Name: ____________________

Unit 3 – Circular Motion and Gravity
Lesson Notes

3.1: Describing Circular Motion

Explain the difference between period and frequency. Then state the symbols and units for each and an equation that relates the two.

Test Your Understanding: Circle to indicate whether each quantity is a frequency, a period, or other.

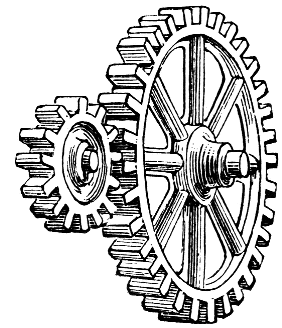
1. The second hand of a clock takes 60 seconds to travel around the clock.	60 is a: (Frequency) (Period) (Other: ____________)
2. The hour hand of a clock travels around the clock 2 times every day.	2 is a: (Frequency) (Period) (Other: ____________)
3. Each second as it orbits, the earth travels 18 miles.	18 is a: (Frequency) (Period) (Other: ____________)
4. The earth rotates 30 times each month.	30 is a: (Frequency) (Period) (Other: ____________)
5. The earth takes 365 days to complete one full orbit around the sun.	365 is a: (Frequency) (Period) (Other: ____________)
6. A trucker averages 600 miles every day.	600 is a: (Frequency) (Period) (Other: ____________)

Explain the difference between degrees and radians.

Explain "arc length" and "arc angle", and state an equation that relates them. Draw a diagram showing the two.

Explain the difference between linear speed and angular speed. Then state the symbols and units for each.

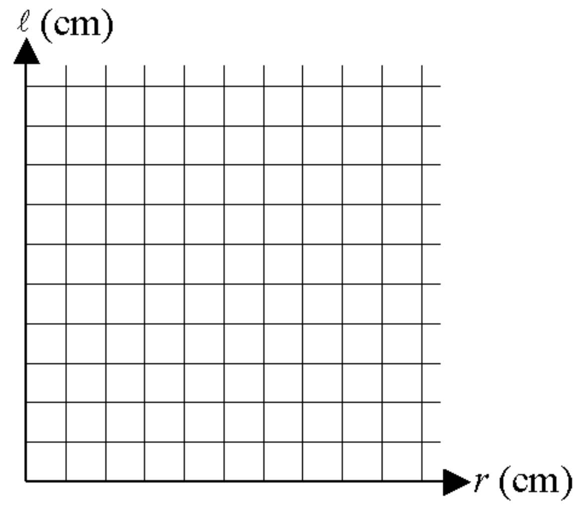
Test Your Understanding: Suppose that two gears rotate at a uniform rate. A large gear has radius r_1 and angular velocity ω_1, and the small gear has radius r_2 and angular velocity ω_2. Write an equation that relates these four quantities. What is the SAME for both of the gears that are turning with their teeth interlocked?



Pivoting Ruler Activity:

On your angle paper, on the next page, pivot the ruler about its big hole next to 29 cm. The ruler's pivot should be on the word "Axis" on the angle paper. Use the ruler to draw four arcs that have the same angle. Then use the ruler to get the radius of each arc, and use the ruler to estimate the length of each arc. Then use the protractor to find the angle of each arc. There are 360 degrees in a circle and 2π radians in a circle. To convert degrees to radians, multiply degrees by the ratio 2π/360.

Arc Radius r (cm)	Arc Length ℓ (cm)	Arc Angle (Degrees)	Arc Angle (Radians)



Linear Regression: ____________________

Plot a graph of the data on the grid. Find a linear regression for the data.

What is the significance of the slope of length vs. radius?

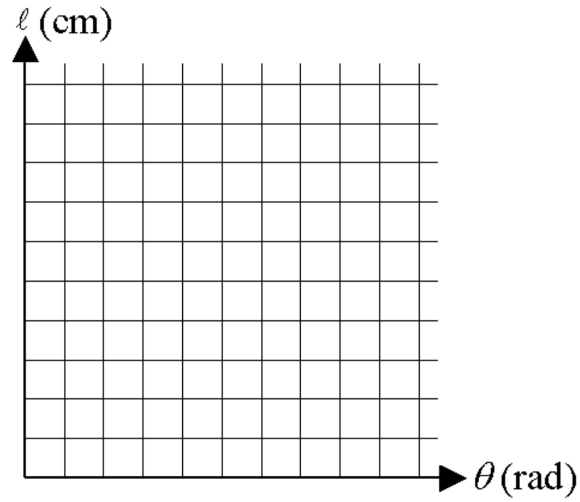
What equation can you write based on this regression?

Bike Wheel Activity:

Measure the radius of the bike wheel. Then roll the bike wheel 90° and measure the distance that the bottom of the wheel has traveled and the distance that the center of the wheel has traveled. Repeat for rotating the bike wheel 180°, 270°, and 360°. There are 360 degrees in a circle and 2π radians in a circle. To convert degrees to radians, multiply degrees by the ratio 2π/360.

