Introduction: Measure for Measure

All experiments to be performed in this laboratory require one or more measurements. A measurement is defined as the ratio of the magnitude (how much) of any quantity to a standard value. The standard value is called a unit. A measurement of any kind requires both a magnitude and a unit.

By their nature, measurements can never be done perfectly. Part of the error in making measurements may be due to the skill of the person making the measurement, but even the most skillful among us cannot make the perfect measurement. Basically this is because no matter how small we make the divisions on our ruler (using distance as an example) we can never be sure that the thing we are measuring lines up perfectly with one of the marks. To put it another way, no matter how fine the measurement, there are always more decimal places that we must estimate. Therefore the judgment of the person doing the measurement plays a significant role in the accuracy and precision of the measurement.

Typically we measure simple quantities of only three types, mass, length, and time. Occasionally we include temperature, electrical charge or light intensity.

It is amazing, but just about everything we know about the universe comes from measuring these six quantities. Most of our knowledge comes from measurements of mass, length, and time alone.

Measuring

The magnitude of the quantity is first determined by using a measuring instrument to compare the unknown quantity to a standard and then expressing the comparison as a number. A number representing the magnitude is not sufficient to express the measurement, however; a unit must also be assigned to the number. For example, to record a person's height simply as 64 is meaningless. We must record it as 64 inches. All measurements, therefore, must be recorded by number and unit.

Many instruments (rulers, balances, clocks, speedometers, thermometers, voltmeters, oscilloscopes, spectrometers, etc.) are used to make measurements. The information obtained from these instruments must be recorded and evaluated in order to obtain a truer value of the properties of the physical quantity. Such terms as least count, significant figures,precision, accuracy, and percent error must be learned and used.

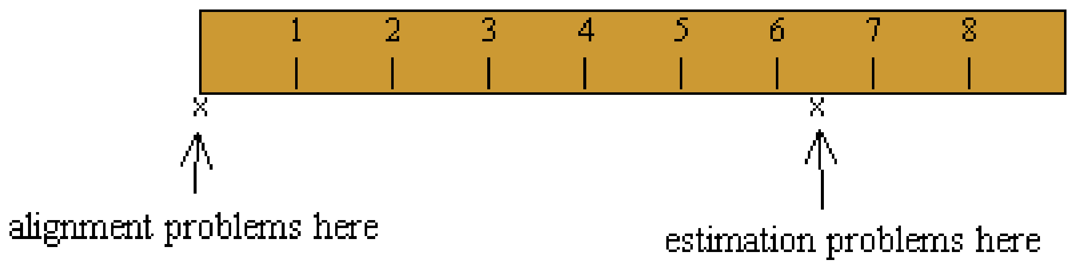
Laboratory experiments require the taking and recording of data using a measuring instrument of one kind or another.The fineness of the measuring scale on the instrument is known as the least count of the instrument.

Definition	The least count is the value of the smallest scale division of the measurement instrument. *Example: a ruler with marks 1 millimeter apart has a least count of 1 millimeter*

The least count of an instrument is the size of the smallest scale division on the instrument. The smaller the least count the more [precise](#) the instrument is said to be. For example a metric ruler which has millimeter marks has a least count of 1 mm or 0.1 cm. and when properly used it can make more [precise](#) measurements than one with centimeter marks.

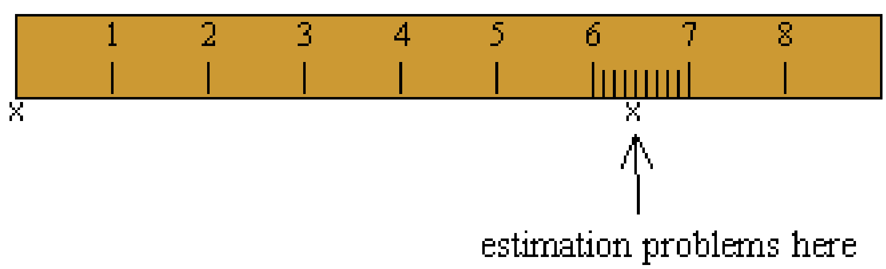
Below are two examples of 'proper' measurements, showing how the last digit is always 'doubtful' or 'uncertain'.

