O

ANDERSON SECONDARY SCHOOL
Prelim Examination 2020
Secondary Four Express and Five Normal

MATHEMATICS 4048 /02

ing Scheme

Question		Solution	Mark	Remarks
1	(a)(i)	$\frac{2px+9qy}{2py+qx} = 3$ $2px + 9qy = 6py + 3qx$ $2px - 3qx = 6py - 9qy$ $x(2p - 3q) = 3y(2p - 3q)$ Since $2p \neq 3q$, $2p - 3q \neq 0$ $x = 3y$ (shown)	[M1] [A1]	
	(a)(ii)	$\frac{x}{y} = 3$ $\frac{x+y}{y} = \frac{x}{y} + 1$ $= 3 + 1$ $= 4$	[M1] [A1]	
	(b)	$\frac{3a+7b}{16a^2-49(a+b)^2} = \frac{3a+7b}{[4a+7(a+b)][4a-7(a+b)]}$ $= \frac{3a+7b}{(11a+7b)(-3a-7b)}$ $= -\frac{1}{11a+7b}$	[M1] [M1] [A1]	

(c)
$$2^{x+3} = 320 - 2^{x+1}$$
$$2^{x+1} + 2^{x+3} = 320$$
$$2^x \times 2 + 2^x \times 2^3 = 320 \quad \text{[M1]}$$
$$2^x (2+8) = 320$$
$$2^x = 32$$
$$= 2^5 \quad \text{[M1]}$$
$$x = \underline{\underline{5}} \quad \text{[A1]}$$

2 (a)
$$\text{Total tickets sold} = \frac{85}{100}(12000)$$
$$= 10200$$
$$\text{Total amount from sales} = 35(3200) + 50(10200 - 3200) \quad \text{[M1]}$$
$$= \$462000 \quad \text{[A1]}$$

(b) Cost of Dec 2019 show for an adult

$$= \frac{100}{80} \times 50 \quad \text{[M1]}$$
$$= \$62.50 \quad \text{[A1]}$$

(c)
$$\text{Profit} = \frac{7}{15}(375000)$$
$$= \$175000 \quad \text{[M1]}$$
$$\text{Compound Interest} = 175000\left(1 + \frac{4/4}{100}\right)^{2\times 4} - 175000 \quad \text{[M1]}$$
$$= \$14499.92 \text{ (to nearest cent)} \quad \text{[A1]}$$

3 (a) Let the radius of circle ABC be r cm.

$OX^2 = OA^2 + AX^2$
$$(6-r)^2 = r^2 + r^2 \quad \text{[M1]}$$
$$36 - 12r + r^2 = 2r^2$$
$$r^2 + 12r - 36 = 0$$
$$r = \frac{-12 \pm \sqrt{12^2 - 4(1)(-36)}}{2} \quad \text{[M1]}$$
$$= \frac{-12 \pm \sqrt{288}}{2} \quad \text{[A1]}$$

Since $r \geq 0$, $r = 2.485$
Radius of circle ABC is 2.49 cm.

(b) [M1]

Angle $AXC = \frac{1}{2}(360^o - 90^o) = 135^o$

Length of minor arc $AC = \frac{135^o}{360^o} \times 2\pi(2.485)$ [M1]

$= 5.855$

$= 5.86$ cm [A1]

(c) Area of shaded region
= Area of sector OCD – Area of minor sector AXC – Area of ΔOAX

$= \frac{45^o}{360^o} \times \pi(6^2) - \frac{135^o}{360^o} \times \pi(2.485^2) - \frac{1}{2} \times 2.485^2$ [M2]

$= 3.7745$

$= 3.77$ cm^2 [A1]

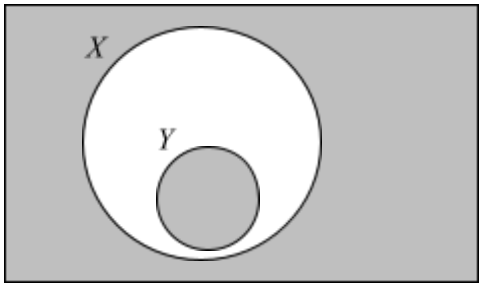
4 (a) (i) $B = \{12, 15, 18, 21, 24, 27, 30\}$ [B1]