Wheel Radius = r = _______________ (cm)

Angle of Rotation (Degrees)	Angle of Rotation θ (Radians)	Distance of wheel bottom (cm)	Distance of wheel center ⍰ (cm)
90			
180			
270			
360			

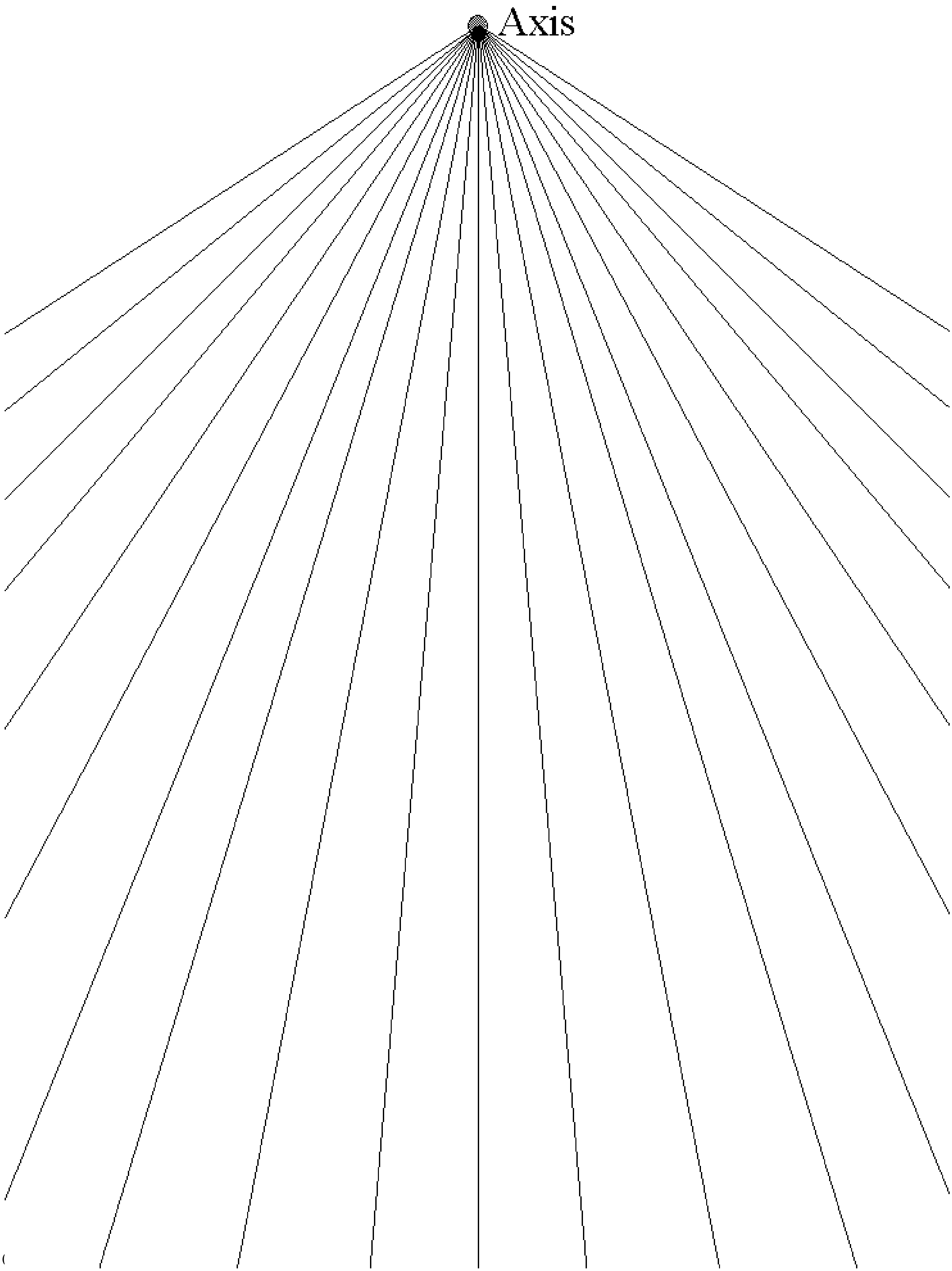


Linear Regression: ____________________

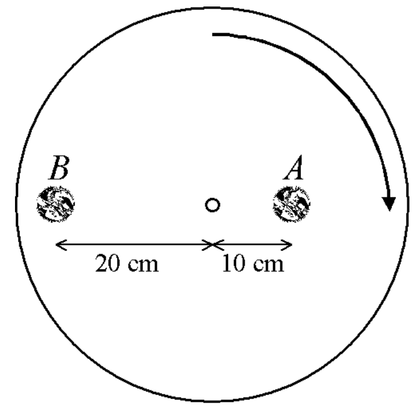
Plot a graph of the data on the grid. Find a linear regression for the data.

What is the significance of the slope of length vs. angle?

What equation can you write based on this regression?


Axis

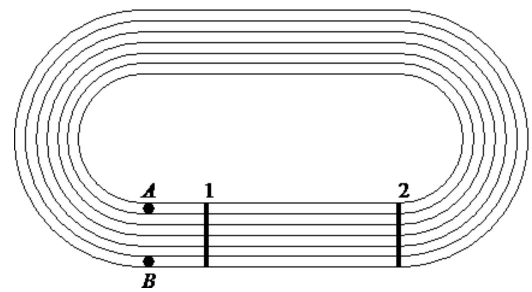
Test Your Understanding: Two coins are on the same rotating turntable. Coin A is 10 cm from the center, and coin B is 20 cm from the center.



Which coin travels the fastest speed v (length per time)? Why?

Which coin has the greater angular speed ω (angle per time)? Why?

Test Your Understanding: Students A and B pass line 1 at the same time, then pass line 2 also at the same time. They then go around the right-side semicircular section of the track.



Which student travels the fastest speed v? Why?

Which student has the greater angular speed ω? Why?

Test Your Understanding: A car with small-radius wheels and a car with large-radius wheels cross a line at the same time. They travel a straight road and cross another line 100 m away at the same time.

