

Ancient Music of 22 Tones conforms to Archyta's Doctrine of Means

Background Information

It is conventional to describe the Middle Octave of music as lying between '1' and '2'. The Higher Octaves lie between '2' to '4', '4' to '8', '8' to '16' and so on. Similarly, the Lower Octaves (below tonic) lie between '1/2' to '1', '1/4' to '1/2', '1/8' to '1/4', '1/16' to '1/8' and so on. We may describe these as the '**natural nodes**' within the overall audio spectrum.

Let us analyze the **intra-spectral space** within an octave. Are there similar '**nodes**' ordained by Mother Nature? Archytas of ancient Greece (*a visionary of his time!*) intuited on such a phenomenon as: "*the power which **tends towards the middle (Centre)**, . . . [in particular, by the **Arithmetic and Harmonic Means**], . . . in the middle of 6 to 12, the ratios 2 : 3 and 3 : 4 come together*". Sounds like a jargon!

During the early Greek era, Pythagoras seems to have made the path-breaking attempt towards emphasizing this concept of "primacy of Means" within the octave: '3/2' [*Arithmetic Mean (AM)*] and '4/3' [*Harmonic Mean (HM)*]; truly masterpieces! However, it appears that this doctrine had not been adequately exploited by the later Pythagoreans. For reasons not known, they had chosen to deviate from the path of Pythagoras and Archytas by following the route of 'cycle of Fifths and Fourths' for postulating the octave tuning theory!

I have made a humble effort **towards extrapolating this important** Doctrine of 'Means'. All possible outcomes of 'Means' (*both Arithmetic and Harmonic*), in a **successive manner**, have been listed. The 'Means' fractions so evolved have been assigned 'group identities' as per prevailing universal convention of the 'Holy Seven-ness of Notes' (*i.e. S, R, G, M, P, D, N, S' of Indian music or C, D, E, F, G, A, B, C' of Western music*) in an octave. The Reader **may please note** that the tonal variants (*i.e. the 'flats' and the 'sharps' of various tones*) so evolved often lie outside the familiar tonal framework of the 12 Tones ET (*Equal Temperament*) system. These are, therefore, being differently identified, in terms of Indian sol-fa notation, with numerical suffixes such as R1, R2, ... G1, G2, ... M1, M2 etc. (*For the benefit of the Western Readers: Sadjā (S) denotes the 'Tonic', Rishabha (R) denotes the 'Seconds', Gandhara (G) denotes the 'Thirds', Madhyama (M) denotes the 'Fourths', Panchama (P) denotes the 'Perfect Fifth', Dhaivata (D) denotes the 'Sixths' and Nishada (N) denotes the 'Sevenths'*).

Figure-1 records the details of progressions of AM and HM, as we move towards the '**Centre**', from the Tonic Sadjā i.e., '1' and octave Sadjā, i.e., '2'.

For example, Arithmetic Mean between '1' and '2' is calculated by the formula: $AM = (a + b) \div 2$ (*where 'a' = '1' and 'b' = '2'*). Therefore, we get $AM = (1 + 2) \div 2$; **AM = 3/2** (i.e. 'P'). Similarly, HM is calculated by the formula: $HM = 2ab \div (a + b)$; by substituting values we get, $HM = 2 * 1 * 2 \div (1 + 2)$; i.e., **HM = 4/3** (i.e. 'M1'). This is illustrated below:

FIGURE-1

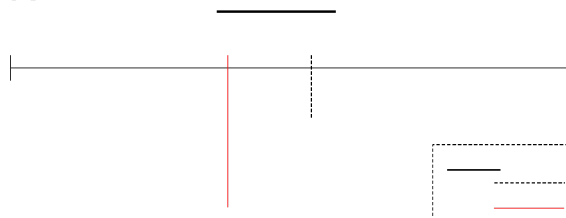


Figure-2. We continue with this thread further towards the next generation of ‘means’: To evaluate the AM between $P = '3/2'$ and octave Sadjja = ‘2’, we apply the formula once again: $AM = (a + b) \div 2$ (where ‘a’ = ‘3/2’ and ‘b’ = ‘2’). Therefore, we get, $AM = (3/2 + 2) \div 2$; i.e., **AM = 7/4** (i.e. N2). Similarly, $HM = 2ab \div (a + b)$, (where ‘a’ = ‘3/2’ and ‘b’ = ‘2’). Therefore, $HM = (2 * 3/2 * 2) \div (3/2 + 2)$; i.e., **HM = 12/7**, (i.e., N1). See the sketch given below: -

