ISRAEL INSTITUTE FOR ADVANCED STUDIES

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Einstein Institute of Mathematics
האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

The 23rd Midrasha Mathematicae:

O-Minimality and its Applications in Diophantine Geometry and Hodge Theory

11-15 December 2022

General Director:
Peter Sarnak, IAS Princeton

Co-directors:
Gal Binyamini, Weizmann Institute of Science
Itay Kaplan, The Hebrew University of Jerusalem
Kobi Peterzil, University of Haifa
Jonathan Pila, University of Oxford
Jacob Tsimerman, University of Toronto

Israel Institute for Advanced Studies
Edmond J. Safra Campus, Givat Ram
The Hebrew University of Jerusalem

Sunday, 11 December

09:30-10:00 Registration

10:00-10:05 Greetings by **Yitzhak Hen**, Director of the IIAS

10:05-11:05 **Tom Scanlon** (University of California, Berkeley) Tutorial I: O-minimality

11:05-11:30 Coffee break

11:30-12:30 **Benjamin Bakker** (University of Georgia): The geometric Andre--Grothendieck period conjecture

12:30-14:00 Lunch break

14:00-15:00 **Hélène Esnault** (Freie Universität Berlin): Integrality of the Betti moduli (joint work with Johan de Jong)

15:00-15:30 Coffee break

15:30-16:30 **Jonathan Pila** (University of Oxford): Ax-Schanuel and Exceptional Integrability

16:30-16:45 Coffee break

16:45-17:45 **Fabrizio Barroero** (Roma Tre University): On the Zilber-Pink conjecture for complex abelian varieties and distinguished categories

18:00 Reception in the IIAS lobby

Monday, 12 December

9:00-9:30 Gathering & coffee

9:30-10:30 **Tom Scanlon** Tutorial II: O-minimality

10:30-11:00 Coffee break

11:00-12:00 **Philipp Habegger** (University of Basel) Tutorial I: Diophantine geometry through the lens of o-minimal geometry

12:00-12:15 Coffee break

12:15-13:15 **Jacob Tsimerman** (University of Toronto) Tutorial I: Hodge theory

13:15-14:30 Lunch

14:30-15:30 **Patrick Brosnan** (University of Maryland): Hodge theory and essential dimension

15:30-16:00 Coffee break

16:00-17:00 **Raf Cluckers** (University of Lille): Title TBA

Tuesday, 13 December

9:00-9:30 Gathering & coffee

9:30-10:30 **Tom Scanlon** Tutorial III: O-minimality

10:30-11:00 Coffee break

11:00-12:00 **Philipp Habegger** Tutorial II: Diophantine geometry through the lens of o-minimal geometry

12:00-12:15 Coffee break

12:15-13:15 **Jacob Tsimerman** Tutorial II: Hodge theory

13:15-14:15 Lunch

14:15 Old City Tour

18:00 Participants' Dinner

20:00 Transportation to the Prima Park Hotel

Wednesday, 14 December

9:00-9:30 Gathering & coffee

9:30-10:30 **Jacob Tsimerman** Tutorial III: Hodge theory

10:30-11:00 Coffee break

11:00-12:00 **Philipp Habegger** Tutorial III: Diophantine geometry through the lens of o-minimal geometry

12:00-12:15 Coffee break

12:15-13:15 **Harry Schmidt** (University of Basel): Uniform effective counting and mixed André-Oort

13:15-14:30 Lunch

14:30-15:30 **Laura Capuano** (Roma Tre University): Multiplicative and linear dependence in finite fields and on elliptic curves modulo primes

15:30-16:00 Coffee break

16:00-17:00 **Gregory Pearlstein** (Texas A&M University): Unimodal singularities and boundary divisors in the KSBA moduli of a class of Horikawa surfaces

Thursday, 15 December

9:00-9:30 Gathering & coffee

9:30-10:30 **Gal Binyamini** (Weizmann Institute of Science): Sharp o-minimality - toward a theory of arithmetically tame structures

10:30-11:00 Coffee break

11:00-12:00 **Tobias Kaiser** (University of Passau): Periods, Power Series, and Integrated Algebraic Numbers

12:00-12:15 Coffee break

12:15-13:15 **Christopher Daw** (University of Reading): An effective geometric Zilber-Pink conjecture via differential algebraic geometry

13:15-14:30 Lunch

14:30-15:30 **Yohan Brunebarbe** (University of Bordeaux): Hyperbolicity in presence of a large local system

15:30-16:00 Coffee break

16:00-17:00 **Bruno Klingler** (Humboldt University of Berlin): On the algebraicity of the Hodge locus

Abstracts

Benjamin Bakker

The geometric Andre--Grothendieck period conjecture

A period integral of a complex algebraic variety is the integral of an algebraic differential form along a topological cycle. These numbers are at the heart of Hodge theory. In this talk I will explain how to prove a version of the Ax--Schanuel conjecture for these period integrals in families, and how it provides a capstone to the advances in the transcendence theory of period maps made over the past decade.

