- ***Logical connectives***:
  - – Negation ¬ NOT
  - – Conjunction ∧ AND
  - – Disjunction ∨ OR
  - – Implication → IMPLIES
  - – Biconditional ↔ IFF

Negation

p	p¬
T	F
F	T

Conjunction

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	p ∨q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR (XOR)

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

Implication

p	q	p q
T	T	T
T	F	F
F	T	T
F	F	T

Understanding Implication

- Assume you have these propositions:
  - p : You exercise three times a week.
  - r : You follow a low-carb diet.
  - q : You lose weight.

p	q	p q	Explanation
T	T	T	It is accepted. If you exercise, then naturally you will lose weight.
T	F	F	It is rejected. Since one of the causes is true, the result must be fulfilled.
F	T	T	It is accepted. There are other causes to lose weight not just exercising.
F	F	T	It is accepted. If the you don't exercise, then you may not lose weight.

Different Ways of Expressing p →q

if p, then q

if p, q

q unless $\neg p$

q if p

q whenever p

q follows from p

p implies q

p only if q

q when p

p is sufficient for q

q is necessary for p

a necessary condition for p is q

a sufficient condition for q is p

Converse, Contrapositive, and Inverse

$q \rightarrow p$ is the **converse** of $p \rightarrow q$

$\neg q \rightarrow \neg p$ is the **contrapositive** of $p \rightarrow q$

$\neg p \rightarrow \neg q$ is the **inverse** of $p \rightarrow q$

Biconditionals

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

p	q			$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Key Logical Equivalences

- Identity Laws: $p \wedge T \equiv p$, $p \vee F \equiv p$
- Domination Laws: $p \vee T \equiv T$, $p \wedge F \equiv F$
- Idempotent laws: $p \wedge p \equiv p$, $p \vee p \equiv p$
- Double Negation Law: $\neg(\neg p) \equiv p$
- Negation Laws: $p \vee \neg p \equiv T$, $p \wedge \neg p \equiv F$

Commutative Laws: $p \wedge q \equiv q \wedge p$, $p \vee q \equiv q \vee p$

- Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
  $(p \vee q) \vee r \equiv p \vee (q \vee r)$

- Distributive Laws: $\big(p \vee (q \wedge r)\big) \equiv (p \vee q) \wedge (p \vee r)$
  $\big(p \wedge (q \vee r)\big) \equiv (p \wedge q) \vee (p \wedge r)$

- Absorption Laws: $\big(p \vee (p \wedge q)\big) \equiv p,$ $\big(p \wedge (p \vee q)\big) \equiv p$

More Logical Equival

TABLE 7 Logical Equivalences Involving Conditional Statements.
$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$\neg(p \rightarrow q) \equiv p \wedge \neg q$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$