1.

Given that $72 = 2^x \times 3^y$, find x and y, given that $x, y \in \mathbb{Z}$

$$72 = 2^x \times 3^y$$
$$2^3 \times 3^2 = 2^x \times 3^y$$
$$x = 3$$
$$y = 2$$

(2)

2.

A rectangle has area $\left(6-\sqrt{3}\right)\ \mathrm{cm}^2$ and the length of one of its sides is $\left(2+\sqrt{3}\right)$ cm. Find the length of the other side.

$$x\left(2+\sqrt{3}\right) = \left(6-\sqrt{3}\right)$$
$$x = \frac{6-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$
$$= \frac{12-6\sqrt{3}-2\sqrt{3}+3}{4-3}$$
$$= 15-8\sqrt{3}$$

(3)

3.

Factorise the following fully

a. $2x^2 + x - 15$

$$(2x-5)(x+3)$$

(1)

b. $8x^3 - 2x$

$$8x^3 - 2x = 2x\left(4x^2 - 1\right)$$
$$= 2x(2x+1)(2x-1)$$

(2)

c. $24x - 27 - 4x^2$

$$24x - 27 - 4x^2 = (2x-9)(3-2x)$$

(2)

4.

a. Find a, b and c such that

$$5 - 6x - x^2 \equiv a(x+b)^2 + c$$

$$\begin{aligned} 5 - 6x - x^2 &= -\left(x^2 + 6x\right) + 5 \\ &= -\left[(x+3)^2 - 3^2\right] + 5 \\ &= -(x+3)^2 + 14 \end{aligned}$$

$$a = -1,\ b = 3,\ c = 14$$

(3)

Given $\mathrm{f}(x) = 5 - 6x - x^2$

b. State the coordinates of the stationary point of $\mathrm{f}(x)$

$(-3, 14)$

(1)

c. State the equation of of the line of symmetry of $\mathrm{f}(x)$

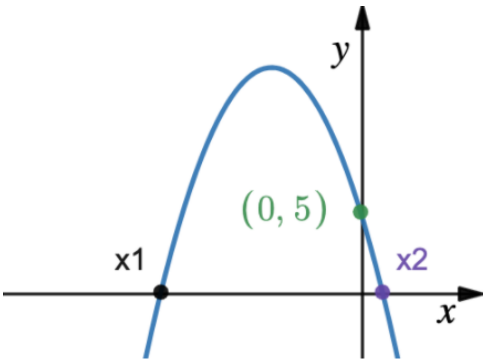
$x = -3$

(1)

d. Sketch $y = \mathrm{f}(x)$ show where the curve cuts the axes.

$(x+3)^2 = 14$

$x_{1,2} = -3 \pm \sqrt{14}$



(3)

5.

Solve the inequalities

a. $(x-5)^2 > 3$

$$x = 5 \pm \sqrt{3}$$
$$x > 5 + \sqrt{3}$$
$$x < 5 - \sqrt{3}$$

(2)

b. $2x^2 + 7x - 4 \geqslant 0$

$$(2x-1)(x+4) \geqslant 0$$
$$x = \frac{1}{2}, -4$$
$$x \leqslant -4,\ x \geqslant \frac{1}{2}$$

(3)

c. $3 - \frac{2}{x} < x$

$$3x - 2 < x^2$$
$$x^2 - 3x + 2 > 0$$
$$(x-1)(x-2) > 0$$
$$x > 2,\ x < 1$$

(3)

6.

Find the range of values of *k* that given the equation has no real roots.

$2x^2 + kx + 1 = 0$

$$k^2 - 4(2)(1) < 0$$
$$\left(k + 2\sqrt{2}\right)\left(k - 2\sqrt{2}\right) < 0$$
$$-2\sqrt{2} < k < 2\sqrt{2}$$

(3)

7.

