BUTLER SCHOOL DISTRICT

High School - Dual Enrollment Pre-Calculus Curriculum

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Adapted from:
New Jersey State Learning Standards

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VISION

The Butler School District's Mathematics Department's objective is to prepare students to think critically, innovate, communicate, and collaborate in an ever-changing world. The Mathematics curriculum provides students with quality, rigorous instruction to help them become better **problem solvers, troubleshooters, and analytical thinkers**. The rich, educational experience provided within the Butler School District will produce young adults with the foundation and expertise they need for the future. It is the goal to challenge each student to develop and extend mathematical proficiency through highest quality mathematics teaching and standard-based assessments that meet the learning needs of each student. Butler Mathematics students will become individuals who persevere in their pursuit of lifelong learning through a culture that appreciates the beauty and usefulness of math.

As a result of a Butler Mathematics education, students will be able to…

- Synthesize mathematical skills across disciplines
- Develop into confident mathematicians
- Learn at their own pace and advance their understanding in a variety of ways
- Collaborate with others and contribute productively and articulately
- Act responsibly and be accountable for actions, in person and online
- Effectively approach, analyze, plan, and apply appropriate strategies for problem solving in ambitious contexts with accommodations for those who need it.
- Persevere through difficult situations and tasks and maintain a growth mindset despite adversity.
- Draw on knowledge from a wide variety of mathematical topics with flexibility to approach the same problem from different mathematical perspectives or represent mathematics in different ways.
- Evaluate situations, draw logical conclusions, and develop, describe and apply solutions.
- Construct and support arguments.
- Evaluate their own reasoning and critique the reasoning of others.
- Assess the reasonableness of a solution with respect to the given construct or problem context.
- Use effective communication to engage in peer collaboration, reflecting on whether or not a solution is viable.
- Create appropriate representations of mathematical situations across a variety of mediums. These models will support the student's ability to demonstrate and explain their mathematical understanding.
- Use mathematical tools to explore and deepen their understanding of mathematical concepts.
- Make effective choices regarding the use of any available tools.

- Make appropriate use of technology as a tool that is constantly changing and evolving.
- Attend precision in their mathematical calculations and in their communication.
- Calculate accurately and efficiently and express numerical answers with a degree of precision that is appropriate to the given context.
- Develop precision in their use of mathematical language.
- Look closely to determine patterns and structures within mathematics.
- Make meaningful connections between their knowledge from previous experiences and the content they are currently exploring.
- Develop deep understandings of mathematical concepts such that these understandings become applicable building blocks for future learning.
- Use their mathematical understandings to make generalizations that apply to various mathematical circumstances.
- Identify patterns in mathematics that can be used to solve problems that are challenging relative to their learning comfort zone.
- Use generalizations to increase the efficiency and manageability of their work.
- Demonstrate growth mindset and grit in effectively approaching ever-rigorous problem solving.
- Apply appropriate strategies with differentiated levels of support.
- Be confident in participating in higher level discussions that will assess and advance the understanding of concepts.
- Learn mathematics through exploring and solving contextual and mathematical problems

COURSE DESCRIPTION

Precalculus is a thoughtful introduction to advanced studies leading to calculus. The course briefly reviews linear equations, inequalities, and systems and moves purposefully into the study of functions. Students then discover the nature of graphs and deepen their understanding of polynomial, rational, exponential, and logarithmic functions. Scaffolding rigorous content with clear instruction, the course leads students through an advanced study of trigonometric functions, matrices, and vectors. The course concludes with a short study of probability and statistics.

This course includes a broad series of lessons and activities that offer a variety of modalities for ultimate student engagement and content retention. Each unit contains a series of lessons that include introduction of content, virtual demonstration of that content, and repeated opportunity to practice that content, along with a quiz per lesson, exam per unit, and final exam at the end of the course.'

Goals

Students will:

1) analyze calculus-related concepts, such as limits, tangent to a curve, and the difference quotient;
2) apply the properties of functions, such as finding the domain and range, determine zeros of functions, symmetries of a function, vertical, horizontal, and oblique asymptotes, use limits to determine end behavior of a function, develop the skills to graph functions, and accurately use a graphing calculator to find transformations of graphs;
3) perform operations of functions - addition, subtraction, multiplication, division, composition, find and graph inverse functions, solve problems involving maximizing and minimizing a quadratic function, and construct mathematical models with functions using real-world applications or formulas;
4) graph exponential and logarithmic functions, accurately use the properties of logarithms and the definition of e, solve exponential and logarithmic equations, and application problems using exponents and logarithms;
5) define trigonometric functions, using the unit circle and right triangle definition, graph trigonometric functions, find the inverse trigonometric functions, verify trigonometric identities and formulas, solve trigonometric equations and solve problems, using real-world applications;
6) develop the ability to analyze, interpret, and apply quantitative information [Quantitative Literacy].

Assessment: Student learning will be assessed through a variety of formative, summative benchmark, and alternative assessments.

Scope and Sequence

SCOPE AND SEQUENCE
(*Pacing Guide*)

Unit of Study	**Estimated Time**
Unit 1: Functions & Graph Properties; Polynomial, Power, and Rational Functions	70 Days
Unit 2: Exponential, Logarithmic, Logistic Functions; Trigonometric Functions;	40 Days
Unit 3: Analytic Trigonometry; Applications of Trigonometry	50 Days
Unit 4: Analytic Geometry, Discrete Math, Systems of Equations and Matrices Introduction to Calculus	20 Days

AFFIRMATIVE ACTION COMPLIANCE STATEMENT

The Butler Public Schools are committed to the achievement of increased cultural awareness, respect, and equity amongst our students, teachers, and community. We are pleased to present all pupils with information pertaining to possible career, professional, or vocational opportunities which in no way restricts or limits options on the basis of race, color, creed, religion, sex, ancestry, national origin, or socioeconomic status.

