Calculus 1 Instructor Guide

Module 8: Contextual Applications of Derivatives

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Learning Outcomes

Detailed Course Learning Outcome Spreadsheet is linked here.

Topic	Student Learning Goals
Limits at Infinity and Asymptotes	• Determine limits and predict how functions behave as x increases or decreases indefinitely • Identify and distinguish horizontal and slanting lines that a graph approaches but never touches • Use a function's derivatives to accurately sketch its graph
Applied Optimization Problems	• Tackle business problems to find the best ways to increase profits, minimize costs, or maximize revenue • Use optimization methods to solve problems involving geometry
L'Hôpital's Rule	• Spot indeterminate forms like 0/0 in calculations, and use L'Hôpital's rule to find precise values • Explain how quickly different functions increase or decrease compared to each other
Newton's Method	• Explain how Newton's method uses repetition to find roots of equations • Recognize when Newton's method does not work • Apply methods that repeat steps to solve different types of mathematical problems
Antiderivatives	• Understand indefinite integrals and learn how to find basic antiderivatives for functions • Use the rule for integrating functions raised to a power • Use antidifferentiation to solve simple initial-value problems

Summary of Module

Background You'll Need

The assumed prerequisite skills:

- Convert from logarithmic form to exponential form
- Define and use the power rule for logarithms to rewrite expressions
- Write expressions for area, perimeter, and volume of geometric figures

Limits at Infinity and Asymptotes

In this section, students delved into the behavior of functions as x approaches infinity or negative infinity, focusing on different function types including polynomial, rational, and transcendental functions. They learned to formally define and calculate limits at infinity, examining horizontal, vertical, and slant asymptotes through graphical and algebraic methods. Interactive exercises allowed them to apply these concepts to solve problems and analyze functions, enhancing their understanding of how functions behave in limits through direct calculation and graphical interpretation.

Applied Optimization Problems

In this section, students explored optimization problems in calculus, focusing on real-world applications like maximizing or minimizing quantities under given constraints. They engaged with problems that involved calculating dimensions for optimal areas or volumes, like gardens or boxes, considering both closed, bounded intervals and unbounded intervals or open domains. Through step-by-step examples, students learned how to apply the extreme value theorem, differentiate functions to find critical points, and use these techniques to solve practical problems, such as determining the cost-effective dimensions of packaging or the best price for maximizing revenue in a business scenario.

L'Hôpital's Rule

In the section students engage with different aspects of function growth rates. They compare functions like $f(x)=x^2$ and $g(x)=x^3$ through tables and limit calculations to understand which function grows faster as x approaches infinity. This practical application is extended to other polynomial, exponential, and logarithmic functions. Using L'Hôpital's rule, students also explore limits that involve quotients of these functions, further deepening their understanding of growth rates in various mathematical contexts. These activities are illustrated with graphs and tables, making the concepts more tangible for students.

Newton’s Method

In this section, students explore Newton's Method as a numerical technique to approximate the roots of functions, particularly when analytic solutions are complex or unavailable. They learn how to apply iterative steps to generate successively closer approximations to the roots, starting from an initial guess. The material presents scenarios where Newton’s Method succeeds in quickly finding roots, alongside examples where it fails due to factors like the derivative being zero or the initial guess leading to cyclic or diverging sequences instead of converging to a root. Additional examples extend the concept to other iterative processes, demonstrating the approach using different functions and exploring their long-term behaviors, including fixed points and cycles, and even chaotic outcomes depending on the setup.

Antiderivatives

In this section, students delve into antiderivatives and indefinite integrals, starting with basic concepts and building up to solving initial-value problems. They practice finding antiderivatives by integrating various functions and learn to use the integral notation effectively. Through interactive examples, students apply these concepts to real-world problems, such as physics scenarios involving motion, where they use antiderivatives to solve differential equations given certain initial conditions.

