

Quantitative Reasoning Instructor Guide

Probability

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Learning Outcomes

[Detailed Course Learning Outcome Spreadsheet is linked here.](#)

Topic	Student Learning Goals
Computing the Probability of an Event	• Describe events in a sample space • Calculate the probability of different types of events • Find the conditional probability of an event
Applications With Probability	• Calculate conditional probability using Bayes' Theorem • Solve counting problems • Calculate the average value of a random event

Summary of Module

Students start with a refresher on fractions and least common multiples to prepare them for the module. The module begins by introducing students to events and outcomes in order to solve basic probability problems. This is expanded upon to determine the probability of independent events and conditional probability. In the last topic of the module, students continue to expand their understanding of probability to solve application-based probability problems using Bayes' Theorem. They are introduced to permutations and combinations as well as the expected value.

Module Resources

Cheat Sheet

 [Probability Cheat Sheet](#)

Worksheets/Handouts

The following are for students who do not wish to purchase/bring in required items:

 [Colored Circles](#)

 [Letter Circles](#)

 [Monty Hall Problem Discussion](#)

 [Probability and Risk Assessment Writing Task](#)

Activity One: Probability Play: Hands-On Exploration with Everyday Items

Evidence-Based Teaching Practice



Caring

Educators encourage students to build relationships with one another in order to establish peer networks of support, and encouragement by assigning random groups for this activity and encouraging collaboration.



Contextualization

Educators help students make sense of theoretical material by demonstrating how it applies to relevant “real world” situations by using examples they encounter in everyday life that relate to probability.



Self-Reflection

Educators create opportunities for students to assess their own work and engage in metacognitive reflection by having students explain how the activity helped them understand probability through self-reflection or an exit survey.

Background

Students will need to bring one of the following items to class: colored candy, deck of cards, or dice. Students who do not own these items and do not wish to purchase them can cut out either the [Colored Circles](#) or [Letter Circles](#). You may wish to bring copies of both of these for students who have forgotten to bring in an item. Students will be completing “interactions” with their item - this is how they will use their item to create probability events. For example, if a student brought in a deck of cards the interaction would be randomly drawing a card from the deck; for dice, it would be rolling the dice; for colored candy or circles, it would be selecting one at random. Students will be paired off with another student with a different item than they brought to class.

Instructions

Time Estimate: 45-60 minutes

1. Conversation starter

In your day-to-day life, where have you encountered instances in which you have needed to determine the probability of something happening?

2. Review

You may wish to briefly review events, outcomes, basic probability, probability of two independent events, and conditional probability.

3. Split the class into groups of 2-3 students

Students should be grouped based on the items they brought to class. There ideally should only be one item type per group. Students will need to be able to write down their ideas either on paper or electronically

4. Guide students through the following questions, discussing the different items as you go.

- Look at the item you brought to class, describe the sample space after one interaction. How does this sample space change if there are two interactions? For those with a deck of cards, frame your sample space around the card colors. How is this sample space different from your neighbors' sample space?

- **Answer:** Answers will vary depending on which item the student brought to class.

Sample Answer: For the deck of cards:

The sample space of the deck of cards after one interaction is:

$$S = \{\text{red, black}\}$$

The sample space of the deck of cards after two interactions is:

$$S = \{(\text{red, red}), (\text{red, black}), (\text{black, black})\}.$$

- Describe 2-3 simple and compound events that could occur after one interaction with your item. Now, try to list 2-3 simple and compound events for your neighbors' item after one interaction.

- **Answer:** Answers will vary depending on which item the student brought to class.

Sample Answer: For the deck of cards:

Examples of simple events after randomly drawing a card from a deck:

drawing the Ace of Spades

drawing a Queen of Hearts card

There are 52 possible simple events

Examples of compound events after randomly drawing a card from a deck:

drawing a red card

drawing a face card

- Select two independent events that can occur during an interaction with your item. For example, if you have colored candy this could be selecting a candy that is a warm color and selecting candy that is red. Solve $P(A \text{ and } B)$ for your two events. Try solving $P(A \text{ and } B)$ for the events your neighbor chose.

- **Answer:** Answers will vary depending on which item the student brought to class and which independent events they have chosen.

Sample Answer: Two independent events that can occur when selecting a card from a deck of cards are - selecting a face card (event A) and selecting a red card (event B). $P(A \text{ and } B)$ for this scenario is:

$$P(A) = 12/52 = 3/13$$

$$P(B) = 26/52 = 1/2$$

$$P(A \text{ and } B) = 3/13 * 1/2 = 6/52 = 3/26$$

- Using the same two events you selected in question 4, solve $P(A \text{ or } B)$ for your two events. Try solving $P(A \text{ or } B)$ for the events your neighbor chose.
- **Answer:** Answers will vary depending on which item the student brought to class and which independent events they have chosen.

