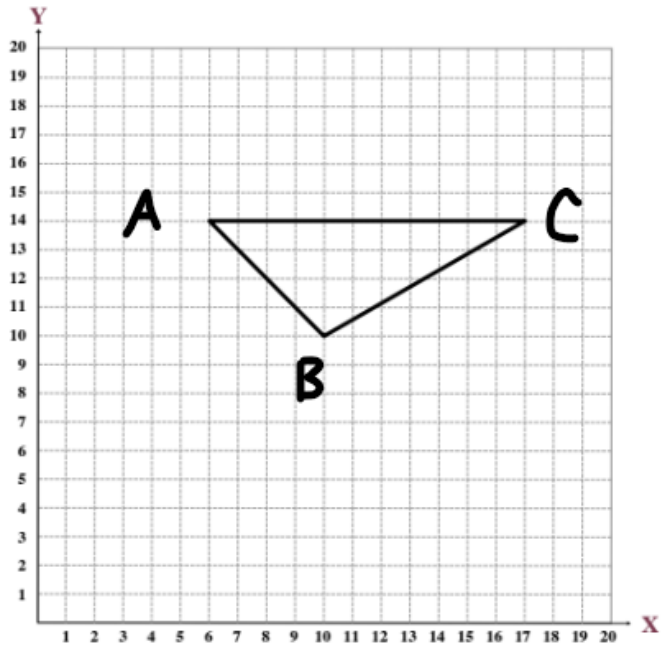
Name: _____
Unit 5: Lesson 4 - Finding an Orthocenter: How To
Geometry

How do you find the orthocenter of a triangle from a graph? Below are steps, and an example using this problem:



STEP 1: Identify the coordinates of all three vertices.

In this case, the coordinates are:
A is (6, 14)
B is (10, 10)
C is (17, 14)

STEP 2: Find the equation of the altitude of one side of the triangle. If any sides are horizontal or vertical, look at those sides first (because it's easiest).

In this case, I will first choose the side AC because it's horizontal. I can easily see that the altitude of side AC connects vertex B to AC with a vertical line. Therefore, the equation of that line is **x = 10**.

STEP 3: Find the equation of the altitude of another side of the triangle.

In this case, it doesn't matter if you choose AB or BC. I will choose AB. For lines that are not horizontal or vertical, here is how you find the altitude:

1. Find the slope of the line using the two endpoints of the segment and the slope formula:
   $m = \frac{y_2 - y_1}{x_2 - x_1}$ For the points A and B, $m = \frac{14-10}{6-10} = \frac{4}{-4} = -1$
2. Find the slope of the line perpendicular to this segment by find the opposite reciprocal of that slope. In this case, the opposite reciprocal of -1 is 1.
3. Plug this perpendicular slope, and the coordinates of the opposite vertex into the point-slope formula:
   In this case, the opposite vertex is C (17, 14) and the perpendicular slope is 1.
   $y - y_1 = m(x - x_1) \rightarrow y - 14 = 1(x - 17)$
4. Use the vertical line method to rewrite this equation in slope-intercept form (you really only need to do this if none of your sides are horizontal or vertical):
   $y - 14 = 1(x - 17)$
   $y - 14 = x - 17$ Distribute the 1
   $y = x - 3$ Add 14 to both sides

STEP 4: Now that you have the equations of two altitudes, you can find the orthocenter by solving this system of equations.

- If both equations are in slope-intercept form, set the right sides equal to each other and solve for x. In other words, remove the y = part from both equations and set the other parts equal to each other (then use the vertical line method to get x by itself).
- In this case, though, you can just plug the x-value from the first equation into the second one:

$y = x - 3$ and $x = 10$

$y = 10 - 3$

$y = 7$

Since x = 10 and y = 7, the coordinate of the orthocenter is (10, 7)