(a)(ii) $A' \cap B' = \{ 10, 11, 13, 14, 17, 19, 22, 23, 25, 26, 29 \}$

$n(A' \cap B') = 11$ [B1]

(a)(iii) $A \cap B = C$ [B1]

(b)(i)



[B1]

(b)(ii) P' [B1] Award B1 if correct answer even if not simplified

5 (a) $AB = \sqrt{7.9^2 + 6^2 - 2(7.9)(6)\cos 130^o}$ [M2]

[A1]

$= 12.62324306$

$= 12.6$ cm (to 3 s.f)

(b) $\dfrac{\sin\ (\text{angle } ADC)}{7.9} = \dfrac{\sin 50°}{6.4}$ [M1]

$\text{angle } ADC = 71.011°$

$= 71.0°$ (to 1 dp) [A1]

(c) $\text{Area of triangle } ABC = \dfrac{1}{2}(7.9)(6)\sin 130°$

$= 18.155 \text{ cm}^2$ [M1]

Let the perpendicular distance from C to AB be d cm.

$\dfrac{1}{2}AB(d) = 18.155$ [M1]

$d = \dfrac{18.155}{0.5(12.623)}$

$= 2.876$

$= 2.88 \text{ cm}$

Shortest distance from C to AB

$= 2.88 \text{ cm}$ [A1]

6 (a) $\mathbf{Q} = \begin{pmatrix} 280 & 320 & 345 \\ 250 & 265 & 280 \\ 190 & 235 & 290 \end{pmatrix}$ [B1]

(b)(i) $\mathbf{R} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ [B1]

(b)(ii) **QR**

$= \begin{pmatrix} 280 & 320 & 345 \\ 250 & 265 & 280 \\ 190 & 235 & 290 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 945 \\ 795 \\ 715 \end{pmatrix}$ [B1]

(c) $S = \begin{pmatrix} 3.80 & 3.50 & 4.00 \end{pmatrix} - \begin{pmatrix} 2.60 & 2.60 & 2.60 \end{pmatrix}$ [B1]

$= \begin{pmatrix} 1.20 & 0.90 & 1.40 \end{pmatrix}$ [shown]

(d)(i) **T = S(QR)**

$= \begin{pmatrix} 1.20 & 0.90 & 1.40 \end{pmatrix} \begin{pmatrix} 945 \\ 795 \\ 715 \end{pmatrix}$ [B1]

$= (2850.50)$

(d)(ii) **T** represents the total amount of profit generated from the sales of AndClean hand sanitizers in all the 3 shops over the three months. [B1]

(e) $1.4 \begin{pmatrix} 345 \\ 280 \\ 290 \end{pmatrix}$ [M1] Accept row matrix,

$= \begin{pmatrix} 483 \\ 392 \\ 406 \end{pmatrix}$ [A1] Award 1M if scalar multiplication poorly expressed but used.

Award 0 if matrix multiplication used

7 (a) $p = -1.6$ [B1]

(b) All points plotted correctly [M1]
Smooth curve passing through all points [M1]
Correct scale used with labels [M1]

(c) Draw line of $y = 2$ [M1] Allow +/- 0.1
$x = 4.5$ [B1]

(d) Correct tangent drawn [M1]

Gradient of tangent $= \dfrac{2.6 - (-3.5)}{4.8 - 2.9}$ [A1] Allow +/- 0.1

$= 3.21$

(e)(i) Correct line drawn [B1]

(e)(ii) $x = 2.9$ [B1]

(e)(iii) $\frac{1}{5}x^3 - \frac{4}{5}x^2 = 4 - 2x$

$\frac{1}{5}x^3 - \frac{4}{5}x^2 + 2x - 4 = 0$

$x^3 - 4x^2 + 10x - 20 = 0$ [M1]

$A = 10,\ B = -20$ [A2]

8 (a) Angle $OEA = 35^\circ$ (Angles in the same segment) [B1]

(b) Angle $AOB = 35^\circ \times 2$ (angle at centre = 2 angles at circumf [M1]

$= 70^\circ$

Reflex $AOB = 360^\circ - 70^\circ$ (Angles at a point)

$= 290^\circ$ [A1]

(c) Angle $OAE = 35°(OA = OE)$ [M1]