Which wheels had the greater linear speed v? Explain.

Which wheels had the greater angular speed ω?

Test Your Understanding: A kid's tricycle has one big wheel in the front and two small wheels in the back. When the kid rides the tricycle, which wheel has greater linear speed? Which wheel has greater angular speed? Explain your reasoning for both answers.

Test Your Understanding: Right now you are moving at hundreds of miles per hour because the Earth is rotating about its axis. Would people in Texas be moving faster, or people in Alaska? Explain how you know.

Test Your Understanding: Right now, you have a linear speed v_{you} (hundreds of miles per hour) and angular speed ω_{you} about the Earth's axis because the Earth is rotating. The moon also has a linear speed v_{moon} and angular speed ω_{moon} about the Earth's axis.

(a) Which is greater: ω_{you} or ω_{moon}? __ ω_{you} __ ω_{moon} ___ Both are the same ___ I need more information
Explain your answer.

(b) Which is greater: v_{you} or v_{moon}? __ v_{you} __ v_{moon} ___ Both are the same ___ I need more information
Explain your answer.

Example: The speedometer in your car is attached to the axle of your car; it is not attached directly to your car's wheels. A friend of mine had a car with 20-inch wheels and the speedometer was calibrated for these wheels. That means that when the speedometer measured 60 mph, the car moved 60 mph. My friend replaced all of his wheels with 24-inch wheels, but didn't change the speedometer. Now when his speedometer measured 60 mph, what was his car doing?

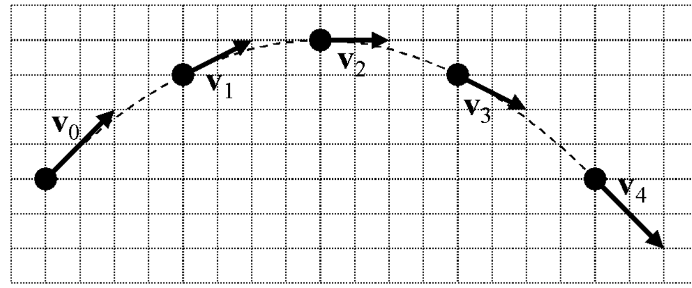
How is velocity directed when an object travels along a curved path?

How can an object have constant speed and yet be accelerating?

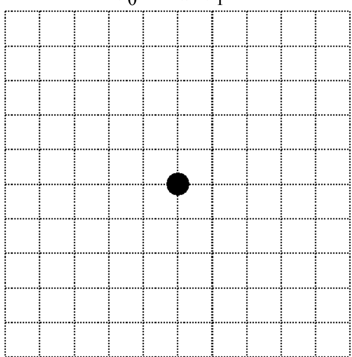
Explain "centripetal acceleration", state some basic equations for centripetal acceleration, and state the direction that centripetal acceleration points in a circular motion.

Example: The diagram below shows an object in projectile motion. The position of the object is shown at regular intervals of time. Each position has an arrow representing the object's velocity at that instant.

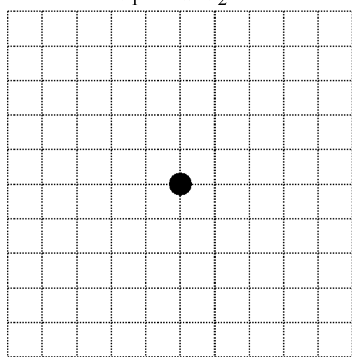
For each pair of vectors, draw both vectors on the dot *exactly to scale*, and draw the vector $\Delta\mathbf{v}$ between them.



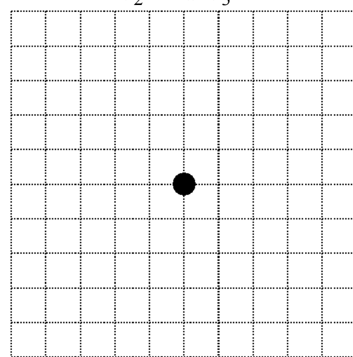
$\mathbf{v}_0$ and $\mathbf{v}_1$



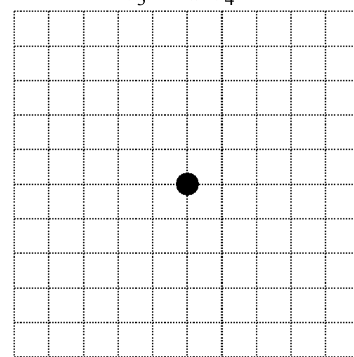
$\mathbf{v}_1$ and $\mathbf{v}_2$



$\mathbf{v}_2$ and $\mathbf{v}_3$



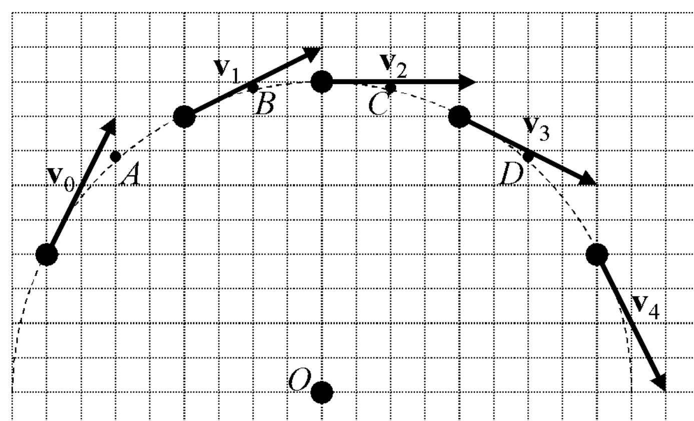
$\mathbf{v}_3$ and $\mathbf{v}_4$



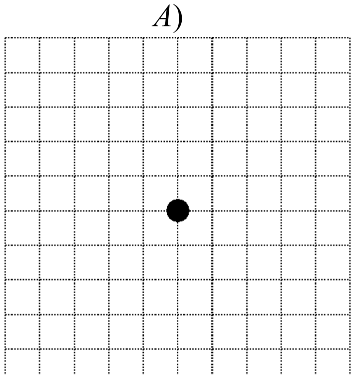
In what direction is the acceleration of EVERY object that is in projectile motion? Remember that the direction of acceleration is the same as the direction of $\Delta\mathbf{v}$.