Solve

$x^2 - xy + y^2 = 7$

$2x + y = 1$

$$x^2 - x(1-2x) + (1-2x)^2 = 7$$
$$x^2 - x + 2x^2 + 1 - 4x + 4x^2 - 7 = 0$$
$$7x^2 - 5x - 6 = 0$$
$$x = \frac{5 \pm \sqrt{25 - 4(7)(-6)}}{2(7)} = \frac{5 \pm \sqrt{193}}{14}$$
$$y = \frac{2 \mp \sqrt{193}}{7}$$

(3)

8.

Given that $t^{\frac{1}{3}} = y,\ y \neq 0$

a. Express $6t^{-\frac{1}{3}}$ in terms of y.

$$6t^{-\frac{1}{3}} = \frac{6}{t^{\frac{1}{3}}} = \frac{6}{y}$$

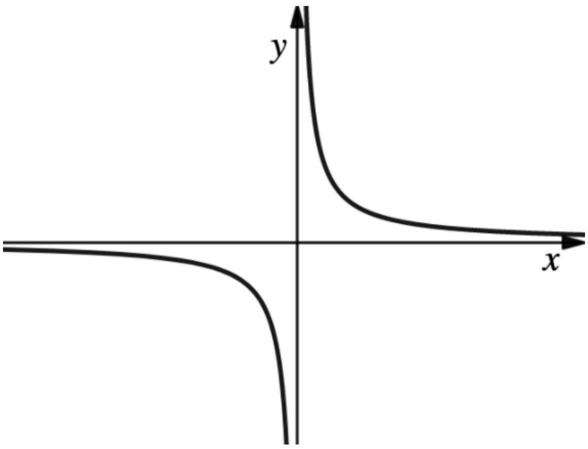
(1)

b. Hence, or otherwise, find the values of t for which $6t^{-\frac{1}{3}} - t^{\frac{1}{3}} = 5$

$$\frac{6}{y} - y = 5$$
$$6 - y^2 = 5y$$
$$y^2 + 5y - 6 = 0$$
$$(y+6)(y-1) = 0$$
$$y = 1, -6$$
$$t = 1, -216$$

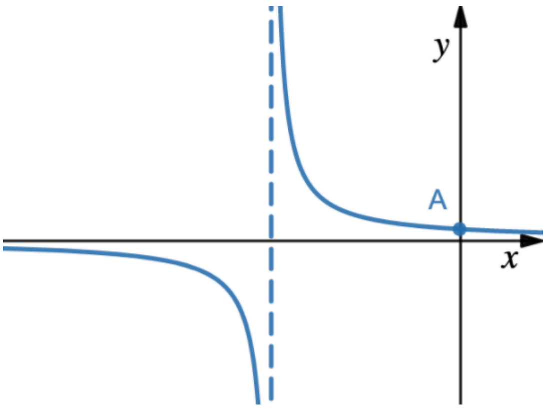
(3)

9.



The figure show the sketch of the curve $\mathrm{f}(x) = \dfrac{1}{x}$

Sketch $y = \mathrm{f}(x+3)$, state the equation of asymptotes and where the curve cuts the axes.

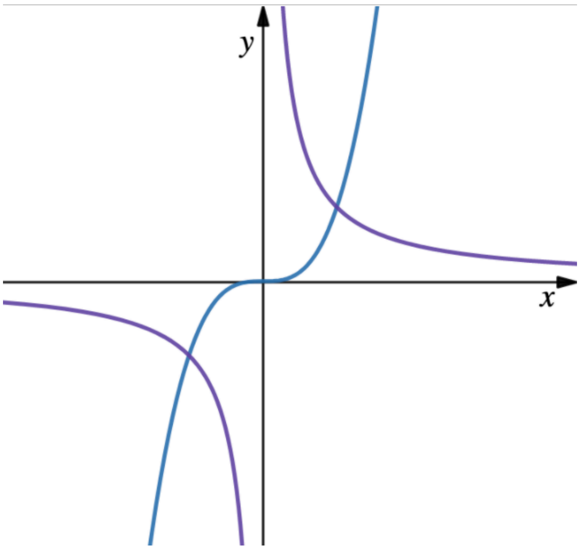


$$A = \left(\frac{1}{3}, 0\right),\ \text{asymptotes}:\ x = -3,\ y = 0$$

(4)

10.