I will also discuss the relationship with the functional version of the Andre--Grothendieck period conjecture, which predicts that all algebraic relations between such periods integrals arise from geometry. This is joint work with J. Tsimerman.

Fabrizio Barroero

On the Zilber-Pink conjecture for complex abelian varieties and distinguished categories

The Zilber-Pink conjecture is a very general statement that implies many well-known results in diophantine geometry, e.g., Manin-Mumford, Mordell-Lang and André-Oort.

I will report on recent joint work with Gabriel Dill in which we proved that the Zilber-Pink conjecture for a complex abelian variety A can be deduced from the same statement for its trace, i.e., the largest abelian subvariety of A that can be defined over the algebraic numbers. This gives some unconditional results, e.g., the conjecture for curves in complex abelian varieties (over the algebraic numbers this is due to Habegger and Pila) and the conjecture for arbitrary subvarieties of powers of elliptic curves that have transcendental j-invariant.

While working on this project we realised that many definitions, statements and proofs were formal in nature and we came up with a categorical setting that contains most known examples and in which (weakly) special subvarieties can be

defined and a Zilber-Pink statement can be formulated. We obtained some conditional as well as some unconditional results.

Patrick Brosnan

Hodge theory and essential dimension

The essential dimension of an algebraic object E over a field L is heuristically the number of parameters it takes to define it. This notion was formalized and developed by Buhler and Reichstein in the late 90s, who noticed at the time that several classical results could be interpreted as theorems about essential dimensions. Since the paper of Buhler and Reichstein, most of the progress on essential dimension has had to do with the essential dimension of versal G-torsors for an algebraic group G. But recently Farb, Kisin and Wolfson showed that interesting theorems can be proved for certain (usually) non-versal torsors arising from congruence covers of Shimura varieties. I'll explain this work, some extensions of it proved by me and Fakhruddin and a conjecture on period maps which generalizes the picture.

Yohan Brunebarbe

Hyperbolicity in presence of a large local system

Serge Lang has proposed several influential conjectures relating different notions of hyperbolicity for projective complex algebraic varieties. For example, he conjectured that the locus swept out by entire curves coincides with the locus swept out by subvarieties not of general type, at least after taking Zariski closures. I will explain that some of these conjectures (including the one above) are true for varieties admitting a large complex local system (e.g. any variety admitting a variation of mixed Hodge structures with a finite period map).

Laura Capuano

Multiplicative and linear dependence in finite fields and on elliptic curves modulo primes

Given n multiplicatively independent rational functions $f_1, ..., f_n$ with rational coefficients, there are at most finitely many complex numbers α such that $f_1(\alpha), ..., f_n(\alpha)$ satisfy two independent multiplicative relations. This was proved independently by Maurin and by Bombieri, Habegger, Masser and Zannier, and it is an instance of more general conjectures of unlikely intersections over tori made by Bombieri, Masser and Zannier and independently by Zilber. We consider a positive characteristic variant of this problem, proving that, for sufficiently large primes, the cardinality of the set of $\alpha \in \mathbb F_p$ such that $f_1(\alpha), ..., f_n(\alpha)$ satisfy two independent multiplicative relations with exponents bounded by a constant K is bounded independently of K and p. We prove analogous results for products of elliptic curves and for split semiabelian varieties $E^n \times \mathbb G_m^n$. This is a joint work with F. Barroero, L. Mérai, A. Ostafe and M. Sha.

Christopher Daw

An effective geometric Zilber-Pink conjecture via differential algebraic geometry

The geometric Zilber-Pink conjecture (GZP), for a subvariety V of a Shimura variety S, can be formulated as follows: the (weakly) optimal subvarieties of V arise as fibres in finitely many algebraic families.

