Discussion 9:

Changing the Order of Integration

Partner up with a classmate as instructed by your teacher.

In this discussion, you and your partner will look at different ways of expressing multiple integrals over general regions. With double integrals, you always have a choice over which variable to address first. In some instances, the choice of variable can change an integral from something impossible into something that is much more manageable.

For each of the following integrals over the given regions, one student should always try to integrate with respect to x first (i.e., $dx\,dy$) and the other student should always integrate with respect to y first ($dy\,dx$). Work together first to set up both integrals, then work out your integrals individually. (Note: Not all of the integrals are possible to find without resorting to a table or numerical methods. If you get stuck, it might be for a very good reason!)

1. Find $\iint_R \frac{x}{1+y^2}\,dA$, where $R = [0,3] \times [0,1]$. Compare your answers. Which solution was easier to find?

2. Find $\iint_D xy^2\,dA$, where D is the region bounded by the x-axis and the lines $x = 2$ and $y = 2x$. Compare your answers. Which solution was easier to find?

3. Find $\iint_D \sqrt{1 + x^2}\,dA$, where D is the region in the first quadrant bounded by the lines $y = 0$, $y = x$, and $x = 3$. Compare your answers. Which solution was easier to find?

4. For the integral $\iint_D x^2 y\,dA$, where D is the region bounded by the y-axis, the line $y = e$, and the curve $y = e^x$, set up the two iterated integrals. Which one looks easier to evaluate? Find the inside integral for each of the two iterated integral, then compare the results. Which one of these integrals looks easier to evaluate? **Bonus:** evaluate these integrals