Example: The diagram below shows an object in uniform circular motion ("uniform circular motion" means motion with constant speed in a circle of constant radius). The position of the object is shown at regular intervals of time. Each position has an arrow representing the object's velocity at that instant.

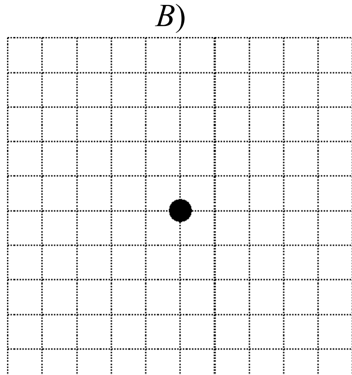
For each pair of vectors, draw both vectors on the dot *exactly to scale*, and draw the vector $\Delta\mathbf{v}$ between them. Then draw an acceleration vector at the lettered point on the diagram to the right.



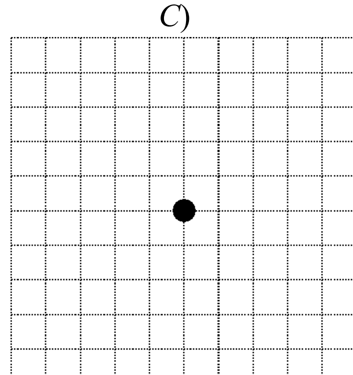
$\mathbf{v}_0$ and $\mathbf{v}_1$ (Draw **a** vector at point *A*)



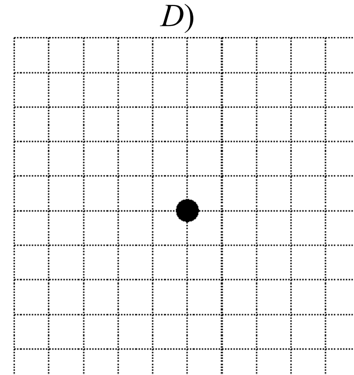
$\mathbf{v}_1$ and $\mathbf{v}_2$ (Draw **a** vector at point *B*)



$\mathbf{v}_2$ and $\mathbf{v}_3$ (Draw **a** vector at point *C*)

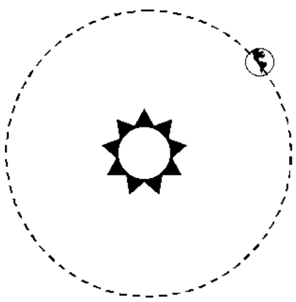
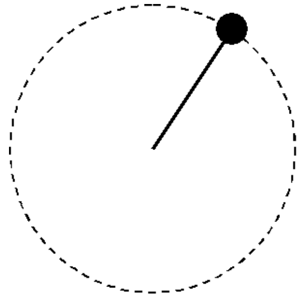
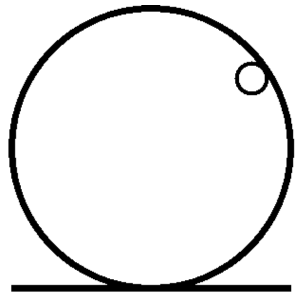
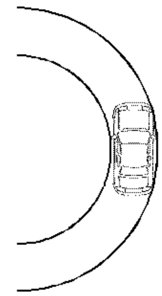
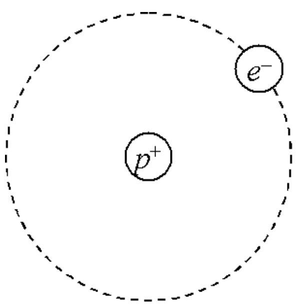
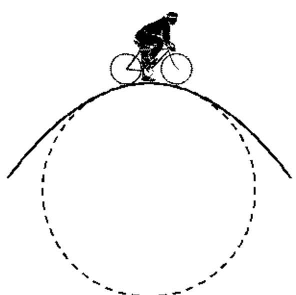
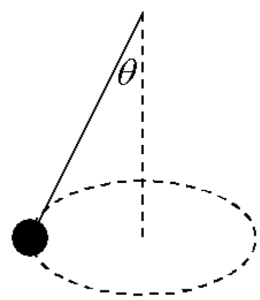
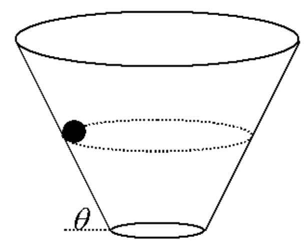


$\mathbf{v}_3$ and $\mathbf{v}_4$ (Draw **a** vector at point *D*)



In what direction is the acceleration of EVERY object that is in uniform circular motion?

Test Your Understanding: All of the following objects move in a circle at constant speed. Draw and label *only the forces that each example asks for*.