Module Resources

Cheat Sheet

- Contextual Applications of Derivatives: Cheat Sheet
- Powering the Future with Calculus Activity
- Powering the Future with Calculus Activity Answer Key
- Exploring Rates of Growth Discussion
- Approximation Methods in Calculus Writing Task

Activity One: Powering the Future with Calculus

Evidence-Based Teaching Practices

Contextualization

Educators help students make sense of theoretical material by demonstrating how it applies it to relevant "real world" situations by creating a scenario where students optimize a solar farm's design using calculus concepts. This practical application helps students understand the real-world relevance of abstract mathematical ideas.

Curiosity

Educators foster students' curiosity by helping them understand that asking questions and making mistakes are necessary parts of learning by encouraging open discussions throughout the activity. Probing questions, reflection sessions, and discussion prompts stimulate students to explore beyond given information and challenge assumptions.

Time on Task

Educators maximize the amount of learning time students spend actively engaged in practice by structuring the activity as a hands-on, collaborative problem-solving challenge. Students spend most of the session actively applying calculus concepts to solve the solar farm optimization problem in groups.

Background

This collaborative, problem-solving challenge integrates key concepts from the module as students take on the role of renewable energy consultants tasked with optimizing a new solar farm's design and placement. Working in groups, students will apply their knowledge of limits, optimization, L'Hôpital's Rule, Newton's Method, and antiderivatives to solve a real-world problem. The challenge is divided into several interconnected parts, each highlighting a different calculus concept.

Instructions

Time Estimate: 75-90 minutes

1. **Conversation starter**

   Why do you think optimizing a solar farm's design is more than just about placing panels in a grid? What real-world factors might complicate this process, and how could calculus help navigate those challenges?

2. **Split class into groups of 3-5 students**

   Students will need to be able to write down their ideas either on paper or electronically. Groups can use a single poster or whiteboard (digitally or provided in the classroom) to write down their final answers.

3. **Distribute the handout**

   Powering the Future with Calculus Activity

4. **Team Problem-Solving**

   Groups should work together to solve each part of the challenge. Encourage students to think creatively and consider real-world constraints. Circulate to provide guidance and ask probing questions.

5. **Presentation Prep**

   Groups should prepare a brief presentation of their solutions and reasoning. This can be done on the poster or whiteboards.

6. **Group Presentations**

   Each group should present their approach and solutions to one part of the challenge. Encourage other groups to ask questions and actively engage in discussions that may follow.

7. **Reflection and Debrief**

   Explore how each calculus concept played a crucial role in addressing the problem. Emphasize the practical applications and how these concepts intersect with other disciplines in real-world scenarios.

Discussion Prompts

- **In Part 1, we calculated limits as time approached infinity for different solar panel designs. How do these mathematical limits relate to real-world performance, and what factors might prevent a solar panel from actually reaching its theoretical limit?**

  Misconception: Students often believe that mathematical models perfectly represent real-world scenarios and that theoretical limits can always be achieved in practice. This misconception can lead to unrealistic expectations and flawed decision-making in applied settings.

- **In the optimization problem, we only considered energy output when determining the optimal spacing between solar panels. What other factors might a real solar farm need to consider, and how might these change our optimization approach?**

  Goal: This discussion aims to broaden students' understanding of real-world optimization problems, encouraging them to consider multiple variables, constraints, and competing objectives that may not be apparent in simplified mathematical models.

- **We used L'Hôpital's Rule to compare the efficiency of solar cell materials at a specific temperature. Can you think of a scenario where L'Hôpital's Rule wouldn't be applicable or might give misleading results?**

  Sample Answer: L'Hôpital's Rule can be misleading or inapplicable in certain scenarios. For example, if repeated applications of the rule still lead to an indeterminate form, or if the limit doesn't exist, the rule may suggest otherwise. It's also unsuitable for functions that are not differentiable at the point of interest. Additionally, in cases with oscillating functions, where the limit of the ratio of derivatives exists but doesn't match the original ratio, L'Hôpital's Rule may give incorrect results. Lastly, applying the rule to one-sided limits without verifying the existence of both one-sided derivatives can lead to inaccuracies.

