Chapter Two

Theoretical Analysis

2.1. Introduction

In this experiment, a hydromechanical system is designed and used to study the amount of waste kinetic energy that could be captured and stored during stopping a mechanical moving part at different speeds and conditions, the hydromechanical system consists of a driving system and a braking system. Fig.2.1 . Shows a schematic for the hydromechanical system used in this experiment.

An electric motor equipped with a torque measuring device is used to drive the system.

The power is transmitted from the motor to the flywheel through another torque measuring device, which measures the torque right before the flywheel.

The braking system consists of a gear pump connected to an electromagnetic clutch which connects the pump to the flywheel when the current delivered to the motor is off.

A flow control valve restricts the oil delivered by the pump; the oil gets stored in a bladder type accumulator.

A pressure gauge is used to record the pressure of the accumulator, the pressure of the accumulator can be controlled using the control valve.

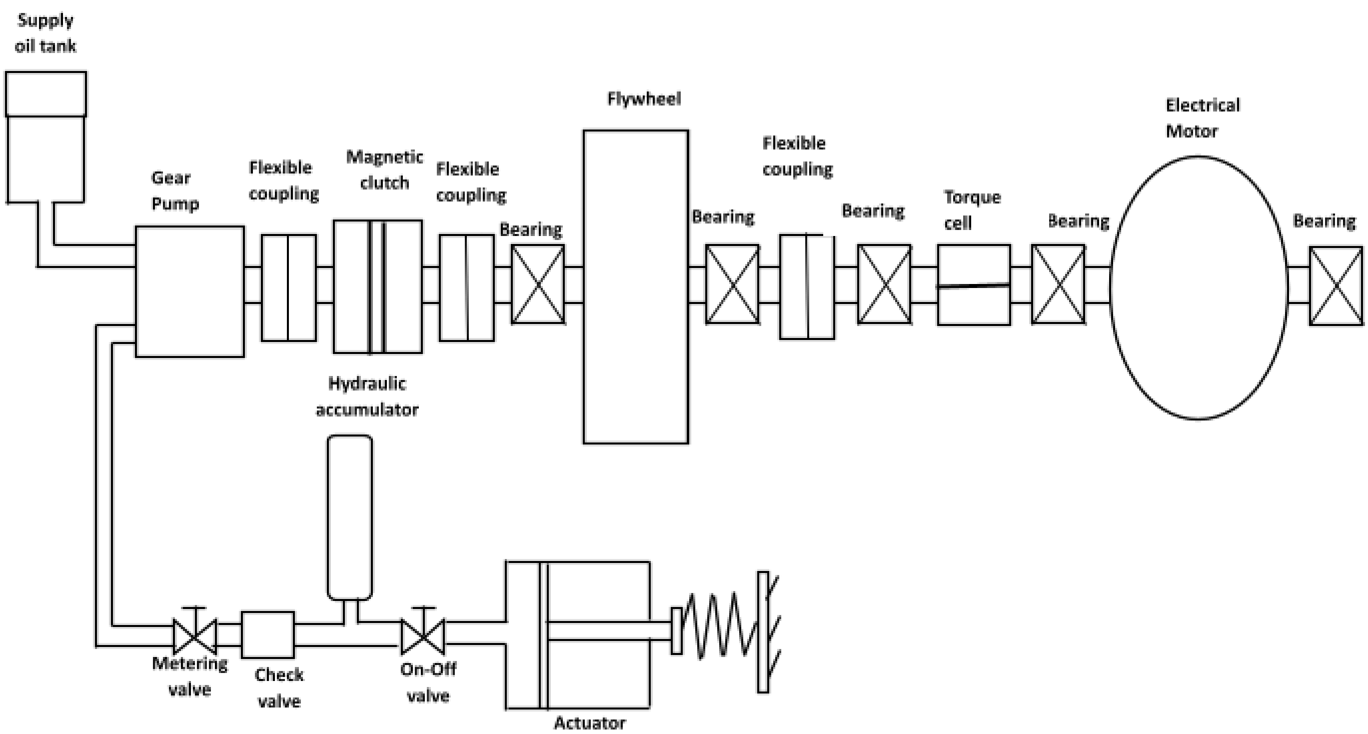

Figure (2.1) a schematic for the hydro-mechanical system.