Angle $BAC = 90° - 35° - 40°$ (Right angle in semi-circle) [A1]

$= 15°$

OR

Angle $BAO = \frac{180° - 70°}{2}(OA = OB)$ [M1]

$= 55°$

Angle $BAC = 55° - 40°$ [A1]

$= 15°$

(d) Angle $CDE = 180° - 40° - 35°$ (angles in opposite segment) [M1]

$= 105°$

Angle $FEB = 105°$ (Corresponding angles, BE parallel to C [A1]

(e) Angle $OED = 180° - 105°$ (adj angles on a straight line) [M1]

$= 75°$

Angle $DOE = 180° - 75° - 75°(OD = OE)$

$= 30°$ [A1]

9 (a)(i) The mean mark of class C was 75. [B1]

Since the standard deviation of the class was zero, all students scored the same marks of 75. [B1]

(a)(ii) Class B had the best performance as the mean mark was the highest at 86.5. [B1]

Class B also had the highest inconsistency in the scores at the standard deviation was the greatest at 2.128. [B1]

(a)(iii) Let the new student's score be a.

New mean $= \dfrac{14\times 75 + a}{15}$ [M1]

$= \dfrac{1050 + a}{15}$ [M1]

$2.30^2 = \dfrac{14\times 75^2 + a^2}{15} - \left(\dfrac{1050 + a}{15}\right)^2$

$1190.25 = 1181250 + 15a^2 - 1102500 - 2100a - a^2$ [M1]

$14a^2 - 2100a + 77559.75 = 0$

$a = \dfrac{-(-2100) \pm \sqrt{(-2100)^2 - 4(14)(77559.75)}}{2(14)}$ [A1]

$a = 84$ or $a = 66$ (to nearest whole number)

(b)(i) P(first draw not an '8') = $\dfrac{7}{8}$ [B1]

b(ii) P(it will take exactly n draws) = $\left(\dfrac{7}{8}\right)^{n-1}\left(\dfrac{1}{8}\right)$ [B1]

b(iii) P(it will take at least n draws) = $\left(\dfrac{7}{8}\right)^{n-1}$ [B1]

(c) Statement is not valid. [A1] No A1 if reason is invalid.

As the balls were **drawn with replacement**, her chances of getting an '8' in the draw is **independent** of her previous draws. [M1]

10 (a) Radius of sphere $= \sqrt[3]{\dfrac{15}{\frac{4}{3}\pi}}$ [M1]

$= 1.5299$

$= 1.53$ cm (to 3 sf) [A1]

(b)(i) Volume of cookie after baking $= \pi(3)^2(0.6)$

$= 16.964$ cm^3 [M1]

Volume of trapped air $= 16.964 - 15$

$= 1.964$

$= 1.96$ cm (to 3 sf) [A1]

(b)(ii) % of trapped air $= \dfrac{1.964}{16.964}\times 100\%$

$= 11.577$

$= 11.6\%$ (to 3 sf) [B1]

(c)(i) Length of $OP = 6$ cm [B1]

(c)(ii) $OQ = \sqrt{6^2 - 3^2}$ [M1]

$= 5.196$

$= 5.20$ cm (to 3 sf) [A1]

(c)(iii) $\frac{TS}{6} = \frac{5.196 + 3}{5.196}$ [M1]

$TS = 9.464$ cm

Length of one side of box = 9.46 cm (to 3 sf) [A1]

11 (a) $(100 - 2x)(55 - 2x) = 105(50 - x)$ [M1]

$2(50 - x)(55 - 2x) = 105(50 - x)$

$(50 - x)[2(55 - 2x) - 105] = 0$

$x = 50$ or $2(55 - 2x) = 105$

$55 - 2x = 52.5$ [M1]

$x = 1.25$

Since $x \leq 27.5$, $x = 1.25$ [M1]

Defrosting is required since the thickness of frost >1 cm. [A1]

(b) Length of box space = 100 + 2(2) + 2(3.5)
= 111 cm [B1]

Height of box space = 100 + 3 + 2(3.5)
= 110 cm [B1]

Width of box space = 55 + 2(2) + 2(3.5)
= 66 cm [B1]

Assumption: The height of the legs of the freezer is negligible. [B1]