INTEGRATED ACCOMMODATIONS AND MODIFICATIONS

Students with IEPs, 504s, and/or Students at Risk of Failure Students read authentic texts and write authentic pieces at their independent and instructional reading levels. Individualized feedback is provided through conferences and small groups. The teacher utilizes visual and multi-sensory methods of instruction in addition to assistive technology when needed. Students are provided with graphic organizers and other scaffolded material. Modification of content and product may be deemed necessary based on student needs. Students are provided with testing accommodations and authentic assessments.

Gifted & Talented Students Students read authentic texts and write authentic pieces at their independent and instructional reading levels. Individualized feedback is provided to the student through conferences and small groups. Students are engaged through inquiry-based instruction to develop higher-order thinking skills. Activities are developed based on student interests and student goals. Students engage in real-world projects and scenarios.

English Language Learners Students read authentic texts and write authentic pieces at their independent and instructional reading levels. Individualized feedback is provided to students through conferences and small groups. Students are pre-taught vocabulary terms and concepts. Teachers engage students through visual learning, including the use of graphic organizers. Teachers use cognates to increase comprehension. The teacher models tasks and concepts, and pairs students learning English with students who have more advanced English language skills. Scaffolding is provided including word walls, sentence frames, think-pair-share, cooperative learning groups, and teacher think-alouds.

21ST CENTURY THEMES & SKILLS

Embedded in many of our units of study and problem based learning projects are the 21st Century Themes as prescribed by the New Jersey Department of Education. These themes are as follows:

- Global Awareness
- Financial, Economic, Business, and Entrepreneurial Literacy
- Civic Literacy
- Health Literacy

CURRICULUM ADDENDA FOR SPECIAL EDUCATION

This curriculum can be both grade and age appropriate for special education students and serves as a guide for the special education teacher in line with the district's written philosophy of special education, as stated within Policy #6700 concerning Programs for Educationally

Disabled Students. Based on the Child Study Team evaluation and consultation with the parent and classroom teacher, an individualized education plan may include modifications to content, instructional procedures, student expectations, and targeted achievement outcomes of this curriculum document in accordance with the identified needs of an eligible student. This educational plan will then become a supplement guide that the classroom teacher, parent, and Child Study Team will use to measure the individual student's performance and achievement.

CURRICULUM ADDENDA FOR ENGLISH LANGUAGE LEARNERS

This curriculum guide is appropriate and is implemented for all students according to age and grade, and is in line with the district's written philosophy of English language acquisition concerning Bilingual Instruction and English as a Second Language Programs. In accordance with the New Jersey Administrative Code 6A:15, the contents herein provide equitable instructional opportunities for English Language Learners to meet the New Jersey Student Learning Standards and to participate in all academic and non-academic courses. Students enrolled in a Bilingual and/or an ESL program may, in consultation with the classroom teacher and Bilingual and/or ESL teacher, receive modification to content, instructional procedures, student expectations and targeted achievement outcomes of this curriculum document in accordance with the students developmental and linguistic needs.

DIVERSITY AND INCLUSION

In alignment with the 2020 NJSLS, the Mathematics Curriculum materials will:

Cultivate respect towards minority groups to foster appreciation of their differences as well as their contributions to the advancement of mathematics

Analyze and appreciate the diverse contributions made in the past (scientifically, economically, politically, and socially) at both the state and federal level as exemplified through mathematics

Examine grade-level texts and resources that simultaneously highlight mathematics as well as the contributions made to it by those of different genders, ethnicities, and abilities.

Employ mathematics as a means of communication — whether in regard to empathy, inclusivity, or advocacy — in an effort to creatively inspire solutions for those with specific needs.

Engage in authentic learning experiences that motivate the acquisition and application of varied perspectives in mathematics

Facilitate the ability to communicate effectively through mathematics while applying content knowledge, interdisciplinary connections, and thinking skills to do so.

Foster active student participation in an inclusive culture that honors mathematicians of all genders, ethnicities, and abilities.

Analyze and develop an understanding of how scientific, economic, political, social, and cultural aspects of society influence new technological and mathematical processes.

Reflect on both personal and non-personal experiences aimed to promote empathy and inclusivity for all regardless of our differences.

UNIT 1
Functions & Graph Properties; Polynomial, Power, and Rational Functions
UNIT SUMMARY
● The objective of this unit is to first introduce students to topics Algebra 2 that are significant to the start of Pre-Calculus. These topics will include and are not limited to linear functions and their graph, complex numbers, modeling, and operations with polynomials.
NEW JERSEY STUDENT LEARNING STANDARDS MATHEMATICS 2023
F.IF.7D-Analyze functions using different representations **A.REI. 8,9**-Solve Systems of Equations **F.BF.1C**-Build a function that models a relationship between two quantities **N.CN.3**-Perform Arithmetic operations with complex numbers **N.CN.A.1 (+):** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers **N.CN.B.4 (+):** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N.CN.B.5 (+):** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically in the complex plane; use properties of this representation for computation. *For example, $(-1+\sqrt{3}i)^3=8$ because $-1+\sqrt{3}i$ has modulus 2 and argument 120^o*. **N.CN.B.6 (+):** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **N.CN.C.8 (+):** Extend polynomial identities to the complex numbers. *For example, rewrite x^2+4 as $(x+2i)(x-2i)$.* **N.CN.C.9 (+):** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **A.APR.C.4 (+):** Prove polynomial identities and use them to describe numerical relationships. *For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythogrean triples.* **A.APR.C.5 (+):** Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in the powers of *x* and *y* for positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. *(Clarification: The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)* **A.APR.D.7 (+):** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **F.BF.A.1c (+):** Composite functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F.BF.B.4b (+): Verify by composition that one function is the inverse of another
F.BF.B.4c (+): Read values of an inverse function from a graph or a table, given that the function has an inverse
F.BF.B.4d (+): Produce an invertible function from a non-invertible function by restricting the domain
F.IF.C.7d (+): Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior

INTERDISCIPLINARY CONNECTIONS

New Jersey Student Learning Standards- Science
HS-ETS1-1. Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.
HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3. Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.
HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

New Jersey Student Learning Standards-Language Arts (2023)
L.VL.11–12.3. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

RI.MF.11–12.6. Synthesize complex information across multiple sources and formats to develop ideas, resolve conflicting information, or develop an interpretation that goes beyond explicit text information (e.g., express a personal point of view, new interpretation of the concept).

W.WR.11–12.5. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

SL.PE.11–12.1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.UM.11–12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.

New Jersey Student Learning Standards: Career Readiness, Life Literacies and Key Skills (2020)

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities
9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving
9.4.12.CT.4: Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes
9.4.12.DC.1: Explain the beneficial and harmful effects that intellectual property laws can have on the creation and sharing of content
9.4.12.GCA.1: Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others
9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

2020 New Jersey Student Learning Standards – Computer Science and Design Thinking

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.1: Use research to design and create a product or system that addresses a problem and make modifications based on input from potential consumers.

21st CENTURY LIFE AND CAREER STANDARDS

Career Readiness, Life Literacies, and Key Skills Practices describe the habits of the mind that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success.

1. Act as a responsible and contributing community member and employee.
2. Attend to financial well-being.
3. Consider the environmental, social and economic impacts of decisions.
4. Demonstrate creativity and innovation.
5. Utilize critical thinking to make sense of problems and persevere in solving them.
6. Model integrity, ethical leadership and effective management.
7. Plan education and career paths aligned to personal goals.
8. Use technology to enhance productivity, increase collaboration and communicate effectively.
9. Work productively in teams while using cultural global competence.

9.1: Personal Financial Literacy A. Civic Responsibility B. Financial Institutions C. Financial Psychology D. Planning and Budgeting E. Risk Management and Insurance F. Civic Financial Responsibility G. Credit Profile H. Economic and Government Influences I. Credit and Debt Management	**9.2: Career Awareness, Exploration & Preparation, and Training** A. Career Awareness (K-2) B. Career Awareness and Planning (3-5) C. Career Awareness and Planning (6-8) D. Career Awareness and Planning (9-12) **9.4 Life Literacies and Key Skills** A. Creativity and Innovation B Critical Thinking and Problem-solving C. Digital Citizenship D. Global and Cultural Awareness E. Information and Media Literacy F. Technology Literacy	**9.3: Career and Technical Education** A. Agriculture B. Architecture C. Arts,A/V, Technology D. Business Management E. Education F. Finance G. Government H. Health Science I. Hospital & Tourism J. Human Services K. Information Tech. L. Law and Public Safety M. Manufacturing N. Marketing O. Science, Technology, Engineering & Math P. Trans./Logistics

TECHNOLOGY STANDARDS	
8.1: Computer Science A. Computing systems B. Networks and the Internet C. Impacts of Computing D. Data & Analysis E. Algorithms & Programming	**8.2 Design Thinking** A. Engineering Design B. Interaction of Technology and Humans C. Nature of Technology D. Effects of Technology on the Natural World E. Ethics & Culture

ENDURING UNDERSTANDINGS	**ESSENTIAL QUESTIONS**
• Find roots, zeros, and x-intercepts • Test for even and odd functions • Inverse Notation and Finding the Inverse of a function • Compose and Decompose Functions • Perform any transformations to the twelve basic functions • Complete the Square • Solve Quadratic Equations • Analyze the correlation coefficient	• Do students understand the characteristics of the twelve basic functions and where they are used in the real world? • Do students have adequate Algebra and Geometry prerequisites to continue into the Pre-Calculus curriculum? • Are students able to model situations from word problems?

• Use regression analysis • Polynomial long and synthetic division • Solve Inequalities using sign charts	

STUDENT LEARNING OBJECTIVES (Students are learning to / Students are learning that)

- F.IF.7D Analyze functions using different representations
- A.REI. 8,9 Solve Systems of Equations
- F.BF.1C Build a function that models a relationship between two quantities
- N.CN.3 Perform Arithmetic operations with complex numbers
- Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

SUGGESTED ACTIVITIES

- Instructional Lecture
- Cooperative Learning
- In Class Group Activities
- Projects
- **F.BF.1C-**Quadratic Emissions Model **Climate**
- A.APR.D.7 Combined Fuel Efficiency **Climate**
- Introduce students to mathematicians from diverse backgrounds, such as Maryam Mirzakhani, Katherine Johnson, or Srinivasa Ramanujan. Discuss their contributions to mathematics and their cultural contexts**. DEI**
- Discuss how precalculus concepts are applied in various professions and industries globally. Highlight examples from different countries or cultures to show the universal applicability of mathematics. **DEI**
  F.IF.B.4 Build a function that models the relationship of the gender pay gap in America **DEI**