2.2. Mechanical system

Fig. 2.2 depicts the entire system, which includes the driving parts (motor, torque measurement device, and flywheel), the gear pump, and the electromagnetic clutch.

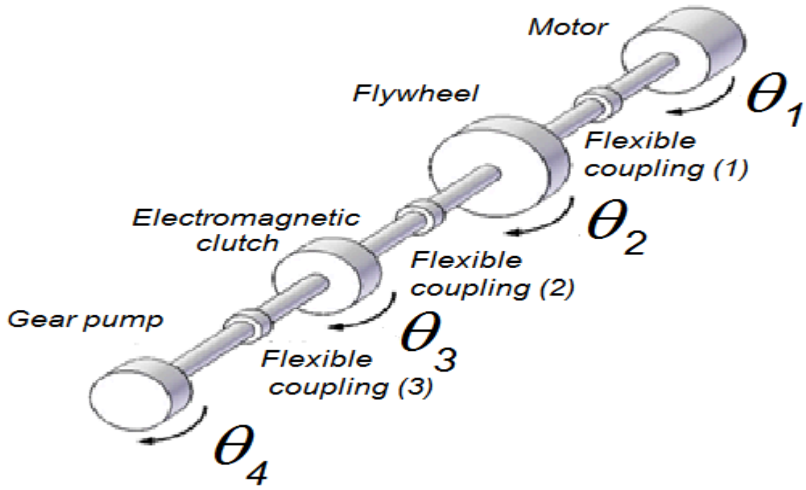

Figure (2.2). The complete mechanical rotational system.

2.2.1 Condition 1 (Electromagnetic clutch disengaged)

This is the driving mechanical system consists of the electric motor, the measuring torque device and the flywheel. The electric motor is linked to a torque measurement device and sends power to the flywheel, which stores kinetic energy. The system achieves steady state with constant angular velocity ($\dot{\Theta}$). The equations of motion are

$$T(t)+k_2(\theta_{o2}-\theta_{o1})=J_m\ddot{\theta}_{o1}$$
$$and,$$
$$-k_2(\theta_{o2}-\theta_{o1})=J_f\,\ddot{\theta}_{o2}$$

These equations may be derived to be as follows,

$$\dot{\theta}_{o1}=\omega\left(1-e^{\frac{-t}{t_o}}\right) \qquad (1)$$

$$and,$$

$$\ddot{\theta}_{o1}=\frac{-t_o\,T(t)}{\left(J_f+J_m\right)}\left(t+t_o(e^{\frac{-t}{t_o}})-t_o\right) \qquad (2)$$

The initial conditions for Eqns. 1 and 2 are as follows:

$$At\ t=0,\ \theta_{o1}=\theta_{o2}=\dot{\theta}_{o1}=\dot{\theta}_{o2}=0$$

To be solved θ_{o1} and θ_{o2} in the domain $t=0\rightarrow t_o$ where $\theta_{o1}=\theta_{o2}$ and $\dot{\theta}_{o1}=\dot{\theta}_{o2}$.

2.2.2 Conditions 2 and 3 (Electromagnetic clutch activated)

When the electric motor's electric current is cut off (braking condition), the electromagnetic clutch connects the driving mechanical system to the gear pump (braking mechanism), and the driving mechanical system and the gear pump slow down to a stop. The equations of motion are

$$k_2(\theta_2-\theta_1)=J_M\ddot{\theta}_1 \qquad (3)$$

$$-k_2(\theta_2-\theta_1)+k_3(\theta_3-\theta_2)=J_f\,\ddot{\theta}_2 \qquad (4)$$

And,

$$-k_3(\theta_3-\theta_2)+k_4(\theta_4-\theta_3)=J_c\,\ddot{\theta}_3 \qquad (5)$$

The initial conditions for Eqns. 3, 4 and 5 are as follows:

$$\begin{aligned} At\ t=t_o:\ &\theta_1=\theta_{o1}\ ,\ \theta_2=\theta_{o2}\ ,\ \theta_3=0\ ,\ \theta_4=0\\ &\dot{\theta}_1=\dot{\theta}_{o1}\ ,\ \dot{\theta}_2=\dot{\theta}_{o2}\ ,\ \dot{\theta}_3=0\ ,\ \dot{\theta}_4=0\end{aligned}$$

2.3 Hydraulic system

Fig. 2.3 depicts the hydraulic system employed in this investigation. The pump draws oil from the tank and pumps it via the metering valve, which controls the flow rate (Q). The oil goes through the check valve, which stops the oil from flowing backward and, as a result, prevents back pressure on the pump. When the ON/OFF valve is closed, the oil gets stored in the receiver where its pressure keeps increasing until it reaches the desired pressure. When the ON/OFF valve is closed, the oil flows from the receiver to the actuator where the stored energy is used, and force is applied on the load.

