Discussion 13:

Finding Taylor Series

There are two main ways to find the Taylor (or Maclaurin) series for a given function: computing the coefficients one-by-one, and relating the series to one that is already known. In certain instances, the second way can be much more efficient. For instance, knowing the Maclaurin series for $sin(x)$ can easily give the series for $sin(2x)$ without needing to go through each of the steps. In this discussion, you and a classmate are going to use these two methods and compare the different efforts required to find the power series representation for a few functions.

Partner up with a classmate as instructed by your teacher and discuss the following questions.

1. For the following functions, find the Maclaurin series in two ways as indicated. Once you have found the series, compare your coefficients and make sure they match. Which way was more effort?

Function	Student 1	Student 2
$f(x) = \frac{1}{(x-1)^2}$	Using the formal definition of Taylor series	Differentiate the Maclaurin series for $\frac{1}{1-x}$
$g(x) = sin^2(x)$	Using the identity that $cos(2x) = 1 - 2sin^2(x)$ and the Maclaurin series for $cos(x)$	Using the formal definition of Taylor series

2. Together, use properties of logarithms to find the Maclaurin series for $h(x) = ln\left(\frac{1+x}{1-x}\right)$. You will need the Maclaurin series of $ln(1 + x)$ to accomplish this. This can either be done by hand or found in Table 6.1 in the textbook.

3. Use five terms from the Maclaurin series from question 2 to approximate $ln\,2$. To do this, use the following steps.

   Step 1: Set $2 = \frac{1+x}{1-x}$ and solve for x to get the value needed to plug into the Maclaurin series. Call this x_0.

Step 2: Evaluate the first five terms of the Maclaurin series at x_0.

4. Taylor's Theorem with Remainder gives a remainder term that bounds how accurate the result is. Use the theorem to determine how many decimal places your estimate is accurate to. This will require taking derivatives and bounding those derivatives near x_0 to find a number M for the remainder term. This is easiest to do using a graphing utility like Desmos.