The doubtful digit is a 'best guess' estimate of the tenth fraction of the smallest scale division on the instrument. For example consider this measurement:



It is "six point something": more than six but less than seven. With certainty we can say it is in the range of 6 to 7. Exactly where in that range is a estimate. We might guess it is 40% of the way, just less than halfway, between 6 and 7 or 6.4.

If we had more marks on the scale we could make a better guess.



It is clear that our estimate was close, but the measurement still does not align with a mark. Not only that, but we are having trouble reading the marks now because they are so fine. Now we can estimate with a higher precision that the measurement is 6.39, although you might say that it is 6.38 or even 6.40. Whatever it is, your best guess is 'correct' although it is not certain.

Note the distinction between 6.4, 6.40, and 6.400. The three numbers indicate that the measurements were made with three different instruments. If you don't understand why they are different carefully read the section on significant figures below.

Measuring Time

Measuring time is a little different than measuring distance.

For one thing, when measuring distance the object stays inone place and you can line up the ruler and take your time (oops!).

When measuring time you only get one chance. Plus, you have to coordinate the starting and stopping of the timer with a sensory cue (sight or sound usually). All of this introduces unique errors into time measurements.

It is usually not possible to make estimates as described in the section on measurements above. The only exception would be using an analog stopwatch, but those are rare in the digital age.

Digital timers do not allow for estimation. The reading on the timer display is rounded to some preset number of digits, usually hundreths of a second.

For that reason, all of the digits of a digital timer are significant.

There is a way around the limitation and errors inherent in timing: make multiple measurements and calculate an average time.

Significant Figures

Values read from the measuring instrument are expressed with numbers known as *significant figures*.

For each measurement made it is important to consider significant figures and to keep in mind the uncertainties involved in measurement. When scientists report the results of their measurements it is important that they also communicate how 'close' those measurements are likely to be. This helps others to duplicate the experiment, but also shows how much 'room for error' there was.

[Obviously, reporting a measurement of 7.956845712581 cm., made with a 25-cent plastic ruler from the supermarket would be absurd. Similarly, using an atomic clock to count minutes would be overkill.]

Definition	**A significant figure (abbreviated s.f.) is one that has been measured with certainty or has been 'properly' estimated.The significant figures in a number includes all digits as read from the instrument plus one doubtful digit.** ***Example: a measurement of 1.2374 meters was made with a stick having millimeter marks.***

Significant digits or significant figures are digits read from the measuring instrument plus one doubtful digit estimated by the observer. This doubtful estimate will be a fractional part of the least count of the instrument.

Examples:

- **203.4 cm: least count of ruler was 1 cm, tenth of centimeter was estimated.**
- **4.07 cm: least count was 0.1 cm. hundredth of centimeter was estimated.**

The *maximum error* in a measurement is expressed as a plus or minus value of one least count.

For example, the length of a wood block is measured to be 15.4 cm +/- 0.1 cm. The maximum error given as + 0.1 cm means the length is between 15.3 cm and 15.5 cm. It also means that the measurement was made with a ruler with smallest marks of 1 centimeter.

Arithmetic With Significant Figures

Once the measurements have been made and recorded, it may be necessary to perform arithmetical operations using the significant figures.

You cannot increase the precision of measurement by doing arithmetic operations such as addition, subtraction, multiplication, division. This can be confusing because the calculator will return a large number of decimal places, especially when performing division.

When adding or subtracting, the last digit retained in the sum or difference should correspond to the number with the smallest number of decimal places.

When multiplying or dividing two or more measurements, the number of significant digits in the final answer can be no greater than the number of significant digits in the measurement with the least number of significant digits.

[Click here to see an example of why this is true for multiplication]

Examples:

- **12.34 cm + 9.8 cm = 22.14 cm., which should be rounded to 22.1 cm. sinc 9.8 has the greater number of decimal places.**
- **16.315 cm x 21.84 cm = 356.3196 cm^2 is rounded to four s.f. and becomes 356.3 cm^2 because there are four s.f. in 16.31**
- **37.91 divided by 6.3 = 6.0174603175 which becomes 6.0 because 6.3 has only two s.f.**

[Here is a tutorial on significant figures if you need further practice.]