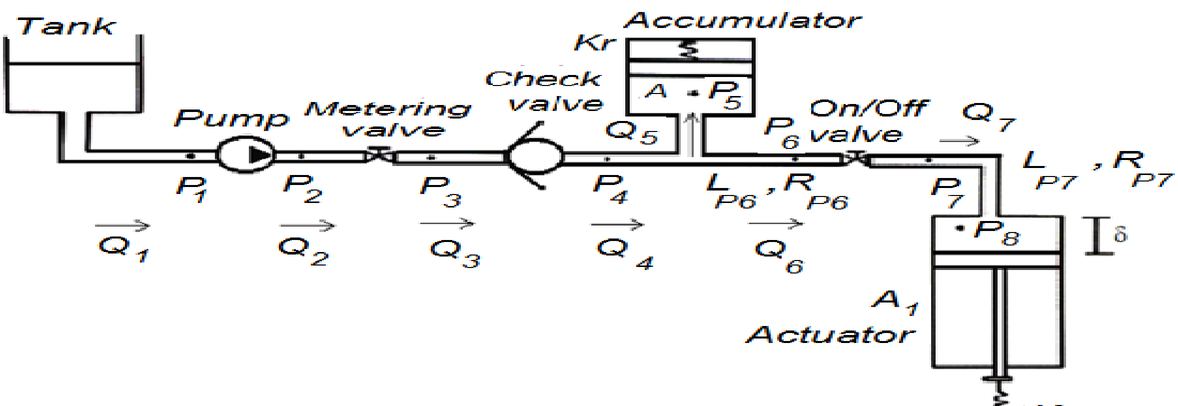

Figure (2.3) the hydraulic system.

The flow is considered as incompressible and isothermal fluid flow and the governing equations depend on the dynamic behavior of the fluid flow through the different elements of the braking mechanism according to the operation conditions.

(i) **The governing equations for the case of electromagnetic clutch is activated and the on/off valve is closed:**

$$Pump: J_p \ddot{\theta}_4 = \left(\frac{1}{\eta_o \dot{\theta}_4}\right) Q_2 \left(p_2 - p_1\right) - k_4\left(\theta_4 - \theta_3\right) \qquad (6)$$

$$Metering\ valve: \ Q_3 = \frac{1}{k_m}\sqrt{\left|p_2 - p_3\right|} \qquad (7)$$

$$Check\ valve: \ Q_4 = \frac{1}{k_c}\left(\left|p_3 - p_4\right|\right) \qquad (8)$$

$$Accumulator: Q_5 = C_5\left(\dot{p}_4 - \dot{p}_5\right) \qquad (9)$$

Where,

$$Q_3 = Q_4 = Q_5 \ and \ C_5 = \frac{A^2}{k_r}$$

(ii) **The governing equations for the case of electromagnetic clutch is activated and the on/off valve is opened:**

$$Actuator: \ddot{\delta} = \left(p_8 - p_{atm}\right)\frac{A_1}{m} - \frac{k_1}{m} \qquad (10)$$

Q_5 May be calculated as

$$Q_5 = A_1 \times \delta \qquad (11)$$

$$Pipe(6): \ p_6 = p_4 - L_{P6}\dot{Q}_6 - R_{P6}Q_6 \qquad (12)$$

$$Pipe(7): \ p_7 = p_6 - \left(Q_5 k_v\right)^2 \qquad (13)$$

And,

$$p_8 = p_7 - \left(L_{P7}\dot{Q}_7 + R_{P7}Q_7\right) \qquad (14)$$

Where,

$$Q_6 = \frac{1}{k_v}\sqrt{\left|p_6 - p_7\right|} \qquad (on / off\ \ valve)$$

$$Q_5 = Q_6 = Q_7$$

With $$p_5 = p_4 \tag{15}$$