot -\frac{17\pi}{3}$

Angle	X value	Y value
0	1	0
$\frac{\pi}{18}$	.98	.17
$\frac{\pi}{9}$	.94	.34
$\frac{\pi}{6}$	.87	.5
$\frac{2\pi}{9}$	.77	.64
$\frac{5\pi}{18}$	.64	.77
$\frac{\pi}{3}$	.5	.87
$\frac{7\pi}{18}$	.34	.94
$\frac{4\pi}{9}$	.17	.98
$\frac{\pi}{2}$	0	1

1. Solve the following equations on $0 \leq \theta \leq 2\pi$

a. $cos\theta = -.64$

b. $cot\theta = -\frac{.17}{.98}$

3. An angle θ has the point (5, -8) on its terminal ray, find...

$sin\,\theta\ =$ $cos\,\theta\ =$ $sec\,\theta\ =$ $cot\,\theta\ =$

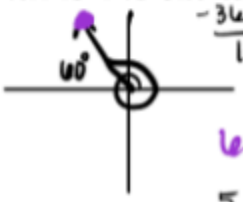
4. If $\csc\theta = -\frac{9}{5}$ and $tan < 0$, find....

$tan\,\theta\ =$ $cos\,\theta\ =$ $cot\,\theta\ =$

5. Ms. Miller answered one of the following questions correctly and two incorrectly. Mark which one is correct and then fix the mistake on the other two questions.

What are the exact values of:

a. What is the sin 480°?

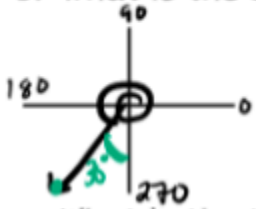


-360
120°

60° $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sqrt{3}/2$

b. What is the cos 600°?



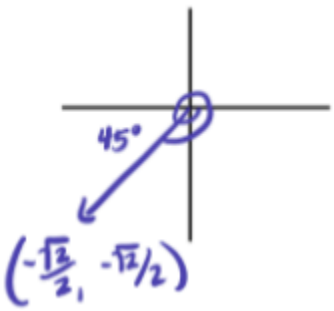
-360
240

30° $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$-\sqrt{3}/2$

c. What is the tan -495°?

+360
135



$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$-\sqrt{2}/2$ / $\sqrt{2}/2$

-1

6. What angle on the unit circle would have this point in radians? Identify the quadrant and the reference angle, if it exists.

a. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ *b.* $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ *c.* $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ *d.* $(0, 1)$

7. Answer these questions fully for all angles in radians. $0 \leq \theta \leq 2\pi$

a. For what value (s) of θ, is $sin\theta = -\frac{\sqrt{3}}{2}$?

b. For what value (s) of θ, is $tan\theta$ is undefined?

c. For what value (s) of θ, is $2cos\theta = -\sqrt{2}$?

d. For what value (s) of θ, is $csc\theta = -2$?

e. For what value (s) of θ, is $cot\theta = \frac{\sqrt{3}}{3}$?

f. For what value (s) of θ, is $2sec\theta + 12 = 14$?

8. State the quadrant in which θ lies.
   a. If $cos\,\theta < 0\ and\ cot\,\theta < 0$ what quadrant is θ in?

   b. Your turn: Write two more questions like part a whose answer is "quadrant 2"