FIGURE-2

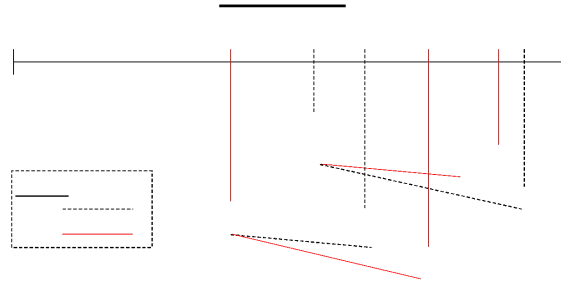
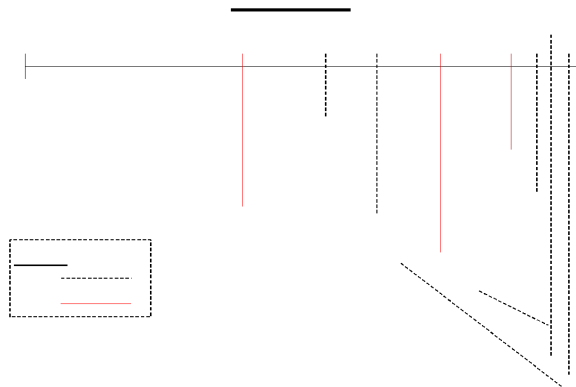


Figure – 3. In this manner, we should continue to evaluate the successive generations of AM and HM for each product. This is illustrated in Figure-3: -

FIGURE-3



Figures-4&5. lists the progressions of AM and HM, as we move towards the ‘Centre’ from the lower octave end, i.e., ‘1’.

FIGURE-4

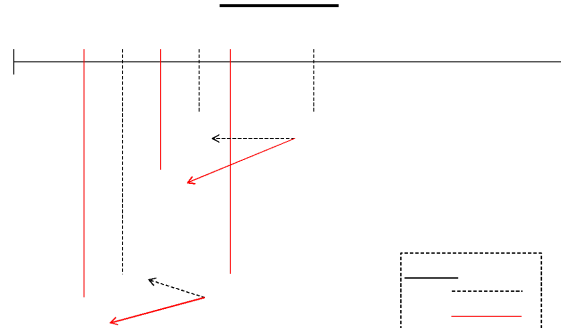
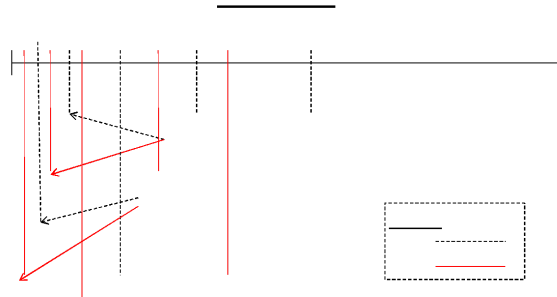


FIGURE-5



Figures – 6&7 give the AM and HM for some more ‘nodes’ that are latent in the above processes (i.e., G2, G4, M2, M3, M4, D1). However, these will surface as we continue to process all the AMs and HMs for the second and third generation products obtained in Figures 1 and 2. *(We should appreciate that each AM and HM, in their own turns, have the option to be processed equally along both directions; i.e. from the higher end of octave as well as from the lower end of the octave):*