GZP was proved by Ren and the first author circa 2017, building on earlier work of Habegger-Pila for products of modular curves, and has since been extended by several authors to more general settings (mixed Shimura varieties, variations of Hodge structures). It has precursors in the settings of abelian varieties, tori, etc. and can be used to reduce the full Zilber-Pink conjecture to arithmetic problems.

In 2021, Binyamini and the first author established a refinement of GZP by (1) constructing a finite collection of algebraic families whose fibres are precisely the weakly optimal subvarieties, (2) obtaining effective degree bounds on the weakly optimal locus and its constituents, and (3) giving an effective procedure for computing the positive dimensional weakly special locus of an explicit family of abelian varieties.

The main ingredients are multiplicity estimates on foliations (in this case due to Gabrielov-Khovanskii) and constructions using the standard principal bundle

associated with a Shimura variety. The hyperbolic Ax-Schanuel conjecture (in this case a theorem due to Mok-Pila-Tsimerman) plays the same role.

We will elaborate on the above and attempt to give an overview of the ideas in this latest work.

Hélène Esnault

Integrality of the Betti moduli (joint work with Johan de Jong)

We show an integrality property of the Betti moduli space over a smooth complex quasi-projective variety X which yields an obstruction for a finitely presented group to be the topological fundamental of such an X. The proof uses the idea developed with Michael Groechenig on how to use companions (the existence of which, proved by L. Lafforgue and Drinfeld, is stemming from the arithmetic Langlands program) in complex geometry. It also uses de Jong's conjecture (the solution to it, proved by Gaitsgory, is stemming from the geometric Langlands program).

Tobias Kaiser

Periods, Power Series, and Integrated Algebraic Numbers

Periods are defined as integrals of semialgebraic functions defined over the rationals. Periods form a countable ring not much is known about. Examples are given by taking the antiderivative of a power series which is algebraic over the polynomial ring over the rationals and evaluate it at a rational number. We follow this path and close these algebraic power series by taking iterated antiderivatives and nearby algebraic and geometric operations. We obtain a system of rings of power series whose coefficients form a countable real closed field. Using techniques from o-minimality we are able to show that every period belongs to this field. In the setting of o-minimality we define exponential integrated algebraic numbers and show that exponential periods and the Euler constant are exponential integrated algebraic numbers. Hence they are a good candidate for a natural number system extending the period ring and containing important mathematical constants.

Bruno Klinger

On the algebraicity of the Hodge locus

Weil (1979) noticed that, according to the Hodge conjecture, the Hodge locus of a polarized variation of Hodge structure of geometric origin should be a countable union of irreducible algebraic varieties. Cattani-Deligne-Kaplan (1995) proved that this is the case, even without the geometricity assumption. In this talk I will discuss the following question: when is the Hodge locus actually algebraic, rather than a countable union of algebraic varieties? Joint work with Baldi and Ullmo.

Gregory Pearlstein

Unimodal singularities and boundary divisors in the KSBA moduli of a class of Horikawa surfaces

Smooth minimal surfaces of general type with K^2 = 1, p_g = 2, and q = 0 constitute a fundamental example in the geography of algebraic surfaces, and the 28-dimensional moduli space M of their canonical models admits a modular compactification $\bar{M}$ via the minimal model program. We describe eight new irreducible boundary divisors in such compactification parametrizing reducible stable surfaces. Additionally, we study the relation with the GIT compactification of M and the Hodge theory of the degenerate surfaces that the eight divisors parametrize.

Thomas Scanlon

Tutorial on O-minimality

Abstract: With this three-lecture tutorial on o-minimality, I will recall some of the history of this subject and highlight some of the key theorems. While I will point out some of the key ideas in the proofs, mostly I will guide you to well-written books and papers where you can learn the subject in detail.

Harry Schmidt

Uniform effective counting and mixed André-Oort

I will talk about joint work with Gal Binyamini, Gareth Jones, and Margaret Thomas. Gao reduced the André-Oort conjecture for mixed Shimura varieties to Galois lower bounds for (pure) Shimura varieties. In our work, we give a reduction argument that reduces André-Oort for families of products of elliptic curves to André-Oort for powers of modular curves. In combination with our counting theorem, this allows for reducing an effective André-Oort for such families to an effective André-Oort for products of modular curves. Moreover, any degree of uniformity is preserved. Our methods allow for uniform and effective bounds in various situations related to Manin-Mumford and (mixed) André-Oort.


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