Earth orbits the sun. Draw the gravity force on Earth. 	Ball on a string is whirled in a circle (seen from above). Draw tension. 	Ball rolls around a vertical track. Draw the normal force. 	Car goes around a curve (seen from above). Draw the friction force. 
An electron orbits a proton in an H-atom. Draw the electric force. 	Biker goes over the top of a hill. Draw weight and normal. 	A rock on a string moves in a horizontal circle. Draw weight and tension. 	Ball rolls along the dotted path inside of the cone. Draw weight and normal. 

When an object moves in uniform circular motion, what is true about the direction of the net force acting on the object?

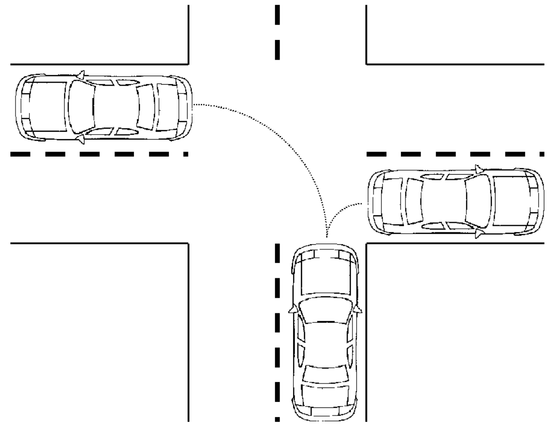
Write an equation expressing Newton's Second Law for the biker going over the hill: ____________________

Write an equation expressing Newton's Second Law for the rock on a string: __________________________

Write an equation expressing Newton's Second Law for the ball in the cone: ___________________________

Explain how to construct a Newton's Second Law equation for an object moving in circular motion.

Example: Consider a car that approaches an intersection to make a turn. The car can either make a left turn or a right turn as shown in the diagram. Assume that the car approaches, makes the turn, and continues forward all without changing its speed.



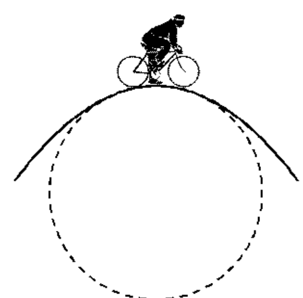
The centripetal force is provided by the force of static friction, which is determined by the relationship $F_{fs} \leq \mu_s F_N$.

Explain why the force of static friction (not kinetic) acts on the car even though the car is in motion.

Explain what happens if the value of mv^2/r is greater than the value of $\mu_s F_N$.

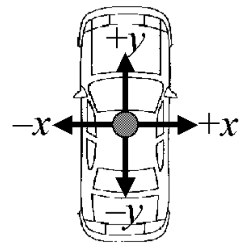
Which turn (left or right) requires the car to slow down more in order to make the turn safely? Explain your answer using appropriate relationships.

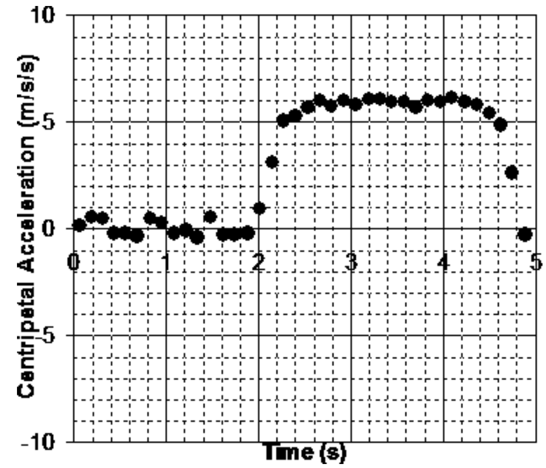
Example: The bicycle-and-rider of mass m rides over a hill with radius of curvature R with constant speed speed v. Draw the forces acting on the bicycle-and-rider and derive an equation for the normal force acting on the rider in terms of m, R, v, and g. Then state two things about the problem that could be changed so that the rider loses contact with the hill and becomes a projectile.



Test Your Understanding: A car takes 3 seconds to make a left turn without changing speed at the intersection of two perpendicular streets. What is the period of the car's circular motion?

Example: An accelerometer is placed in a car. The accelerometer measures the acceleration of the car (in the car's frame of reference) along the two axes (x and y) shown. The car is initially traveling along a straight path. The car then makes a 90° turn without changing speed. The acceleration along the x-axis is shown in the graph. The acceleration measured along the y-axis is not shown.



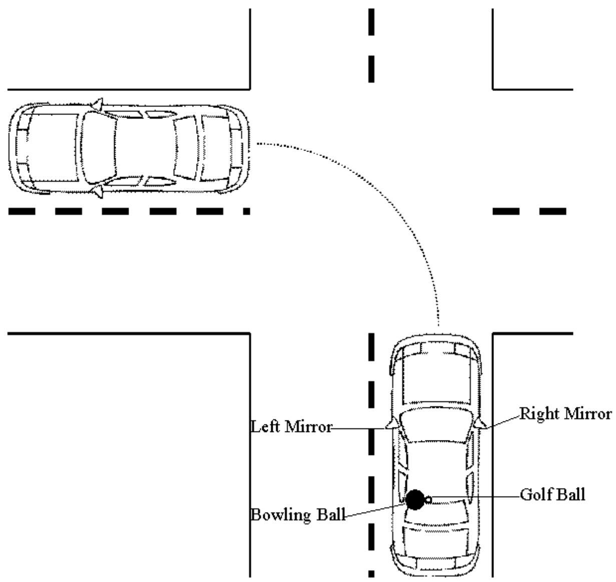


What would the graph of the acceleration along the y-axis look like as a function of time? What evidence from the question statement would indicate this?