EVIDENCE OF LEARNING

Formative Assessments:	**Summative Assessment:**
Classroom Discussion Exit Slip Checklists Peer Assessment Vocabulary Quizzes Rubrics Participation and teacher observation Mini Whiteboard Responses Think-Pair-Share Concept Map Classroom Poll	Unit Tests End-of-Book Test NJGPA Test
Benchmark Assessment: Unit Benchmarks (3x/year)	**Alternative Assessments:** Project Portfolio

INSTRUCTIONAL RESOURCES

Core Instructional Resource:	Teacher Created Materials:	Supplemental Resources:
Demana,Waits,Foley, Kennedy Precalculus-Graphical, Numerical, Algebraic	Warmups, Exit tickets Guided notes PowerPoints	**Math XL; Illustrative Mathematics; National Council of Teachers of Mathematics; Khan Academy; Desmos; Math Warehouse; Mathematics Assessment Resource Services (MARS); Learn Zillion; New Jersey Center for Teaching and Learning; Edulastic; Nearpod;Ed-Puzzles;Quizizz ;Kahoot**

INTEGRATED ACCOMMODATIONS AND MODIFICATIONS

Special Education:
Provide modified notes and access to extra copies online
Provide oral reminders and check student work during independent work time
Model skills/techniques to be mastered
Check and sign assignment planner
Preferential seating
Pair visual prompts with verbal presentations
Modified or scaffolded homework and classwork
Extended time as needed
Provide graphic organizers and study guides

English Learners:
Provide scaffolded assignments and assessments
Pair visual prompts with visual presentations
Check and sign assignment planner
Native Language translation (peer, online assistive technology, translation device, bilingual dictionary)
Extended time for assignment and assessment as needed
Highlight key vocabulary
Use graphic organizers
Provide verbal and written directions
Preferential seating with a English-speaking peer

At Risk of Failure:
Check and sign assignment planner

Encourage class participation and reinforce skills
Model skills and assignments
Extended to time to complete class work
Preferential seating
Provide extra help outside of class and 1:1 instruction when needed
Communicate regularly with students' other teachers
Provide positive feedback for tasks well done
Encourage student to proofread assessments and projects and ask for teacher proofreading of large writing assignments

Gifted and Talented:
Pose higher-level thinking questions
Provide higher level reading and writing materials for literacy based activities
Probe student to extend thinking beyond the text or connect two or more texts
Provide alternate or project-based assessments and assignments

Students with 504 Plans
Provide extended time as needed
Modify length of writing assignment
Provide short breaks within the lesson
Provide scaffolding for students
Utilize graphic organizers

UNIT 2
Complex Numbers/ Exponential and Logistic Functions/ Trigonometric Functions and their properties
UNIT SUMMARY
● The objective of this unit is to build on the first unit and continue analyzing the Exponential and Logistic Functions. The trigonometric functions will also be introduced and be analyzed through its properties and its behaviors. ● One of the primary interdisciplinary connections that can be made in this unit is with finance, speech, and physics. Many of the problems that are introduced are similar to problems that are introduced in a basic physics course.
NEW JERSEY STUDENT LEARNING STANDARDS MATHEMATICS 2023
● F.TF.3,4 Extend the domain of trigonometric functions using the unit circle ● F.TF 6,7 Model periodic phenomena with trigonometric functions ● F.TF.9 Prove and apply trigonometric identities ● **F.BF.B.5 (+):** Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents

- F.TF.A.1 (+) Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F.TF.A.2 (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as the radian measures of angles traversed counterclockwise around the unit circle
- F.TF.A.1 3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for
- $\frac{\pi}{3}, \frac{\pi}{4}, and \frac{\pi}{6}$ and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x, \pi + x and 2\pi - x$ in terms of their values for x, where x is any real number.
- F.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

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7. Plan education and career paths aligned to personal goals.
8. Use technology to enhance productivity, increase collaboration and communicate effectively.
9. Work productively in teams while using cultural global competence.

9.1: Personal Financial Literacy	**9.2: Career Awareness, Exploration & Preparation, and Training**	**9.3: Career and Technical Education**
J. Civic Responsibility K. Financial Institutions L. Financial Psychology M. Planning and Budgeting N. Risk Management and Insurance O. Civic Financial Responsibility P. Credit Profile Q. Economic and Government Influences R. Credit and Debt Management	G. Career Awareness (K-2) H. Career Awareness and Planning (3-5) I. Career Awareness and Planning (6-8) J. Career Awareness and Planning (9-12) **9.4 Life Literacies and Key Skills** A. Creativity and Innovation B Critical Thinking and Problem-solving C. Digital Citizenship D. Global and Cultural Awareness K. Information and Media Literacy L. Technology Literacy	Q. Agriculture R. Architecture S. Arts,A/V, Technology T. Business Management U. Education V. Finance W. Government X. Health Science Y. Hospital & Tourism Z. Human Services AA. Information Tech. BB. Law and Public Safety CC. Manufacturing DD. Marketing EE. Science, Technology, Engineering & Math FF. Trans./Logistics

TECHNOLOGY STANDARDS	
8.1: Computer Science A. Computing systems B. Networks and the Internet C. Impacts of Computing D. Data & Analysis E. Algorithms & Programming	**8.2 Design Thinking** A. Engineering Design B. Interaction of Technology and Humans C. Nature of Technology D. Effects of Technology on the Natural World E. Ethics & Culture

ENDURING UNDERSTANDINGS	**ESSENTIAL QUESTIONS**
● Angle Measure Conversion ● Finding Arc Length	● Do students understand the characteristics of the exponential and

• Evaluating Trigonometric functions algebraically, graphically, and numerically • Finding Inverses and applying them to real world situations	trigonometric functions and where they are used in the real world? • Are students able to represent exponential functions algebraically, graphically, and numerically? • Are students able to model situations from word problems?

