Conceptual Curriculum Map (CCM)

Content Area **Mathematics** **Course** **Precalculus 31**

Grade Level **9 - 12**

Version 2: Curriculum Mapping in conjunction with Long-Term Outcomes

Units 0-1 Functions and Their Properties: Polynomials and Rationals	**Long-Term Outcomes/Transfer Goals:** **TRANSFER GOALS** *Students will be able to independently use their learning to…* ● Analyze and model mathematical relationships in authentic and varied contexts, make informed decisions, and draw conclusions. ● Construct viable arguments, critique the reasoning of others, and communicate ideas precisely using the language of mathematics. ● Share diverse ideas and perspectives, ask questions, and respectfully engage with peers while working towards a common goal. ● Persevere, think strategically/flexibly, and reflect and revise thinking in order to solve complex problems		
	Standards	**Conceptual Overview**	**Rationale**
Focus & Timeframe Polynomial Functions and Rational Functions (28 days)	**HSF.IF.C.7.C** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **HSF.IF.C.7.D** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **HSF.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **HSF.IF.B.4** For a function that models a relationship	The degree and sign of the leading term of a polynomial determines the end behavior of the polynomial function. Students will understand how characteristics of functions including increasing/decreasing behavior, concavity, and end behavior affect the shape of the graph. Students will understand the relationship between the end behavior of a rational function and horizontal asymptotes. Students will understand how to find and interpret both local/relative and absolute/global extrema (minimum/maximum) for polynomial functions.	Students begin this unit by extending their prior knowledge of Polynomials from Algebra II, such as what a polynomial is, and how certain key characteristics are displayed and used in various functions. Students review real numbers, absolute value, equations for lines, complex numbers, and systems of equations - the foundation for the rest of the course. Students investigate this throughout the unit, as well as learn other new characteristics from parent functions they may not have discovered before. This will then segue into a discovery of rational functions, and

	between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **HSF.BF.B.3** Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Students will understand symmetry related to both even and odd functions.	how they differ from polynomials. Students will use these parent functions of polynomials and rationals to define new graphs, explore new connections, and identify unique relationships that exist between polynomial functions, as well as those that exist between rational functions.
Unit 2 Exponentials and Logarithms	**Long-Term Outcomes/Transfer Goals:** **TRANSFER GOALS** *Students will be able to independently use their learning to…* ● Analyze and model mathematical relationships in authentic and varied contexts, make informed decisions, and draw conclusions. ● Construct viable arguments, critique the reasoning of others, and communicate ideas precisely using the language of mathematics. ● Share diverse ideas and perspectives, ask questions, and respectfully engage with peers while working towards a common goal. ● Persevere, think strategically/flexibly, and reflect and revise thinking in order to solve complex problems		
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Focus & Timeframe Exponential Functions and Logarithmic Functions (11 days)	**HSF.IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$. **HSA.SSE.B.4** Derive the formula for the sum of a finite geometric	Students will understand how to write explicit rules for arithmetic and geometric sequences. Students will be introduced to summation notation to find sums of series. Students will use sequences and series to model and solve real life problems.	Students will understand how arithmetic and geometric sequences work, and be able to connect these sequences to linear and exponential functions. Due to this connection, students will be able to pair arithmetic sequences with linear functions, and geometric with exponential functions in order to model and solve real world problems.

	series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.** **HSF.BF.B.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **HSF.IF.C.7.E** Graph exponential and logarithmic functions, showing intercepts and end behavior. **HSF.LE.A.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. **HSA.CED.A.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. **HSF.IF.8b**	Students will become familiar with exponential functions, and use them to model both exponential growth and decay. Students will understand "e" as the base of growth rate for all continually growing processes. They will use exponential models to make predictions about the dependent variable in contexts including compound interest, radioactive decay, and human memory. Students will use linear, quadratic, and exponential regression equations to model real world scenarios. Students will write equations for compositions of functions, and make connections between the composition of functions and inverse functions. Students will understand that a logarithm represents the inverse of an exponent.	Understanding these connections will help students succeed in this unit, as well as giving students the required background knowledge to succeed in Calculus. This study of exponential and logarithmic functions will prepare students for the variety of real-world problems in which exponential and logarithmic functions are used to model growth and decay. Students' understanding of inverse functions will be reinforced by studying exponential and logarithmic functions together. Working with logarithms and logarithmic functions is also an essential skill in Calculus that is developed in this unit.

	Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. **HSF.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **HSF.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.		
Unit 3 Trigonometry and Polar Functions	**Long-Term Outcomes/Transfer Goals:** **TRANSFER GOALS** *Students will be able to independently use their learning to…* ● Analyze and model mathematical relationships in authentic and varied contexts, make informed decisions, and draw conclusions. ● Construct viable arguments, critique the reasoning of others, and communicate ideas precisely using the language of mathematics. ● Share diverse ideas and perspectives, ask questions, and respectfully engage with peers while working towards a common goal. ● Persevere, think strategically/flexibly, and reflect and revise thinking in order to solve complex problems		

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Focus & Timeframe The Unit Circle, Trigonometric Functions, Identities, and Equations (22 days)	**CCSS.MATH.CONTENT.HSF.TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **CCSS.MATH.CONTENT.HSF.TF.B.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **CCSS.MATH.CONTENT.HSF.TF.B.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **CCSS.MATH.CONTENT.HSF.TF.C.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. **CCSS.MATH.CONTENT.HSF.TF.C.9** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	Students learn how to evaluate and graph the trigonometric functions sine and cosine. Students are introduced to radian measure and the unit circle. Students learn how to find trigonometric ratios of an acute angle by drawing a right triangle and of any angle by drawing a unit circle and a reference angle. Students learn how to graph and evaluate the reciprocals and inverses of trigonometric functions. Students use trig ratios to solve problems in a variety of contexts such as mechanics, biology, and navigation. Students will use equivalent trigonometric identities arising from Pythagorean, angle sum, and double angle identities to solve equations. Students will identify key features of the polar graphs of circles, roses, and limacons including symmetry, intercepts, domain and range, and maximum and minimum values.	Students will have been introduced to trigonometric functions, radians, and the unit circle in Geometry. This unit builds on students' prior knowledge by using the unit circle to identify exact values of trigonometric functions evaluated for angles in radians which is necessary for Calculus. It also introduces graphs of trigonometric functions, and modeling of real-world periodic phenomena. Understanding graphs and working with trigonometric identities to solve equations will also prepare students to work with trigonometric functions later in Calculus.

Unit 4 Parametrics, Vectors, and Matrices	**Long-Term Outcomes/Transfer Goals:** **TRANSFER GOALS** *Students will be able to independently use their learning to…* ● Analyze and model mathematical relationships in authentic and varied contexts, make informed decisions, and draw conclusions. ● Construct viable arguments, critique the reasoning of others, and communicate ideas precisely using the language of mathematics. ● Share diverse ideas and perspectives, ask questions, and respectfully engage with peers while working towards a common goal. ● Persevere, think strategically/flexibly, and reflect and revise thinking in order to solve complex problems		
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Focus & Timeframe Polar Coordinates, Equations, and Graphs; and Parametric Equations (10 days)	**HSF.IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **HSF.IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **HSF.BF.B.3** Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Students will identify key characteristics of a parametric planar motion function that are related to position, direction, and rate of change. Students will identify key characteristics of a vector, and perform basic operations on vectors. Students will be able to find sums, differences, products, inverses and determinants of matrices. Students will learn how linear transformations and matrices are related. Students will learn how to construct and use matrices and matrix multiplication to model probability scenarios	Converting to parametric and polar equations and understanding the polar coordinate system helps students think more flexibly about working with equations and modeling, and is relevant to real-world phenomena such as the path of an object and engineering applications. This understanding and the skills developed will be useful in Calculus and other advanced math classes. Matrix algebra provides a powerful technique to manipulate large data sets and solve the related problems that are modeled by the matrices. Transformations are powerful mathematical tools. If we can show that our transformation is a linear

			transformation, we can represent it using a transformation matrix.
Unit 5 **An Introduction to Calculus**	**Long-Term Outcomes/Transfer Goals:** **TRANSFER GOALS** *Students will be able to independently use their learning to…* ● Analyze and model mathematical relationships in authentic and varied contexts, make informed decisions, and draw conclusions. ● Construct viable arguments, critique the reasoning of others, and communicate ideas precisely using the language of mathematics. ● Share diverse ideas and perspectives, ask questions, and respectfully engage with peers while working towards a common goal. ● Persevere, think strategically/flexibly, and reflect and revise thinking in order to solve complex problems		
	Standards	**Conceptual Overview**	**Rationale**
Focus & Timeframe Limits, Derivatives, and Integrals (4 days)	**2.B** Identify mathematical information from graphical, numerical, analytical, and/or verbal representations. **3.B** Identify an appropriate mathematical definition, theorem, or test to apply. **4.B** Use appropriate units of measure. **1.E** Apply appropriate mathematical rules or procedures, with and without technology.	Students will be able to analyze rates of change (average/ instantaneous) which are fundamental to understanding physics, economics, engineering, and even history. Students will learn how to find the "instantaneous velocity" and connect the concept to tangent lines and derivatives. Students will learn how to apply the "distance equals rate times time" to a continuously changing rate. Students will learn how to find distance from a constant velocity and from a changing velocity, and understand how these concepts connect to the area problem that has many applications in science, history, and economics.	Looking at two central problems of motion much as Newton and Leibniz did, connecting them to geometric problems involving tangent lines and areas helps students think more flexibly about calculus. The language of limits used in this unit to describe asymptotes, end behavior, and continuity, will also help students develop skills that will be useful in Calculus and other advanced math classes.