FIGURE- 6

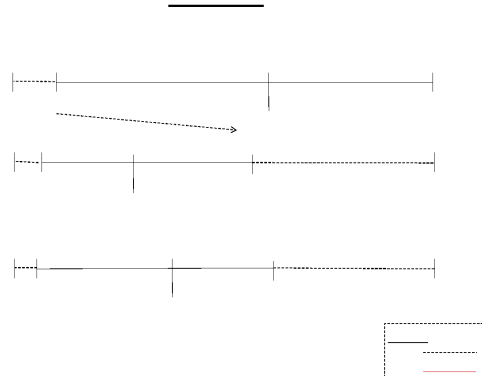
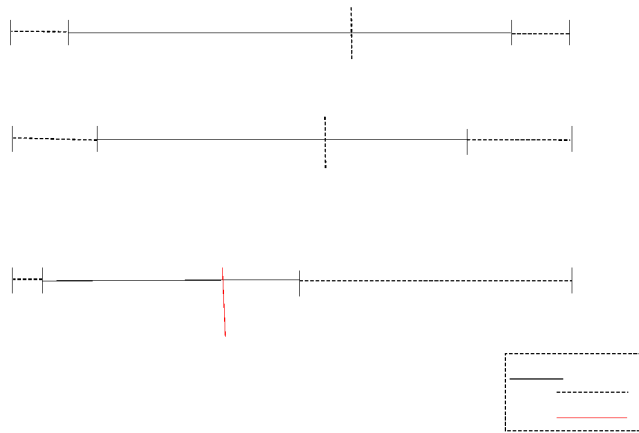


FIGURE- 7



The following are the highlights:

- Within the octave, as we proceed downwards, i.e., from '2' (i.e., *Octave Sadjja*) to '1' (*Tonic Sadjja*), the progression of the 'Means' follow a clear hierarchical pattern. The series begins with the first set of 'Means', i.e., the perfect 'fifth' ($3/2$) and the musical 'fourth' ($4/3$), that may be described as the 'first generation' products (Figure – 1).
- The second generation of 'Means', in turn, gives rise to four more 'Means' (Figure- 2): one set comprises of one variant of the seventh ($7/4$) and another variant of the seventh ($12/7$) as evolved from the 'fifth' (i.e. $3/2$); another set comprises of the two variants of 'sixths' $5/3$ and $8/5$ as evolved from the 'fourth' (i.e. $4/3$).
- The third generation of 'Means' effectively amounts to two: Two more variants of sevenths: N4 (i.e., $11/6$) and N3 (i.e., $8/5$). (See Figure – 3). We may progress further with the fourth generation of 'Means' in a similar manner and so on. How many such 'generations' of 'Means' exist? There seems to be an 'Infinity' over there!
- Similarly, within the octave, as we proceed upward from '1' towards the centre, the progression of the 'Means' reveals the presence of another hierarchical pattern. The first generations of 'Means' are again only two: i.e., the 'fifth' ($3/2$) and the 'fourth' ($4/3$). The second generation of 'Means', in turn, gives rise to four more 'Means': the two 'thirds' $5/4$ and $6/5$ as evolved from the 'fifth' ($3/2$) and the two 'seconds' $8/7$ and $7/6$ as evolved from the 'fourth' ($4/3$). (See Figure – 4). The third generation of 'Means' effectively sum up to four: i.e., the variants of 'seconds': fractions $9/8$ and $10/9$ (as evolved from the 'major third' $5/4$), and fractions $11/10$ and $12/11$ (as evolved from the 'minor third' $6/5$). (See Figure – 5). We may progress this to the fourth generation of 'Means' and further. How many such 'generations' of 'Means' exist in this approach? There is another 'Infinity' here too!

Now the next question is how many “generations” of ‘Means’ are ‘necessary’ as well as ‘sufficient’ for actualizing ‘melody’ in music? Here comes Plato’s concern for ‘self-limitation’ in the interest of containing this ‘**threatening Infinity**’! How do we determine this ‘self-limiting’ factor?