Does the car make a left turn or a right turn? What evidence from the graph indicates this?

Use the graph to determine both the speed of the car as it turns and the radius of the turn. Each step of mathematical work must be accompanied by words explaining what is being done and what evidence from the graph is being taken.

Why there is no centrifugal force: Consider a car that makes a very fast left turn as shown below. In the back seat are a bowling ball and a golf ball. Assume the car does not roll over.



Draw the path that the left mirror takes during the turn.

Draw the path that the right mirror takes during the turn.

Draw the path that the bowling ball and the golf ball take during the turn. Assume the back seat is frictionless.

Are the balls "forced to the outside of the circle"? Explain.

Explain "tangential acceleration" (a_t) and "centripetal acceleration" (a_c) in a way that is easy to understand. What is the difference between the two?

If someone says "6 m/s^2 means gaining 6 m/s of speed every second", is the 6 m/s^2 a tangential acceleration or a centripetal acceleration? Explain why.

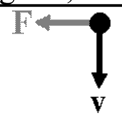
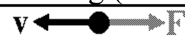
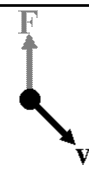
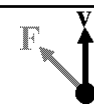
If someone says "An object's tangential acceleration is 3 m/s^2 and its centripetal acceleration is 4 m/s^2; what is its net acceleration?" then what would the correct answer be? Why?

Test Your Understanding: Describe what an object does in each of the following cases. As part of your description, state whether the path is straight or curved, and whether the speed increases, decreases, or remains the same.

- An object has no a_t and no a_c.
- An object has a_t that is in the same direction as velocity, and no a_c.
- An object has a_t that is in the opposite direction as velocity, and no a_c.
- An object has no a_t but does have a_c.
- An object has a_t that is in the same direction as velocity, and has a_c.

- An object has a_t that is in the opposite direction as velocity, and has a_c.

Test Your Understanding: Each of the following dots represents a car driving forward (in the driver's frame of reference). Each dot shows the velocity vector *v* and the net force vector *F* acting on the car as seen from above. State whether the car is speeding up, slowing down, or driving with constant speed, and whether the car is turning left, turning right, or not turning (in the driver's frame of reference).

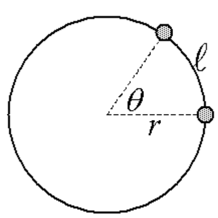
Test Your Understanding: In each box, draw a motion diagram showing an object moving with either constant speed, speeding up, or slowing down, and having or not having centripetal acceleration. Draw the dots with one color, draw at least five dots in each diagram. On each dot, draw velocity vectors labeled *v* in a different color. Draw acceleration vectors in a third color. If there is either a_t or a_c, label the acceleration vector either a_t or a_c. If there are both types of acceleration, label the acceleration vector *a*, and draw components for tangential and centripetal acceleration and label them a_t and a_c.

	Constant Speed	Speeding Up	Slowing Down
No Centripetal Acceleration			

Yes Centripetal Acceleration			

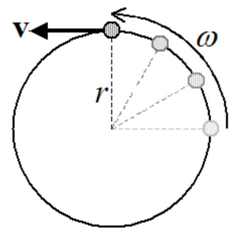
What is rotational acceleration? Explain what it is in your own words, state the symbol, units, and a basic equation.

Each diagram shows three letters. Under each diagram, write the correct equation that relates the three letters.



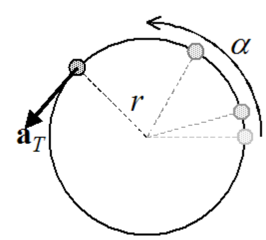
When an object rotates an angle θ radians, the object travels a length x meters.

Equation:



When an object's rate of rotation is ω radians/second, the object's speed is v meters/ second.

Equation:



When an object's rate of rotation increases by α rad/sec every second, the object's speed increases a_T m/s every second.

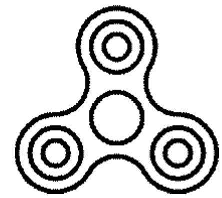
Equation:

The table below lists the four equations for translational kinematics. Write the four corresponding equations for rotational kinematics to complete the table.

<u>TRANSLATIONAL</u>	↔	<u>ROTATIONAL</u>

Constant Velocity	$x = vt + x_0$	$\leftrightarrow$		Constant Angular Velocity
Constant Acceleration	$v = at + v_0$	$\leftrightarrow$		Constant Angular Acceleration
	$x = \frac{1}{2}at^2 + v_0 t + x_0$	$\leftrightarrow$		
	$v^2 = 2ax + v_0^2$	$\leftrightarrow$		

