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MODULE 1 - Ratios and Unit Rates

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. c. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Level	Skill Level Description	Standard	Sample Task	Eureka Lessons
4.0	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	6.RP.A.3d		
3.5	Calculates and compares rates and ratios in different forms (tables, tape diagrams, double number lines) to answer real-world problems.	6.RP.A.3		
3.0	- Solve unit rate problems including those involving unit pricing and constant speed.Solves unit rate problems--pricing, speed. - Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity);	6.RP.A.3b 6.RP.A.3c	3b. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? 3c. 30% of a quantity means 30/100 times the quantity	

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	solve problems involving finding the whole, given a part and the percent.			
2.5	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	6.RP.A.3		Topic A Lesson 3-6
2.0	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. Plots ordered pairs from a ratio table on the coordinate plane.	6.RP.A.3a		
1.5	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. Describes a ratio relationship using rate and ratio language.	6.RP.A.2	For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”2	Topic A Lessons 7-8
1.0	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. Writes a ratio that describes the relationship between two quantities.	6.RP.A.1	For example, “The ratio of wings to beaks in the bird house at the zoo was 2: 1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”	Topic A Lessons 1-2
0.5	- Plots points correctly on a coordinate plane. - Understands that a fraction represents division of the numerator by the denominator.	5.GA.1 5.NFB.3		
0.0	Minimal understanding of the coordinate plane. Minimal understanding of multiplicative or additive comparison.	4.OA.A.2		
Pacing	**Topic A**: Combine Lessons 1&2, Teach Lessons 3-8			

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MODULE 2 - Arithmetic Operations Including Division of Fractions

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.

6.NS.B.2 Fluently divide multi-digit numbers using the standard algorithm.

6.NS.B.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Module 2 - Learning Progression #1

Level	Skill Level Description	Standard	Sample Task	Eureka Lessons
4.0	Interpret and represent word problems with an equation and a drawing	6.NS.A.1	For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	
3.5	Compute quotients of fractions using equations	6.NS.A.1		
3.0	Compute quotients of fractions using visual models	6.NS.A.1		
2.5	Divide fractions by whole numbers.	5.NF.B.7		
2.0	Multiply fractions by whole numbers		For example, express 36 + 8 as 4 (9 + 2).	

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	Find the greatest common factor and least common multiple within 100.	6.NS.B.4		
1.5	Find greatest common factor and least common multiple within 36	6.NS.B.4		
1.0	Find factors and multiples within 100	4.OA.B.4		
0.5	Find factors and multiples within 36	4.OA.B.4		
0.0	Minimal understanding of multiplication facts within 100.	4.OA.B.4		

Module 2 - Learning Progression #2				
Level	**Skill Level Description**	**Standard**	**Sample Task**	**Eureka Lessons**
4.0	Fluently divide multi-digit decimals using the standard algorithm for each operation.	6.NS.B.3		
3.5	Fluently multiply multi-digit decimals using the standard algorithm for each operation.	6.NS.B.3		
3.0	Fluently divide multi-digit whole numbers using the standard algorithm.	6.NS.B.2		
2.5	Use a model to divide whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations,and/or the relationship between multiplication and division.	5.NBT.B.6		

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2.0	Fluently add, subtract multi-digit decimals using the standard algorithm for each operation.	6.NS.B.3		
1.5	Fluently add, subtract multi-digit decimals using a model for each operation.	5.NBT.B.6		
1.0	Add and subtract multi-digit whole numbers using the algorithm.	3.NBT.A.2		
0.5	Add and subtract multi-digit whole numbers using a model.	2.OA.C.3 2.OA.C.4		
0.0	Minimal knowledge adding and subtracting whole numbers without support.			

MODULE 3 - Rational Numbers

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

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d. Distinguish comparisons of absolute value from statements about order.
6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Level	Skill Level Description	Standard	Sample Task	Eureka Lessons
4.0	- Solves real-world and mathematical problems by graphing points in all four quadrants in all four quadrants of the coordinate plane. Includes use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. - Writes, interprets, and explains statements of order for rational numbers in real-world contexts.	6.NS.C.8 6.NS.C.7b		
3.5	- Uses absolute value to interpret statements involving positive and negative quantities in real world contexts. - Reflects points over axes in all four quadrants of the coordinate plane. - Compares positive and negative quantities in real world contexts using absolute values. Interprets absolute value as magnitude for a positive and negative quantity in real-world contexts. - Distinguishes comparisons of absolute value from statements about order.	6.NS.C.5 6.NS.C.6b 6.NS.C.7c 6.NS.C.7d	6.NS.C.7c For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars. 6.NS.C.7d For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars	
3.0	- Plots points in all four quadrants of a coordinate plane.	6.NS.C.6c 6.NS.C.7a 6.NS.C.7b	6.NS.C.7a For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right. 6.NS.C.7b For example, write −3°C > −7°C to	

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	- Understands ordering and absolute value of rational numbers using their distance from zero on a number line. - Writes and interprets inequality statements comparing positive and negative numbers.		express the fact that −3°C is warmer than −7°C.	
2.5	Describes positive and negative numbers in real-world contexts. (ex. elevation, charges and debits)	6.NS.C.5	(e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge)	
2.0	Plots positive and negative points on a single number line.	6.NS.C.6.c		
1.5	Understands that the opposite of the opposite of a number is that number.	6.NS.C.5		
1.0	Understands that positive and negative numbers are used together to describe quantities having opposite directions or values. Recognizes that opposite signs mean numbers are on opposite sides of zero on a number line.	6.NS.C.5		
0.5	Plots ordered pairs within the first quadrant on a coordinate plane.	5.G.1		
0.0	Has minimal understanding of ordering positive rational numbers.			