STUDENT LEARNING OBJECTIVES (Students are learning to / Students are learning that)

- Define sine, cosine, tangent, cosecant, secant, and cotangent functions.
- Understand these functions as ratios of sides in a right triangle.
- Explore their properties and behaviors, including periodicity and symmetry.
- Graph sine, cosine, and tangent functions.
- Understand amplitude, period, phase shift, and vertical shift.
- Graph transformations of these functions (e.g., stretching, compressing, reflecting).
- Prove and apply fundamental trigonometric identities (e.g., Pythagorean identities, co-function identities).
- Simplify trigonometric expressions using identities.
- Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- Write a function that describes a relationship between two quantities.
- Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

SUGGESTED ACTIVITIES

Math Starters! 2nd Edition
Desmos Graphing Calculator (desmos.com)
Graphing Whiteboards
Youtube demonstrations

- Assign each group a different cultural or historical context to research how trigonometric functions were used or developed in that culture. Examples could include:
  -Ancient Greek mathematics and astronomy
  -Islamic contributions to trigonometry during the medieval period
  -Trigonometry in ancient Indian mathematics
  -Trigonometric functions in Maya civilization for astronomy
  -Contemporary applications in different parts of the world (e.g., engineering in Japan, navigation in Polynesian cultures) **DEI**
- F.TF.A.4 (+)-Speedy Sea Current **Climate**

- F.TF.A.4 (+)-A New Renewable **Climate**

EVIDENCE OF LEARNING	
Formative Assessments: Classroom Discussion Exit Slip Checklists Peer Assessment Vocabulary Quizzes Rubrics Participation and teacher observation Mini Whiteboard Responses Think-Pair-Share Concept Map Classroom Poll	**Summative Assessment:** Unit Tests End-of-Book Test NJGPA Test
Benchmark Assessment: Unit Benchmarks (3x/year)	**Alternative Assessments:** Project Portfolio

INSTRUCTIONAL RESOURCES		
Core Instructional Resource: Demana, Waits, Foley, Kennedy Pre-Calculus-Graphical, Numerical, Algebraic	**Teacher Created Materials:** Warmups, Exit tickets Guided notes PowerPoints	**Supplemental Resources:** **Math XL; Illustrative Mathematics; National Council of Teachers of Mathematics; Khan Academy; Desmos; Math Warehouse; Mathematics Assessment Resource Services (MARS); Learn Zillion; New Jersey Center for Teaching and Learning; Edulastic; Nearpod;Ed-Puzzles;Quizizz ;Kahoot**

INTEGRATED ACCOMMODATIONS AND MODIFICATIONS
Special Education: Provide modified notes and access to extra copies online Provide oral reminders and check student work during independent work time Model skills/techniques to be mastered Check and sign assignment planner Preferential seating Pair visual prompts with verbal presentations

Modified or scaffolded homework and classwork
Extended time as needed
Provide graphic organizers and study guides

English Learners:
Provide scaffolded assignments and assessments
Pair visual prompts with visual presentations
Check and sign assignment planner
Native Language translation (peer, online assistive technology, translation device, bilingual dictionary)
Extended time for assignment and assessment as needed
Highlight key vocabulary
Use graphic organizers
Provide verbal and written directions
Preferential seating with a English-speaking peer

At Risk of Failure:
Check and sign assignment planner
Encourage class participation and reinforce skills
Model skills and assignments
Extended to time to complete class work
Preferential seating
Provide extra help outside of class and 1:1 instruction when needed
Communicate regularly with students' other teachers
Provide positive feedback for tasks well done
Encourage student to proofread assessments and projects and ask for teacher proofreading of large writing assignments

Gifted and Talented:
Pose higher-level thinking questions
Provide higher level reading and writing materials for literacy based activities
Probe student to extend thinking beyond the text or connect two or more texts
Provide alternate or project-based assessments and assignments

Students with 504 Plans
Provide extended time as needed
Modify length of writing assignment
Provide short breaks within the lesson
Provide scaffolding for students
Utilize graphic organizers

UNIT 3

Analytic Trigonometry, Applications of Trigonometry

UNIT SUMMARY

- The objective of this unit is to go further into the topics of trigonometry and begin the real world connections. The topics which will mainly be focused on in this unit are the applications of trig functions and the laws of trigonometric functions

NEW JERSEY STUDENT LEARNING STANDARDS MATHEMATICS 2023

F.IF.C.7.f (+) Graph trigonometric functions, showing period, midline, and amplitude.
F.TF.B.5 (+) Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F.TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F.TF.C.8 (+) Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$ or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$ or $tan(\theta)$ and the quadrant of the angle.
F.TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
G.SRT.D.9 (+) Derive the formula $A = \frac{1}{2}absin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

INTERDISCIPLINARY CONNECTIONS

New Jersey Student Learning Standards- Science
HS-ETS1-1. Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.
HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3. Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.
HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

New Jersey Student Learning Standards-Language Arts (2023)
L.VL.11–12.3. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

RI.MF.11–12.6. Synthesize complex information across multiple sources and formats to develop ideas, resolve conflicting information, or develop an interpretation that goes beyond explicit text information (e.g., express a personal point of view, new interpretation of the concept).

W.WR.11–12.5. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

SL.PE.11–12.1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.UM.11–12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.

New Jersey Student Learning Standards: Career Readiness, Life Literacies and Key Skills (2020)

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.CT.4: Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes

9.4.12.DC.1: Explain the beneficial and harmful effects that intellectual property laws can have on the creation and sharing of content

9.4.12.GCA.1: Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

2020 New Jersey Student Learning Standards – Computer Science and Design Thinking

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
8.2.12.ED.1: Use research to design and create a product or system that addresses a problem and make modifications based on input from potential consumers.