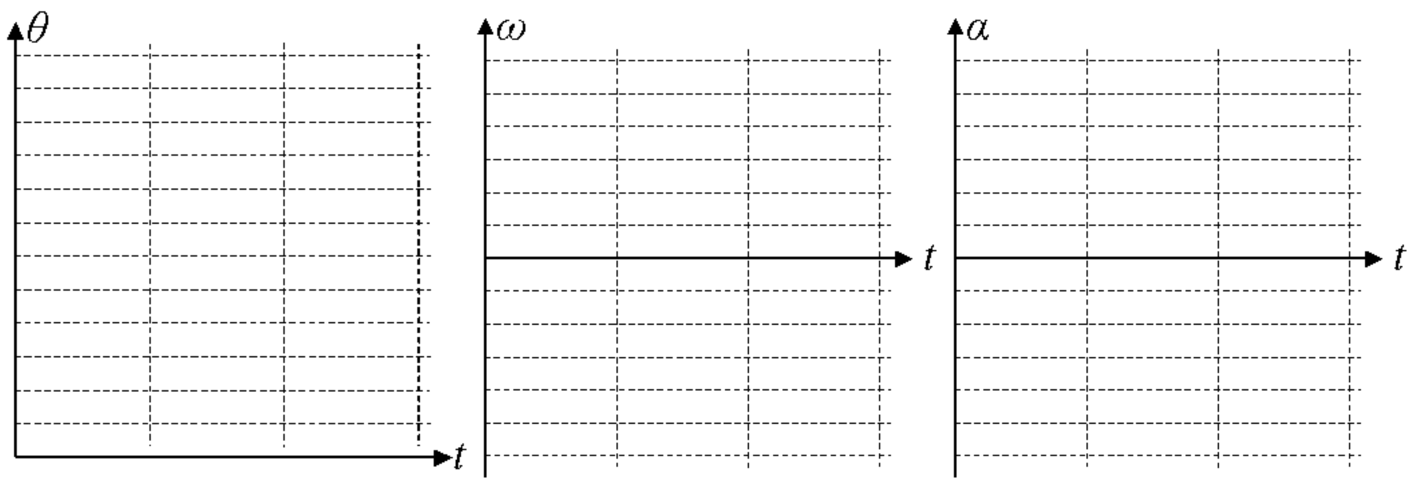
Fidget Spinner Activity: You are given a fidget-spinner that looks like the one shown to the right. Your teacher turns on a strobe light that flashes with a frequency of __________ Hz. When you spin the spinner, it slows down as it spins. At one particular instant of time, the spinner appears to be "standing still" in the strobe light. After this instant, it takes ___________ seconds for the spinner to come to rest.



(a) Calculate the angular speed of the spinner at the instant it appears to be standing still in the strobe light. Explain where the quantities that you use for the calculation come from.

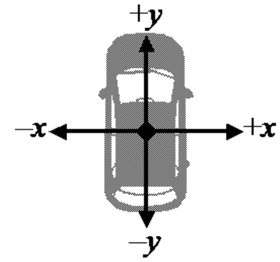
(b) Using the angular speed and the time it takes for the spinner to stop, calculate the angular acceleration of the spinner as its rotation slows down.

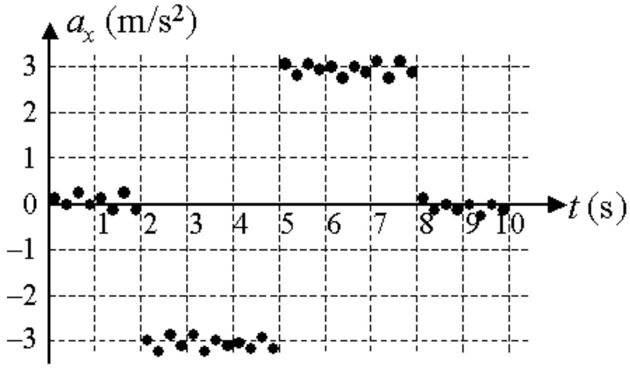
Example: A yo-yo is released from rest. The yo-yo falls and unrolls from its string as it accelerates toward the ground. After 1 second, the yo-yo reaches the end of its string. The yo-yo continues to spin but slows down over the course of another 2 seconds until it stops rotating. Sketch graphs of angular displacement θ, angular velocity ω, and angular acceleration α as functions of time for the yo-yo during these 3 seconds.

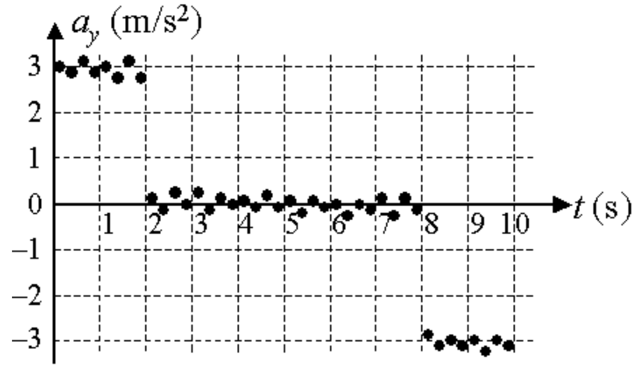


Example: Almost all smart phones come equipped with a device called an "accelerometer", which can measure the phone's acceleration in any direction. Apps can be downloaded to use the accelerometer to measure the phone's acceleration in any of the three dimensions.

Suppose a physicist attaches their smart phone to the bottom of their car. The car is initially at rest and facing north in an empty parking lot. The accelerometer measures acceleration in the x-direction (left and right, always in the driver's frame of reference) and y-direction (forward and backward in the driver's frame of reference) as indicated in the diagram to the right over a ten-second interval. The graphs of the accelerometer's output are shown below.







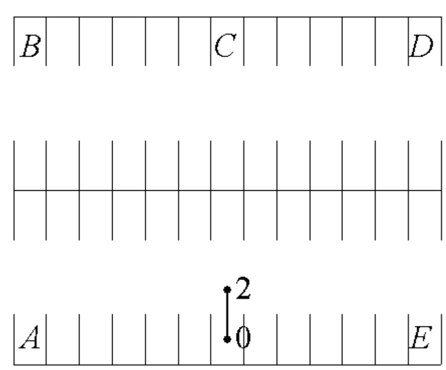
(a) Calculate the following. For each quantity, explain how the graphs are used to make the calculation.

i. The speed of the car at time $t = 7$ seconds

ii. The radius of curvature of the car's circular motion at time $t = 7$ seconds

(b) Briefly explain in words how the driver operates the car during the ten-second interval. Discuss the use of the accelerator pedal, brake pedal, and steering wheel.

