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B. Sc. (Hons Maths) (Semester – 1st)

ANALYSIS-I

Subject Code: BMATS1103

Paper ID: 19131203

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a. State Archimedean property of real numbers.
- b. State Bolzano-Weierstrass theorem.
- c. Discuss the convergence of the series $\sum \frac{1}{(n+1)^2}$
- d. Discuss the convergence nature of the sequence $\left\{ \frac{1}{n2^n} \right\}$
- e. Show that the function defined by
 $f(x) = \begin{cases} x \sin \frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$ is continuous at $x = 0$.
- f. State alternating series test for an infinite series.
- g. Define Cauchy sequence.
- h. State Darboux's theorem.
- i. State Cauchy convergence criterion.
- j. Evaluate:

$$\frac{xe^x - \log \log (1+x)}{x}$$

Section – B

(5 marks each)

Q2. Show that the sequence $\left\{ \frac{3n-1}{4n+5} \right\}$ converges to $\frac{3}{4}$.

Q3. Discuss the convergence of the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$$

Q4. Show that a function which is continuous on a closed interval is also uniformly continuous on that interval.

Q5. Examine the validity of the hypothesis and conclusion of Rolle's theorem for $f(x)$ on $[0, 2]$, where

$$f(x) = 1 - (x - 1)^{\frac{2}{3}}$$

Q6. Test the convergence of the series

$$\sum \frac{n^2-1}{n^2+1} x^n, \quad x > 0$$

Section – C

(10 marks each)

Q7. State and prove Bolzano – Weierstrass theorem.

Q8. Show by applying Cauchy convergence criterion that the sequence $\{a_n\}$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n+1} \text{ is not convergent.}$$

Q9. Discuss the convergence of the series:

$$\sum \frac{(2n)!}{n!n!}$$