21st CENTURY LIFE AND CAREER STANDARDS

Career Readiness, Life Literacies, and Key Skills Practices describe the habits of the mind that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success.

1. Act as a responsible and contributing community member and employee.
2. Attend to financial well-being.
3. Consider the environmental, social and economic impacts of decisions.
4. Demonstrate creativity and innovation.
5. Utilize critical thinking to make sense of problems and persevere in solving them.
6. Model integrity, ethical leadership and effective management.
7. Plan education and career paths aligned to personal goals.
8. Use technology to enhance productivity, increase collaboration and communicate effectively.
9. Work productively in teams while using cultural global competence.

9.1: Personal Financial Literacy

A. Civic Responsibility
B. Financial Institutions
C. Financial Psychology
D. Planning and Budgeting
E. Risk Management and Insurance
F. Civic Financial Responsibility
G. Credit Profile
H. Economic and Government Influences
I. Credit and Debt Management

9.2: Career Awareness, Exploration & Preparation, and Training

A. Career Awareness (K-2)
B. Career Awareness and Planning (3-5)
C. Career Awareness and Planning (6-8)
D. Career Awareness and Planning (9-12)

9.4 Life Literacies and Key Skills

A. Creativity and Innovation
B Critical Thinking and Problem-solving
C. Digital Citizenship
D. Global and Cultural Awareness
E. Information and Media Literacy
F. Technology Literacy

9.3: Career and Technical Education

A. Agriculture
B. Architecture
C. Arts,A/V, Technology
D. Business Management
E. Education
F. Finance
G. Government
H. Health Science
I. Hospital & Tourism
J. Human Services
K. Information Tech.
L. Law and Public Safety
M. Manufacturing
N. Marketing
O. Science, Technology, Engineering & Math
P. Trans./Logistics

TECHNOLOGY STANDARDS		
8.1: Computer Science A. Computing systems B. Networks and the Internet C. Impacts of Computing D. Data & Analysis E. Algorithms & Programming	**8.2 Design Thinking** A. Engineering Design B. Interaction of Technology and Humans C. Nature of Technology D. Effects of Technology on the Natural World E. Ethics & Culture	
ENDURING UNDERSTANDINGS	**ESSENTIAL QUESTIONS**	
• Find the Product and Quotient of Complex Numbers • Prove Identities using trigonometric identities • Solve problems using the law of sines and the law of cosines • Trigonometric functions model periodic phenomena and are essential in describing oscillatory motion, wave behavior, and cyclic patterns in nature and technology. • Trigonometric functions exhibit periodicity, symmetry, and specific ranges and domains, allowing for the analysis and prediction of behavior in various contexts. • Trigonometric identities and equations facilitate the simplification and manipulation of expressions, enabling the solution of equations and inequalities involving trigonometric functions. • The study of trigonometry fosters critical thinking skills by requiring students to analyze problems, formulate strategies, and apply trigonometric concepts to model and solve practical problems in diverse contexts.	• Do students understand the characteristics of the trigonometric functions and where they are used in the real world? • Are students able to represent trigonometric functions algebraically, graphically, and numerically? • Are students able to model situations from trigonometric problems and apply them to the world of physics? • To what extent can trigonometric functions model real-world phenomena, and what are the limitations of these models? • How does trigonometry contribute to understanding periodic behavior in nature, such as tides, seasons, and biological rhythms?	
STUDENT LEARNING OBJECTIVES (Students are learning to / Students are learning that)		
• Solve equations involving trigonometric functions.		

- Solve trigonometric equations over specified domains.
- Apply identities and algebraic techniques to solve more complex equations.
- Solve problems involving right triangles, such as finding side lengths and angles.
- Solve problems involving oblique triangles using the Law of Sines and the Law of Cosines.
- Apply trigonometric functions to model periodic phenomena, such as simple harmonic motion and waves.
- Define and understand inverse trigonometric functions.
- Graph and apply inverse trigonometric functions to solve equations involving trigonometric functions.

SUGGESTED ACTIVITIES

Math Starters! 2nd Edition
Desmos Graphing Calculator (desmos.com)
Graphing Whiteboards
Youtube demonstrations

- F.TF.B.7 (+) Solar Sign Curve **Climate**
- Assign each group a specific cultural or historical context to research (e.g., ancient Egypt, ancient Greece, medieval Islamic mathematics, Chinese trigonometry). **DEI**
- Encourage students to research how trigonometric concepts (like sine, cosine, tangent) were applied or discovered in that culture. **DEI**
- They should look for practical applications (e.g., astronomy, architecture, navigation) and theoretical developments. **DEI**

EVIDENCE OF LEARNING

Formative Assessments:	**Summative Assessment:**
Classroom Discussion Exit Slip Checklists Peer Assessment Vocabulary Quizzes Rubrics Participation and teacher observation Mini Whiteboard Responses Think-Pair-Share Concept Map Classroom Poll	Unit Tests End-of-Book Test NJGPA Test
Benchmark Assessment: Unit Benchmarks (3x/year)	**Alternative Assessments:** Project Portfolio

INSTRUCTIONAL RESOURCES

Core Instructional Resource:	**Teacher Created Materials:**	**Supplemental Resources:**
Demana, Waits, Foley, Kennedy Pre-Calculus-Graphical, Numerical, Algebraic	Warmups, Exit tickets Guided notes PowerPoints	**Math XL; Illustrative Mathematics; National Council of Teachers of Mathematics; Khan Academy; Desmos; Math Warehouse; Mathematics Assessment Resource Services (MARS); Learn Zillion; New Jersey Center for Teaching and Learning; Edulastic; Nearpod;Ed-Puzzles;Quizizz ;Kahoot**