Sketch on the same axes $y = \frac{1}{x}$ and $y = x^3$, state with a reason the number of real roots of the equation $x^3 - \frac{1}{x} = 0$



2 intersect, 2 solutions

(3)

11.

Given $A(1,2),\ B(-3,8)$

find

a. the midpoint of AB

$$M = \frac{A+B}{2} = \left(\frac{1-3}{2}, \frac{2+8}{2}\right) = (-1,5)$$

(1)

b. the gradient of AB

$$m = \frac{2-8}{1+3} = -\frac{3}{2}$$

(1)

c. the equation of the perpendicular bisector of AB in the form $ax + by + c = 0$, where *a, b* and *c* are integers.

$$y - 5 = \frac{2}{3}(x+1)$$
$$3y - 15 = 2x + 2$$
$$2x - 3y + 17 = 0$$

(3)

12.

By completing the square for the x terms and for the y terms, find the centre and radius of the circle.

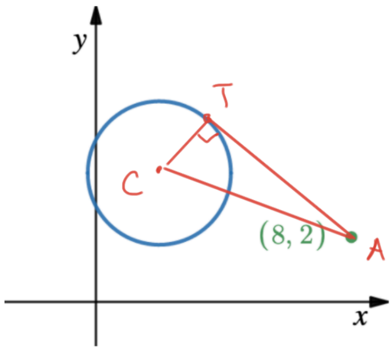
$$x^2+6x+y^2-2y=15$$
$$(x+3)^2-3^2+(y-1)^2-1^2=15$$
$$(x+3)^2+(y-1)^2=5^2$$
$$\text{centre}=(-3,1),\ \text{radius}=5$$

(3)

13.

Find the lengths of the tangests from the given points to the circle

$$x^2+y^2-4x-8y-5=0;\ (8,2)$$



$$(x-2)^2-2^2+(y-4)^2-4^2=5$$
$$(x-2)^2+(y-4)^2=25$$
$$CT=5$$
$$CA=\sqrt{(2-8)^2+(2-2)^2}=6$$
$$TA=\sqrt{6^2-5^2}=\sqrt{11}$$

(4)

14.

Given $f(x) = x^3 - 3x^2 - 4x + 12$

a. Use factor theorem to show $(x-2)$ is a factor of $f(x)$

$$f(2) = 2^3 - 3(2)^2 - 4(2) + 12 = 0$$
$$\therefore (x-2) \text{ is a factor of } f(x)$$

(1)

b. Use algebraic long division to find the other factors of $f(x)$

$$\begin{array}{r|l} & x^2 - x - 6 \\ x-2 & x^3 - 3x^2 - 4x + 12 \\ & x^3 - 2x^2 \\ \hline & -x^2 - 4x + 12 \\ & -x^2 + 2x \\ \hline & -6x + 12 \\ & -6x + 12 \\ \hline & 0 \end{array}$$

$$f(x) = (x-2)(x^2 - x - 6)$$
$$= (x-2)(x-3)(x+2)$$

(3)

c. State the set values of x such that $f(x) \leqslant 0$

$x \leqslant -2,\ 2 \leqslant x \leqslant 3$

(2)

15.

a. Write $\binom{n}{3}$ in terms of n

$$\binom{n}{3} = \frac{n!}{(n-3)!3!} = \frac{n(n-1)(n-2)}{6}$$

(1)

b. Expand $(2-x)^{10}$, in ascending order powers of x, as far as the term in x^3

$$(2-x)^{10} = 2^{10} + \binom{10}{1}2^9(-x)^1 + \binom{10}{2}2^8(-x)^2 + \binom{10}{3}2^7(-x)^3$$
$$= 1024 - 5120x + 11520x^2 - 15360x^3$$

(3)

c. Find the coefficient of x^5 for $(3-2x)^{11}$

coefficient of $x^5 = \binom{11}{5}3^6(-2)^5 = -10,777,536$

(2)