

Rational Functions

Let $f(x) = a + \frac{9}{x-b}$.

The line $x = 3$ is a vertical asymptote to the graph of f .

a Write down the value of b .

The graph of f has a y -intercept at $(0, 4)$.

b Find the value of a .

c Write down the equation of the horizontal asymptote of the graph of f .

An Internet security service charges a 20 AUD set-up fee and then a monthly charge of 10 AUD.

a Construct a model to show the average monthly cost.

b Sketch a graph of the model.

c Use your model to find how long you have to use the service to average 15 AUD per month.

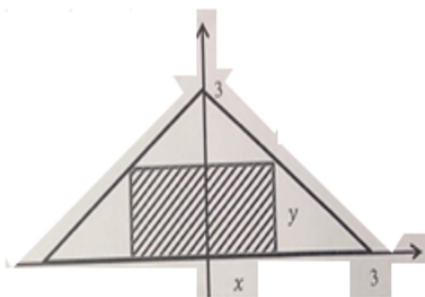
d Tim uses the service for many years at the same price. What will his average cost per month get closer and closer to?

Quadratic Equations

Find all real solutions of the equation $2m^4 - 5m^2 + 2 = 0$.

Find all solutions, expressed exactly, to the equation $w^{\frac{1}{2}} = 4w^{\frac{1}{4}} - 2$

Find the maximum area of the shaded rectangle.



Applied Quadratics

The main span of the Clifton Suspension Bridge in Bristol, UK is about 194 m long. The height of the towers is 26.2 m. The supporting cable for the main span of the bridge touches the road deck at the centre of the span.

- 1 The parabola shown in the graph models the supporting cable of the main span of the Clifton Suspension Bridge and the green line segments represent the towers.



Write down the values of r , s and t .

- 2 Consider the three forms of the equation of a quadratic function you have studied. For each of these forms the values of some of the parameters are equal to r , s or t .

$$y = a(x - h)^2 + k \quad y = ax^2 + bx + c \quad y = a(x - p)(x - q)$$

Replace the appropriate parameters in each equation with the values of r , s and t .

- 3 For which two of the three forms of quadratics from 2 would it be easier to find the other missing parameters? Find the missing parameter in each of these two. Then, convert both models to **general form** in order to show that they are equivalent.

- 4 Find the height of the supporting cable from the road deck at a point 24 m from a tower.

Suppose there are vertical cables with length 2.75 m, attaching the supporting cable to the road deck. Find the distance of these cables from the left-hand tower.

You have created a TikTok channel, filming all videos in the school bathroom. On your channel, you sell premium hair gel. At 14 euro per package, you sell 23 units per day. Based on some research, you estimate that for every price increase of 3 euro, you will sell 2 fewer packages.

- a) If x represents the number of price increases, write an expression for price after x price increases.
- b) Write an expression for the number of units sold following x increases in price.
- c) Show that the revenue as a function of price increases is given by $R(x) = -8x^2 + 160x + 1792$ $R(x) = -6x^2 + 41x + 322$.
- d) Use your model to:
- State the y-intercept. Describe what this point represents.
 - Find the vertex. Describe what this point represents.
 - Find the x-intercepts. Describe what these points represent.
- e) Sketch the graph of $R(x)$.

Associated with this small business are costs. You estimate that costs will be 348 euro when one price increase is applied to the original price **and** the costs will be 396 euro after 7 price increases.

- f) Show that the cost function is $C(x) = 8x + 340$
- g) Add $C(x)$ to your sketch of $R(x)$.
- h) Show that the profit function is $P(x) = -6x^2 + 33x - 18$
- i) Find the number of price increases that will lead to maximum profit.
- Label the gap between revenue and costs that leads to maximum profit on your sketch.
- j) Solve the inequality $C(x) > R(x)$. Describe what this represents.