A “self-limit” placed at the integer value of ‘12’ appears to be most optimal for following reasons: -

- Firstly, the hierarchical compulsions. Within the octave, as we proceed downwards from ‘2’ to ‘1’, the octave splits into the ‘fifth’ and the ‘fourth’, by way of the first-generation products of ‘Means’. As we proceed further to evolve the second-generation products, the ‘fifth’ splits itself into the major ‘third’ and the minor ‘third’. Correspondingly, the ‘fourth’ splits itself into two variants of ‘seconds’ (i.e., ‘7/6’ and ‘8/7’). For evolving the third-generation products, the major ‘third’ splits itself into the two variants of ‘seconds’ (i.e., ‘9/8’ and ‘10/9’) and the minor ‘third’ splits itself into the two variants of ‘seconds’ (i.e., ‘11/10’ and ‘12/11’). In this manner, the guiding rule seems to be: *“whenever a tone splits itself into its constituent ‘Means’, the latter recedes to a hierarchical status below that of the former”*. In the ambience of this Rule, let us examine further. The next fraction in the queue that has become due for ‘splitting’ happens to be ‘7/6’ which is a variant of the ‘second’. But there is no hierarchy below the ‘seconds’ within an octave for accommodating the ‘Means’ products of the ‘Seconds’. Thus, we have reached a self-limiting condition.
- The six variants of the ‘seconds’ evolved through this ‘Means Doctrine’ of Archytas [i.e., ‘12/11’ (151 cents), ‘11/10’ (165 cents), ‘10/9’ (182 cents), ‘9/8’ (204 cents), ‘8/7’ (231 cents) and ‘7/6’ (267 cents)] have another unique feature! They act as ‘atomic intervals’ in deciding the hierarchy of any Note within the octave. As per this Doctrine, every Tone in the octave is resolvable in terms of the ‘atomic intervals’ of the ‘seconds.’ For example:
 - The minor ‘third’ (‘6/5’, 316 cents) is ‘12/11’ * ‘11/10’; i.e., 151 + 165 = 316 cents.
 - The major ‘third’ (‘5/4’, 386 cents) is ‘10/9’ * ‘9/8’; i.e., 182 + 204 = 386 cents.
 - The perfect ‘fourth’ (‘4/3’, 498 cents) is ‘8/7’ * ‘7/6’; i.e., 231 + 267 = 498 cents).
 - The perfect ‘fifth’ (‘3/2’, 702 cents) is ‘9/8’ * ‘8/7’ * ‘7/6’; i.e., 204 + 231 + 267 = 702 cents.
 - The major ‘sixth’ (‘5/3’, 884 cents) is ‘10/9’ * ‘9/8’ * ‘8/7’ * 7/6; i.e., 182 + 204 + 231 + 267 = 884 cents.
 - The major ‘seventh’ (‘11/6’, 1049 cents) is ‘11/10’ * ‘10/9’ * ‘9/8’ * 8/7 * 7/6; i.e., 165 + 182 + 204 + 231 + 267 = 1049 cents.
 - The octave C1200 (2/1) (cents) is 12/11 * 11/10 * 10/9 * 9/8 * 8/7 * 7/6; i.e., 151 + 165 + 182 + 204 + 231 + 267 = 1200 cents.
 - While reaching octave C’, we realize that we have arrived at the limiting condition for two reasons: firstly, the resources of all the six ‘seconds’ have been fully used up; secondly, the entire octave spectral length of 1200 cents get accounted for. We have to, therefore, infer that the zone of

‘Seconds’ evolved by the Means Doctrine, can accommodate only six ‘Means’ and no more. This is another self-limiting condition.