INTEGRATED ACCOMMODATIONS AND MODIFICATIONS

Special Education:
Provide modified notes and access to extra copies online
Provide oral reminders and check student work during independent work time
Model skills/techniques to be mastered
Check and sign assignment planner
Preferential seating
Pair visual prompts with verbal presentations
Modified or scaffolded homework and classwork
Extended time as needed
Provide graphic organizers and study guides

English Learners:
Provide scaffolded assignments and assessments
Pair visual prompts with visual presentations
Check and sign assignment planner
Native Language translation (peer, online assistive technology, translation device, bilingual dictionary)
Extended time for assignment and assessment as needed
Highlight key vocabulary
Use graphic organizers
Provide verbal and written directions
Preferential seating with a English-speaking peer

At Risk of Failure:
Check and sign assignment planner
Encourage class participation and reinforce skills
Model skills and assignments
Extended to time to complete class work
Preferential seating
Provide extra help outside of class and 1:1 instruction when needed

Communicate regularly with students' other teachers
Provide positive feedback for tasks well done
Encourage student to proofread assessments and projects and ask for teacher proofreading of large writing assignments

Gifted and Talented:
Pose higher-level thinking questions
Provide higher level reading and writing materials for literacy based activities
Probe student to extend thinking beyond the text or connect two or more texts
Provide alternate or project-based assessments and assignments

Students with 504 Plans
Provide extended time as needed
Modify length of writing assignment
Provide short breaks within the lesson
Provide scaffolding for students
Utilize graphic organizers

UNIT 4
Systems of Inequalities, Analytic Geometry in Two and Three Dimensions, Discrete Mathematics
UNIT SUMMARY
The objective of this unit is to prepare students for the introduction of Calculus using the tangent problem. Students will also go into depth learning about conic sections, matrices, vectors, polar, and representing them numerically, algebraically, and graphically. In addition, students will have an introduction to discrete mathematics.
NEW JERSEY STUDENT LEARNING STANDARDS MATHEMATICS 2023
G.GPE.3 Translate between the geometric description and the equation for a conic section. G.GMD.2 Explain volume formulas and use them to solve problems S.MD.1,2,3,4 Calculate expected values and use them to solve problems S.MD. 5a, 5b Use probability to evaluate outcomes of decisions N.VM.A.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v,|v|,||v||,v). N.VM.A.2 (+) Find components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.A.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.B.4 (+) Add and subtract vectors. N.VM.B.4.a Add vectors end-to-end, component wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

N.VM.B.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
N.VM.B.4.c Understand vector subtraction v-w as v+(-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
N.VM.B.5 (+) Multiply a vector by a scalar.
N.VM.B.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c($v_x, v_y) = (cv_x, cv_y)$.
N.VM.B.5.b Compute the magnitude of a scalar multiple cv using ||cv||=|c|v. Compute the direction of cv knowing that when |c|v≠0, the direction of cv is either along v (for c>0) or against v (for c<0).
N.VM.C.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
N.VM.C.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
N.VM.C.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
N.VM.C.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N.VM.C.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.C.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.C.12 (+) Work with 2x2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
A.REI.C.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.
A.REI.C.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3x3 or greater).

INTERDISCIPLINARY CONNECTIONS

New Jersey Student Learning Standards- Science
HS-ETS1-1. Analyze a major global challenge to specify qualitative and quantitative criteria and constraints for solutions that account for societal needs and wants.
HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
HS-ETS1-3. Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.
HS-ETS1-4. Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

New Jersey Student Learning Standards-Language Arts (2023)

L.VL.11–12.3. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.

RI.MF.11–12.6. Synthesize complex information across multiple sources and formats to develop ideas, resolve conflicting information, or develop an interpretation that goes beyond explicit text information (e.g., express a personal point of view, new interpretation of the concept).

W.WR.11–12.5. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.

SL.PE.11–12.1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

SL.II.11–12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.PI.11–12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

SL.UM.11–12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.

New Jersey Student Learning Standards: Career Readiness, Life Literacies and Key Skills (2020)

9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas

9.4.12.CI.2: Identify career pathways that highlight personal talents, skills, and abilities

9.4.12.CT.1: Identify problem-solving strategies used in the development of an innovative product or practice

9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving

9.4.12.CT.4: Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes

9.4.12.DC.1: Explain the beneficial and harmful effects that intellectual property laws can have on the creation and sharing of content

9.4.12.GCA.1: Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others

9.4.12.TL.2: Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

2020 New Jersey Student Learning Standards – Computer Science and Design Thinking

8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.

8.2.12.ED.1: Use research to design and create a product or system that addresses a problem and make modifications based on input from potential consumers.

21st CENTURY LIFE AND CAREER STANDARDS

Career Readiness, Life Literacies, and Key Skills Practices describe the habits of the mind that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success.

1. Act as a responsible and contributing community member and employee.
2. Attend to financial well-being.
3. Consider the environmental, social and economic impacts of decisions.
4. Demonstrate creativity and innovation.
5. Utilize critical thinking to make sense of problems and persevere in solving them.
6. Model integrity, ethical leadership and effective management.
7. Plan education and career paths aligned to personal goals.
8. Use technology to enhance productivity, increase collaboration and communicate effectively.
9. Work productively in teams while using cultural global competence.