Sample Answer: Two independent events that can occur when selecting a card from a deck of cards are - selecting a face card (event A) and selecting a red card (event B). $P(A \text{ or } B)$ for this scenario is:

$$P(A) = 3/13$$

$$P(B) = 1/2$$

$$P(A \text{ and } B) = 3/26$$

$$P(A \text{ or } B) = 3/13 + 1/2 - 3/26 = 8/13$$

- Now we must select two events that occur across two interactions with your item. For example, if you have dice this could be the first die that shows a 4 (event A), and the sum of the dice is 7 (event B). Solve $P(A|B)$ for your two events. Try solving $P(A|B)$ for the events your neighbor chose.

- **Answer:** Answers will vary depending on which item the student brought to class and which events they have chosen.

Sample Answer: If you have dice and the first die shows a 4 (event A), and the sum of the dice is 7 (event B), $P(A|B)$ is calculated as follows.

$$P(A \text{ and } B) = P(B) * P(A|B)$$

$$P(B) = \frac{\text{Number of favorable outcomes for B}}{\text{Total Possible outcomes}} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \text{ and } B) = \frac{\text{Number of favorable outcomes for A and B}}{\text{Total Possible outcomes}} = \frac{1}{36}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

Discussion Prompts

- **What is the difference between the AND, OR, and NOT statements in probability?**

Misconception: Students will sometimes mix up the AND and OR statements when computing probability problems. Try and have them discuss probability in the real world that uses these statements to discuss their differences.

- **Consider the role of probability in fields such as insurance, healthcare, or even game design. How do you think professionals in these areas use probability to make informed decisions or predictions?**

Goal: The goal of this question is to prompt students to think about the role of probability in various professional fields. It encourages them to apply their understanding of probability to specific industries, considering how it can be used in real-world professional contexts to inform decisions and predict outcomes.

- **Sometimes our intuition about probability can be misleading. Can you discuss an aspect of today's activity that challenged your intuitive understanding of probability?**

Goal: This question aims to make students reflect on their preconceptions about probability and how the activity might have challenged those. It helps students to recognize that intuition can sometimes be at odds with mathematical reality, promoting a deeper understanding of probability.

Reflection

After the activity, we recommend that students complete exit cards. Have each student write on a piece of paper one key concept they learned from the activity and one concept they have questions about. Below are some suggestions for students:

- What was the most surprising thing you learned about probability during today's activity? Why?
- How would you explain the concept of independent and dependent events to someone who has never studied probability before?
- Can you describe a situation from today's activity where the actual outcome differed significantly from the expected outcome? What does this tell you about probability?
- Reflecting on today's activity, how do you think understanding probability could be useful in your personal life or future career?
- Reflect on today's exploration of probability. Which of these topics covered did you find most challenging and why?

Online Variation

Students can answer the questions provided using their items at home on a discussion board. They can then look at other students' discussion boards to see the different items used.

Assignments

Monty Hall Problem Discussion

In this discussion, students will explore the famous Monty Hall probability puzzle, where contestants choose between three doors, one concealing a car and two hiding goats. Students will first share their intuitive response to whether they would stay with their original door choice or switch after the host reveals a goat behind one of the unchosen doors. Using probability calculations, they'll determine that staying with the original door provides a $\frac{1}{3}$ chance of winning while switching increases the probability to $\frac{2}{3}$. Students will analyze why this counterintuitive result causes strong reactions, explain how the problem illustrates conditional probability concepts, and discuss why many people (including mathematicians) initially rejected the correct solution. Through peer engagement, students will compare their initial instincts with classmates, verify probability calculations, and reflect on how this deceptively simple problem challenges our intuitive understanding of probability while demonstrating the importance of systematic mathematical analysis in scenarios where intuition often leads us astray.

[Monty Hall Problem Discussion](#)

We ask that you make your own copy to edit and adjust to fit the needs of your classroom

Probability and Risk Assessment Writing Task

In this writing task, students will explore how probability applies to real-world risk assessment by analyzing mortality statistics from different causes. Students will calculate the probability of dying from various causes including car accidents, motorcycle accidents, tornadoes, and skydiving, expressing answers as fractions and in scientific notation. They'll distinguish between general population probabilities and personalized risk factors, explaining why these calculations might differ based on individual circumstances. Students will perform comparative risk analyses between activities like skydiving versus driving and motorcycle riding versus driving, identifying what additional information would be needed for more accurate comparisons (such as exposure time and participant numbers). The assignment concludes with a reflection on why human risk perception often differs from statistical probability, referencing cognitive biases that affect risk assessment and providing a personal example where probability calculations could influence decision-making.

[Probability and Risk Assessment Writing Task](#)

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