(c) The diagram shows the car at point 0 in its initial parking space in the parking lot. Five parking spaces are labeled A through E. The point labeled 2 is the location of the car at time $t = 2$ s; the path that the car takes between $t = 0$ and $t = 2$ s is drawn with the straight line that connects the points.



i. Which of the labeled parking spaces does the car end up in at time t = 10 seconds? _____

ii. Draw the rest of the car's path between $t = 2$ s and $t = 10$ s. On the path, label two points "5" and "8" representing the location of the car at times $t = 5$ s and $t = 8$ s, respectively.

<u>3.2: Universal Gravitation</u>

What makes a force a "fundamental force"?

Name the three fundamental forces and briefly explain each one in a sentence.

The normal force is actually a manifestation of which of the forces you just listed? Explain why.

Name two other forces we have learned about that are not fundamental forces.

We learned that the equation for the force of gravity is $F_g = mg$. Why is this equation not "universal"? Under what circumstances can we use this equation?

State the equation for the Law of Universal Gravitation. State the name of the constant G and its value and units. State specifically what m_1 and m_2 represent, and what r represents.

What type of relationship exists between the force of gravity and the distance of the object's separation?

Test Your Understanding: What is the force of gravity between the earth (mass 5.98×10^{24} kg) and the moon (mass 7.35×10^{22} kg) if the distance between them is 3.84×10^{8} m?

On which object is this force applied? ___ Earth ___ Moon
Which object has greater acceleration because of this force? ___ Earth ___ Moon

Test Your Understanding: Planet *X* and its moon *Y* exert a gravitational force *F* on each other. Determine the new gravitational force *F*' in each of the following cases.

The mass of *X* doubled.	The mass of *Y* doubled.	The mass of both *X* and *Y* doubled.
The distance between doubled.	The distance between is 1/3 as much.	Both masses doubled, and the distance between them is tripled.

What is the equation that gives the gravitational field strength *g* on the surface of a planet of mass *M* and radius *R*? Show how this equation is derived from the two equations for the force of gravity.

What are the two units that *g* can have? Explain what each of the two units physically represent or communicate.

If the radius of a planet is multiplied by *N*, what is the volume of the planet multiplied by? Explain why using a basic equation from geometry.

Test Your Understanding: The gravitational field strength on the Earth is 10 N/kg. What is the gravitational field strength on another planet where…

(a) The radius is half that of the Earth, but the density is the same?

(b) The radius is twice that of the Earth, but the density is half as much?

(c) The radius is three that of the Earth, and the density is twice as much?

What basic equation do we always write when we deal with a circular orbit? Explain where this equation comes from. Also state another equation that relates orbital speed v to orbital period T and orbital radius R.

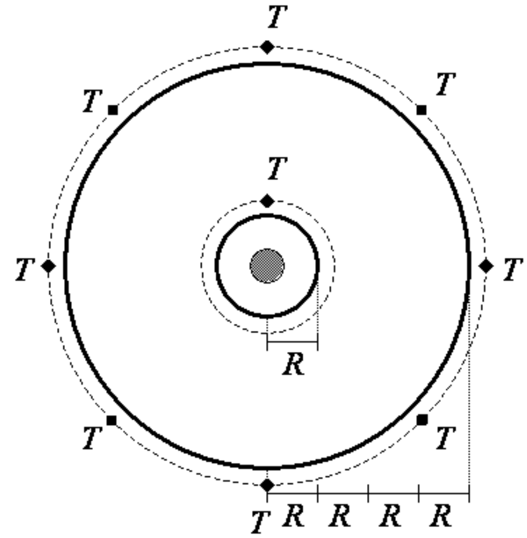
An object of mass m orbits a planet of radius M with orbital radius R. Use the equation you wrote above to derive equations for:

(a) The speed v that the object has as it orbits.

(b) The period T of the object's orbit.

Test Your Understanding: A satellite's orbit is called "geosynchronous" if the period of the orbit is exactly one day, so that the satellite is always above the same point on the Earth's surface. Determine the radius of orbit and the speed of a geosynchronous satellite. Use $M_{Earth} = 5.98 \times 10^{24}$ kg and $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ and a calculator.

Write Kepler's Third Law (known as the square-cube law) as a short sentence and as a ratio equation.



Four times the radius results in eight times the orbital period.

Show the derivation of Kepler's Third Law.

Test Your Understanding: Suppose you know that the Moon orbits the Earth with a radius of 3.84×10^8 m and takes 27.3 days to go around the Earth. A geosynchronous satellite takes one day to go around the Earth. What is the radius of a geosynchronous satellite?

Test Your Understanding: Earth's distance from the Sun is defined as 1 astronomical unit (AU) and Earth takes one year to go around the Sun. Saturn is 9 AU from the Sun. How long does Saturn take to go around the Sun?

Example: Suppose you have data for the period *T* of the orbits of five of Jupiter's moons. You also know the radius *R* of each moon's orbit. What data should be plotted to give a straight-line fit to the data? What would the slope of the graph be?

Horizontal Variable: _____ Vertical Variable: _____ Slope: _______________

Objects in orbit appear weightless. This is NOT because there is zero gravity. In the space below, explain why there can never be zero gravity. Then explain why objects that are in orbit appear weightless.