MODULE 4: Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

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a. Write expressions that record operations with numbers and with letters standing for numbers. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

6.EE.A.3 Apply the properties of operations to generate equivalent expressions.

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Reason about and solve one-variable equations and inequalities.

6.EE.B.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.

6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Level	Skill Level Description	Standard	Sample Task	Eureka Lessons
4.0	- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. - Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world	6.EE.B.8 6.EE.B.5 6.EE.C.9		

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	mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. - Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.		For example, in a problem involving motion at constant speed,list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	
3.5	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers.	6.EE.B.7		
3.0	- Write numerical expressions (in story problems) involving whole number exponents and variables (letters that stand for numbers.) - Identify when two expressions are equivalent. (ex. 3y is y+y+y). - Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	6.EE.A.1 6.EE.A.4 6.EE.B.6		
2.5	- Evaluate numerical expressions involving whole-number exponents and variables (letters that stand for numbers.)	6.EE.A.1 6.EE.A.2a 6.EE.A.3		

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	- Apply the properties of operations to generate equivalent expressions.			
2.0	Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations.)	6.EE.A.2c		
1.5	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient): view one or more parts of an expression as a single entity. (2(8+7) is the product of two factors)	6.EE.A.2b		
1.0	- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. - Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. - Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	5.OA.A.2 5.OA.B.3 5.NBT.A.2		
0.5	- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number	5.G.A.1 4.OA.B.4 4.MD.C.5 4.MD.C.7		

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	indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). - Find all factor pairs for a whole number in the range 1-100. - Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. - Recognize angle measures as additive.			
0.0	- Apply properties of operations as strategies to add and subtract. (Commutative & Associative properties) - Apply properties of operations as strategies to multiply and divide. (Commutative & Associative properties)	1.OA.B.3 3.OA.B.5		

MODULE 5: Area, Surface Area and Volume Problems

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

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6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V=lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Level	Skill Level Description	Standard	Sample Task	Eureka Lessons
4.0	*Apply area, surface area, and volume formulas and nets of shapes below to real world problems.*	6.G.A.1 6.G.A.2 6.G.A.3 6.G.A.4		
3.5	- Find the area of ***special quadrilaterals, and polygons*** by composing into rectangles or decomposing into triangles and other shapes. - ***Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures***. - Draw polygons in the coordinate plane given coordinates for the vertices; ***use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.*** - Solve real-world and mathematical problems by writing and solving equations of the form *x+p=q* and *px=q* for cases in which, p, q, and x, are all nonnegative rational numbers.	6.G.A.1 6.G.A.4 6.G.A.3 6.EE.B.7		
3.0	- Use the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths.	6.G.A.2 6.G.A.1 6.G.A.3		

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	- Find the area of ***right triangles, other triangles*** by composing and decomposing into separate shapes. - **Draw polygons in the coordinate plane given coordinates for the vertices.** - ***Identify three-dimensional figures using nets made up of rectangles and triangles.*** - Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	6.G.A.4 6.NS.C.8		
2.5	Show that the volume is the same as would be found by multiplying the edge lengths of the prism.	6.G.A.2		
2.0	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths.	6.G.A.2		
1.5	- Recognize volume as an attribute of solid figures and understand concepts of volume measurement. - Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. - Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.	5.MD.C.3 5.MD.C.4 5.MD.C.5		
1.0	- Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number	5.G.A.1 5.G.A.2		

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	indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond. - Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.			
0.5	- Apply the area and perimeter formulas for rectangles in real world and mathematical problems. - Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	4.MD.A.3 4.G.A.2		
0.0	- Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. - Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. - Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.	1.G.A.2 2.G.A.2 3.G.A.2		

MODULE 6: Statistics

Develop understanding of statistical variability.

6.SP.A.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the

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answers.

6.SP.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.A.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.B.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

6.SP.B.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measure of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Level	Skill Level Description	Standard	Sample Task	Eureka Lessons
4.0	Quantitative measure of center, variability, deviation and shape of data distribution.	6.SP.A.5c 6.SP.A.5d		
3.5	Interpret data for center and variation.	6.SP.A.3		
3.0	Display (provided) numerical data in number line, dot plot, histogram, and box plots.	6.SP.A.4 6.SP.A.5b		
2.5	Describe center and shape of data. (Include vocabulary of mean, median, deviation, range.)	6.SP.A.2 6.SP.A.5a		
2.0	Recognize whether a question would give you a sample of data on which to base statistics.	6.SP.A.1		
1.5	Write, read, and evaluate expressions in which letters	6.EE.A.2		

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	stand for numbers.			
1.0	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.	5.MD.B.2		
0.5	- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. - Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	5.NBT.B.6 5.NBT.B.7		
0.0	Fluently multiply multi-digit whole numbers using the standard algorithm.	5.NBT.B.5		