- **Throughout this challenge, we've applied calculus to a renewable energy scenario. How might professionals in fields like engineering, environmental science, or economics use these calculus concepts in their work?**

  Misconception: Students often view calculus as a purely theoretical mathematical subject with limited real-world applications. They may fail to recognize how fundamental calculus concepts underpin many practical problem-solving approaches across various professional fields.

Reflection

After the activity, we recommend that students complete exit cards. Have each student write on a piece of paper one key concept they learned from the activity and one concept they have questions about. Below are some suggestions for students:

- Which calculus concept (limits, optimization, L'Hôpital's Rule, Newton's Method, or antiderivatives) do you feel most confident applying after today's activity, and why?
- Describe one real-world scenario, aside from the solar farm example, where optimization using calculus could be essential.
- What was the most challenging part of today's problem-solving challenge, and how did your group overcome it?
- How did your understanding of L'Hôpital's Rule evolve through applying it in this activity?
- If you were to explain Newton's Method to a peer who missed today's session, how would you describe its importance in finding solutions to complex problems?
- Reflecting on your group's collaboration, what strategies did you use that were most effective in tackling the interconnected parts of the challenge?

Online Variation

For synchronous online classes, use breakout rooms in Zoom or Teams to divide the class into groups of 3-5 students. Each group can collaborate using shared digital tools like Google Docs, Jamboard, or Miro to work through the challenge. After the problem-solving phase, groups can present their solutions using screen sharing, allowing for interactive discussion and feedback from the entire class. The reflection and debrief can take place in the main session, where students share insights and discuss how calculus concepts applied to the real-world problem.

For asynchronous online courses, provide students with access to a shared document or platform where they can collaborate on the problem at their own pace. Set deadlines for each part of the challenge, and require students to submit their group's work by a specific date. Encourage groups to record short video presentations or write detailed summaries of their solutions, which can then be posted to a discussion board for peer review and feedback. The reflection and debrief can be done through a class-wide discussion board where students share their experiences and insights on the application of calculus concepts.

Assignments

Exploring Rates of Growth Discussion

In this discussion, students will analyze how functions grow or decay as variables approach infinity, comparing different growth rates in real-world contexts. Students will select an application domain such as population dynamics, computing efficiency, finance, environmental science, or technology adoption where comparing growth rates is meaningful. They'll identify two or more functions with different growth rates that model phenomena in their chosen area and use L'Hôpital's Rule to mathematically determine which function grows faster. Students will create visualizations to demonstrate these comparisons and explain what the mathematical results mean in real-world terms. In their posts, students will introduce their chosen domain, describe the specific functions being compared, show their mathematical work using L'Hôpital's Rule, and explain the practical implications of their findings. After posting, they'll engage with classmates by identifying connections between different application domains, asking questions about mathematical analyses, and suggesting extensions to other functions or situations.

Exploring Rates of Growth Discussion

We ask that you make your own copy to edit and adjust to fit the needs of your classroom

Approximation Methods in Calculus Writing Task

In this writing task, students will analyze and compare two important approximation techniques: Newton's Method and L'Hôpital's Rule. Using a cubic function, students will first apply Newton's Method to approximate one of its roots, creating a table showing how successive approximations converge and discussing scenarios where the method might fail with different initial values. Next, they'll construct a rational function that leads to an indeterminate form and apply L'Hôpital's Rule to evaluate its limit, verifying their result using an alternative method. Students will then compare these approximation techniques, discussing the types of problems each method solves, their prerequisites, strengths, limitations, and the iterative nature of Newton's Method versus the direct application of L'Hôpital's Rule. The assignment concludes with students identifying real-world scenarios where each method could be applied, explaining how they would set up mathematical models, interpret results, and account for limitations or assumptions.

Approximation Methods in Calculus Writing Task

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