Rounding

The following procedures are usually sufficient to round off significant figures

If the last significant digit on the right is less than 5, drop it.

Example: 12.363 rounded to 4 significant figures is 12.36

If the last significant digit is 5 or greater, drop it and increase the preceding digit by one.

Example: 15.47 rounded to 3 significant figures is 15.5

If there is no decimal point apply procedure 1 or 2 but change the last digit to zero

Example: 642 rounded to 2 significant figures is 640

More Examples

Round off the following numbers to two significant figures.

- **247 is rounded to 250- Since the last digit on the right is greater than 5, drop it and increase the preceding digit by one, for the result 250.**
- **243 is rounded to 240 - Since the last digit on the right is less than 5, drop it and insert zero instead, for the result 240.**
- **27.16 is rounded to 27. Since the last digit on the right is greater than 5, drop it and increase the preceding digit. This leaves, for the result 28.**

These procedures will be sufficient for most cases.

ZEROS AND SIGNIFICANT FIGURES

A zero can be significant sometimes and not others. It is quite logical, most of the time.

- Zero is always a significant digit when it is between the decimal point and another digit.

- **Example: 14.002 has 5 significant figures.**
- **Example: 60.2 has 3 significant figures**
- **Example: 0.001 has 3 significant figures. The zeros to the right of the decimal point are significant while the zero to the left of the decimal point is not..**

- Zero is significant if it is to the right of the decimal point

- **Example: 29.20 has 4 significant figures**
- **Example: 7.020 has 4 significant figures**

- Zero is usually not a significant digit if there is no decimal point.

- **Example: 240 has 2 significant figures**
- **Example: 1000 has 1 significant figure**

One exception is when a zero is obtained by rounding

- **Example: 249.8 is rounded to 3 significant figures to become 250. Here the zero is significant.**

Another exception is when the zero represents the estimated final digit. This is ambiguous because when you see the number as written it is not clear what that final zero represents.

The ambiguity of the zero can be resolved by using powers of ten notation, also called scientific notation.

If this seems complicated you are partly correct. It can be complicated, but it is necessary because it relates to the reader of your lab report how precise your measurements were. There is a big difference between the statements, "The length of my ruler is 10 centimeters" and "the length of my ruler is 10.00 centimeters". If you don't understand the difference go back and read the section on least count and significant figures again.

Units of Measurement

Simply recording a numerical value for the measured quantity is not sufficient to express a physical quantity. A unit must also be indicated. For example, a measurement is taken for the length of the laboratory table and recorded as 183. This number has no real meaning unless expressed with a unit. A correct recording would be 183 centimeters (cm). All measurements should be made in metric units, or converted to metric. The United States is the only country in the world that has not officially adopted the metric units as a standard.

Below are common units used in the metric system (also called the SI system.) Units in capital letters are the common ones.

LENGTH		
Unit	Abbreviation	Meters
MILLIMETER	mm	0.001
CENTIMETER	cm	0.01

METER	m	1.0
KILOMETER	km	1000.0

MASS		
Unit	Abbreviation	grams
MILLIGRAM	mg	0.001
GRAM	g	1
KILOGRAM	kg	1000

Measuring Uncertainty in Measurements

The process of taking any measurement always involves some uncertainty.

Such uncertainty is usually called experimental error. It is expressed as a ratio of the amount of error compared to the size of the thing being measured. For example, an error of 1 meter is not much when we are measuring the distance from the earth to the moon but it is a large error when measuring the length of a table which is two meters long.

Two equivalent methods are used to calculate the amount of error.

(1) When an accepted or standard value of the physical quantity is known, the percent error is calculated to compare an experimental measurement with a standard.

$$\text{Percent Error} = 100 \times \frac{(\text{Experimental value} - \text{Standard value})}{\text{Standard value}}$$

(2) Percent difference is calculated (in a comparison of two or more experimental measurements). This is used when no standard exists, or when it is desired to mesure the precision of an experiment.

$$\text{percent difference} = \left(\frac{\text{Largest Value} - \text{Smallest Value}}{\text{Average Value}}\right)$$

When calculating percent error or percent difference it is the number that is important. We will ignore the positive or negative sign.