- Let us hypothetically create a hierarchy for the ‘infra-seconds’ in the “No man’s land” that separates the tonic from the ‘Seconds’. This zone would now serve as the ‘parking space’ for accommodating the set of 12 HM and AM fractions, begotten from the ‘seconds’: i.e. the fractions ‘13/12’ (139 cents), ‘14/13’ (128 cents) (*evolved by splitting the ‘second’ 7/6*), ‘15/14’ (119 cents), ‘16/15’ (112 cents) (*evolved by splitting the ‘second’ 8/7*), ‘17/16’ (105 cents), ‘18/17’ (99 cents) (*evolved by splitting the ‘second’ 9/8*), ‘19/18’ (94 cents), ‘20/19’ (89 cents) (*evolved by splitting the ‘second’ 10/9*), ‘21/20’ (85 cents), ‘22/21’ (81 cents) (*evolved by splitting the ‘second’ 11/10*), ‘23/22’ (77 cents), and ‘24/23’ (74 cents) (*evolved by splitting the ‘second’ 12/11*). Again this ‘parking space’ labeled as ‘infra-seconds’ is saturated. How do we say that? The confirmation is done by adding the cent values of all these 12 members and we arrive at the total of 1200 cents (i.e., the total octave spectral space). We can theoretically extend this exercise by creating the next hierarchy of say, ‘infra-infra-seconds’ and accommodate another 24 ‘Means’. It can be further continued with the next hierarchy of ‘infra-infra-infra-seconds’ with 48 ‘Means’ and so on. But what do we observe while undertaking such a laborious exercise? The AMs and HMs start bunching together and also there is hardly any tangible difference in the value of a fraction and its AM and HM! ***There is total blur. Also, there is heavy clutter occurring in the “No man’s land”!*** When we commenced with the octave, the AM (‘3/2’) and HM (‘4/3’) were located widely apart. These added musical values within the octave. The second and third generations derived from the fractions ‘3/2’ and ‘4/3’ have also created good value for the musical ear, by giving rise to the ‘thirds’ and ‘Seconds’. But once we cross this delicate threshold of the ‘seconds’, the system collapses into a major blur, fit to be only discarded! This is the status of the progression of ‘Means’ as we continue to move downwards from ‘2’ to ‘1’ within the octave.

What about the status of the progression of ‘Means’ as one proceeds upwards from ‘1’ to ‘2’ within the octave? Here also, we find that the first, second and third generation of ‘Means’ do add significant “musical values” by giving rise to the ‘sixths’ and the ‘Sevenths’. Continuing the progression beyond the ‘sevenths’ once again poses the similar cultural dilemma by way of creation of more hierarchical zones, in the form of, say, the ‘ultra-sevenths’, ‘ultra-ultra-sevenths’, ‘ultra-ultra-ultra-sevenths’ and so on. Such zones of ‘Means’ do create similar clutters in the “No man’s land” that separates the ‘sevenths’ from the octave upper boundary! Earnest McClain’s phraseology (in his Paper titled: ‘On the possible origins of the 22 sruti system of Hindu musicology’) serves as an apt quote in this context: **“sins of the fathers accumulate to the third and fourth generations”**! Therefore, there is a case to do away with the further progression of ‘Means’ beyond the limits of ‘Sevenths’.

22 fractions which are evolved based on Archytas Doctrine of ‘Means’ are unique as they belong to a family comprising of the ‘simplest’ of fractions and it is not

mathematically feasible to generate a similar family of fractions simpler than these. In all these fractions, Plato's "self-limiting" factor (set at the limit of '12') may be found incorporated by way of design; i.e., the integer '12' is never exceeded either in the numerator or in the denominator.

These are the same 22 'god-fractions' of the Sumerian Pantheon as brought out earlier in my blog: 'Gods of Sumeria symbolize Ancient Music' (<http://sumeriangods.blogspot.com>). The relevance of Archytas 'Means' Doctrine has now come out as a new 'construct' to reinforce my earlier contention that our Ancient Music was based on 22 'simple fractions'.

These are also the same fractions which are identifiable through the ancient Indian musicological doctrines of Sadjagraha, Madhyama-graha and Murchanas. Please see my Blog: <http://synopsis-nambirajan.blogspot.com>.

In my view, identification of all logical nodes within an octave is an 'a priori' requirement, before one ventures into the task of carving out a tuning theory for the octave. In this important aspect, the early Greeks had erred; they seemed to have attempted to postulate a tuning theory, in great haste, immediately after the identification of the first generation 'Means' of the octave (i.e., the Perfect 'fifth' and the 'fourth') and even without waiting to explore the Pythagoras/Archytas doctrine to the fullest extent! If only had they shown greater restraint in their pursuit of their valuable doctrines, they might not have missed the opportunity of uncovering our 'Ancient Music of 22 Tones', long back!

Please visit my web site: <http://www.22sruti.com> for more details about my work on "Ancient Music of 22 Tones".

May I solicit comments from the Viewers please? My contact tele: 91 9673998967, 98501 21834. My e-mail id: snnambirajan@rediffmail.com. My postal address: Srinivasan Nambirajan, A-7/ 103, Florida Estate, Keshav Nagar, Mundhwa, Pune-411036.

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