9.1: Personal Financial Literacy

S. Civic Responsibility
T. Financial Institutions
U. Financial Psychology
V. Planning and Budgeting
W. Risk Management and Insurance
X. Civic Financial Responsibility
Y. Credit Profile
Z. Economic and Government Influences
AA. Credit and Debt Management

9.2: Career Awareness, Exploration & Preparation, and Training

M. Career Awareness (K-2)
N. Career Awareness and Planning (3-5)
O. Career Awareness and Planning (6-8)
P. Career Awareness and Planning (9-12)

9.4 Life Literacies and Key Skills

A. Creativity and Innovation
B Critical Thinking and Problem-solving
C. Digital Citizenship
D. Global and Cultural Awareness

9.3: Career and Technical Education

GG. Agriculture
HH. Architecture
II. Arts,A/V, Technology
JJ. Business Management
KK. Education
LL. Finance
MM. Government
NN. Health Science
OO. Hospital & Tourism
PP. Human Services
QQ. Information Tech.
RR. Law and Public Safety
SS. Manufacturing
TT. Marketing
UU. Science, Technology, Engineering & Math
VV. Trans./Logistics

	Q. Information and Media Literacy R. Technology Literacy	

TECHNOLOGY STANDARDS	
8.1: Computer Science A. Computing systems B. Networks and the Internet C. Impacts of Computing D. Data & Analysis E. Algorithms & Programming	**8.2 Design Thinking** A. Engineering Design B. Interaction of Technology and Humans C. Nature of Technology D. Effects of Technology on the Natural World E. Ethics & Culture
ENDURING UNDERSTANDINGS	**ESSENTIAL QUESTIONS**
● Use the head-minus tail rule for vectors ● Resolve a Vector ● Computing a derivative from data ● Finding a probability of an event ● Multiplication Principle of Probability ● Conditional Probability Formula ● Finding the Sum of a Geometric and Arithmetic Sequence ● How to sketch ellipses, parabolas, circles, and hyperbolas ● Solving systems of equations/inequalities algebraically	● Do students understand how to represent conic sections graphically and apply them in real-life situations? ● Do students understand the basic properties of derivatives, integrals, and limits? ● Are students able to model situations from situations involving discrete mathematics?

STUDENT LEARNING OBJECTIVES (Students are learning to / Students are learning that)

- Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
- Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- Solve problems involving velocity and other quantities that can be represented by vectors.
- Add, subtract, and multiply matrices of appropriate dimensions.
- Represent a system of linear equations as a single matrix equation in a vector variable.
- Translate between the geometric description and the equation for a conic section.
- Explain volume formulas and use them to solve problems
- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

SUGGESTED ACTIVITIES

Math Starters! 2nd Edition
Desmos Graphing Calculator (desmos.com)
Graphing Whiteboards
Youtube demonstrations

- Assign each group a specific cultural or practical context to research (e.g., indigenous navigation techniques, ancient astronomy, artistic representations, modern technological applications). Encourage students to investigate how polar coordinates and vectors are used or represented in their assigned context. **DEI**
- They should look for practical applications, historical developments, and any cultural significance related to these mathematical concepts. **DEI**
- N.VM.A.3 (+) Hurricane Direction Shift **Climate**

EVIDENCE OF LEARNING

Formative Assessments:	**Summative Assessment:**
Classroom Discussion Exit Slip Checklists Peer Assessment Vocabulary Quizzes Rubrics Participation and teacher observation Mini Whiteboard Responses Think-Pair-Share Concept Map Classroom Poll	Unit Tests End-of-Book Test NJGPA Test
Benchmark Assessment: Unit Benchmarks (3x/year)	**Alternative Assessments:** Project Portfolio

INSTRUCTIONAL RESOURCES

Core Instructional Resource:	**Teacher Created Materials:**	**Supplemental Resources:**
Pre-Calculus- Graphical, Numerical, Algebraic By: Demana, Waits, Foley, and Kennedy	Warmups, Exit tickets Guided notes PowerPoints	**Math XL; Illustrative Mathematics; National Council of Teachers of Mathematics; Khan Academy; Desmos; Math Warehouse; Mathematics Assessment Resource Services (MARS); Learn Zillion; New Jersey Center for Teaching and Learning; Edulastic; Nearpod;Ed-Puzzles;Quizizz**

		;Kahoot

INTEGRATED ACCOMMODATIONS AND MODIFICATIONS

Special Education:
Provide modified notes and access to extra copies online
Provide oral reminders and check student work during independent work time
Model skills/techniques to be mastered
Check and sign assignment planner
Preferential seating
Pair visual prompts with verbal presentations
Modified or scaffolded homework and classwork
Extended time as needed
Provide graphic organizers and study guides

English Learners:
Provide scaffolded assignments and assessments
Pair visual prompts with visual presentations
Check and sign assignment planner
Native Language translation (peer, online assistive technology, translation device, bilingual dictionary)
Extended time for assignment and assessment as needed
Highlight key vocabulary
Use graphic organizers
Provide verbal and written directions
Preferential seating with a English-speaking peer

At Risk of Failure:
Check and sign assignment planner
Encourage class participation and reinforce skills
Model skills and assignments
Extended to time to complete class work
Preferential seating
Provide extra help outside of class and 1:1 instruction when needed
Communicate regularly with students' other teachers
Provide positive feedback for tasks well done
Encourage student to proofread assessments and projects and ask for teacher proofreading of large writing assignments

Gifted and Talented:
Pose higher-level thinking questions
Provide higher level reading and writing materials for literacy based activities
Probe student to extend thinking beyond the text or connect two or more texts
Provide alternate or project-based assessments and assignments

Students with 504 Plans
Provide extended time as needed

Modify length of writing assignment
Provide short breaks within the lesson
Provide scaffolding for students
Utilize graphic organizers