Parametric Equations and Modelling

Q1.

The curve C has parametric equations

$x = 3t - 4$, $y = 5 - \frac{6}{t}$, $t > 0$

(a)   Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b)   Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c)   Show that the cartesian equation for C can be written in the form

$y = \frac{ax + b}{x + 4}$, $x > -4$

where a and b are integers to be determined.

(3)

(Total for question = 8 marks)

Q7.

A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(Total for question = 3 marks)

Q4.

A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $t = \frac{\pi}{6}$

(4)

(b) Find a cartesian equation for C in the form

$$y = \mathrm{f}(x), \; -k \leq x \leq k,$$

stating the value of the constant k.

(3)

(c) Write down the range of $\mathrm{f}(x)$.

(2)

(Total 9 marks)

Q14.

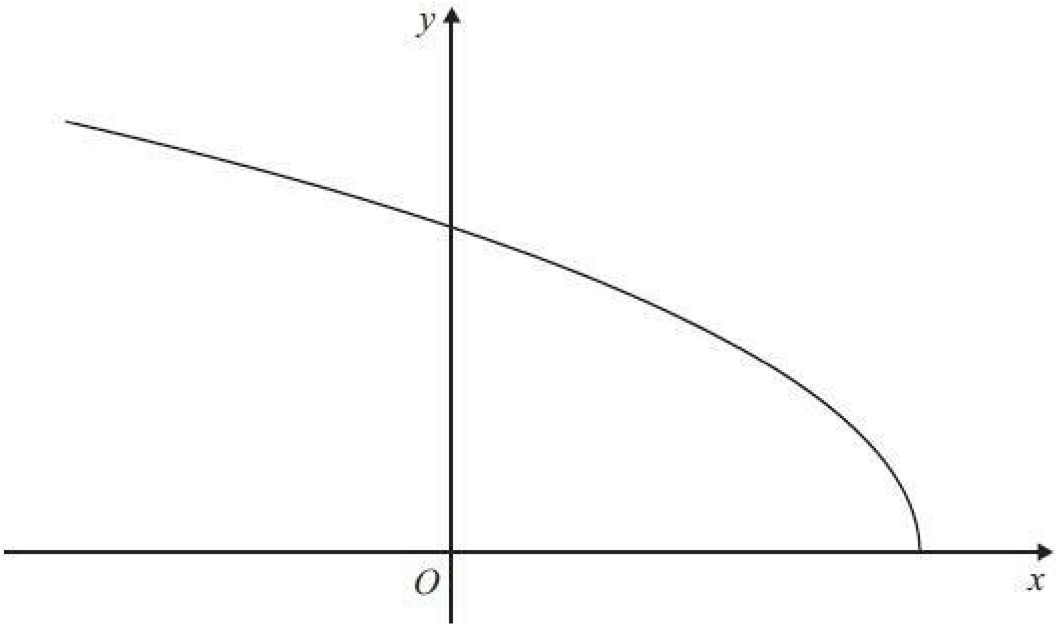

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \dfrac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant *k*.

(4)

(c) Write down the range of f(*x*).

(2)
(Total 10 marks)

Q2.

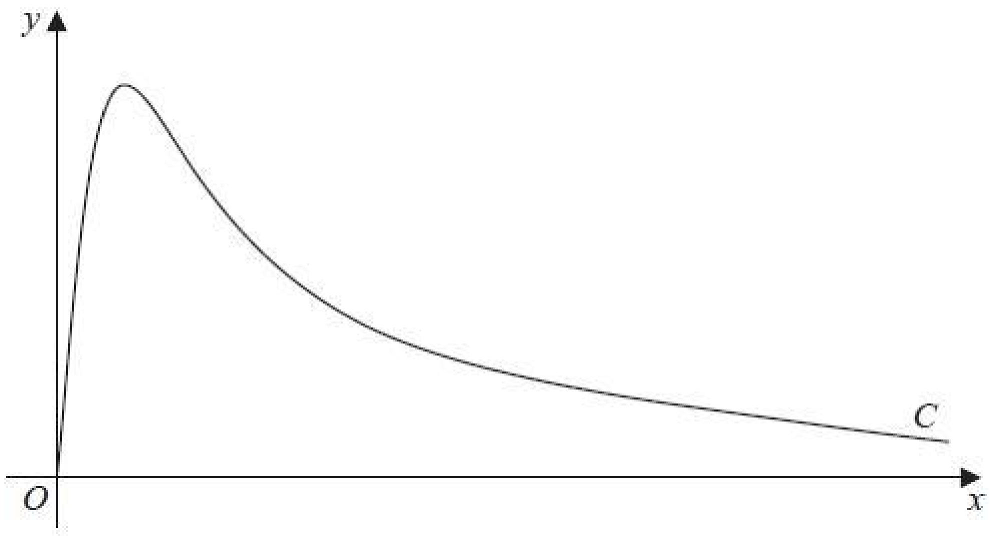

Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point *P* lies on *C* and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point *P*.

Give your answer as a simplified surd.

(4)

The point *Q* lies on the curve *C*, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(b) Find the exact coordinates of the point *Q*.

(2)

(Total for question = 6 marks)

Q6.

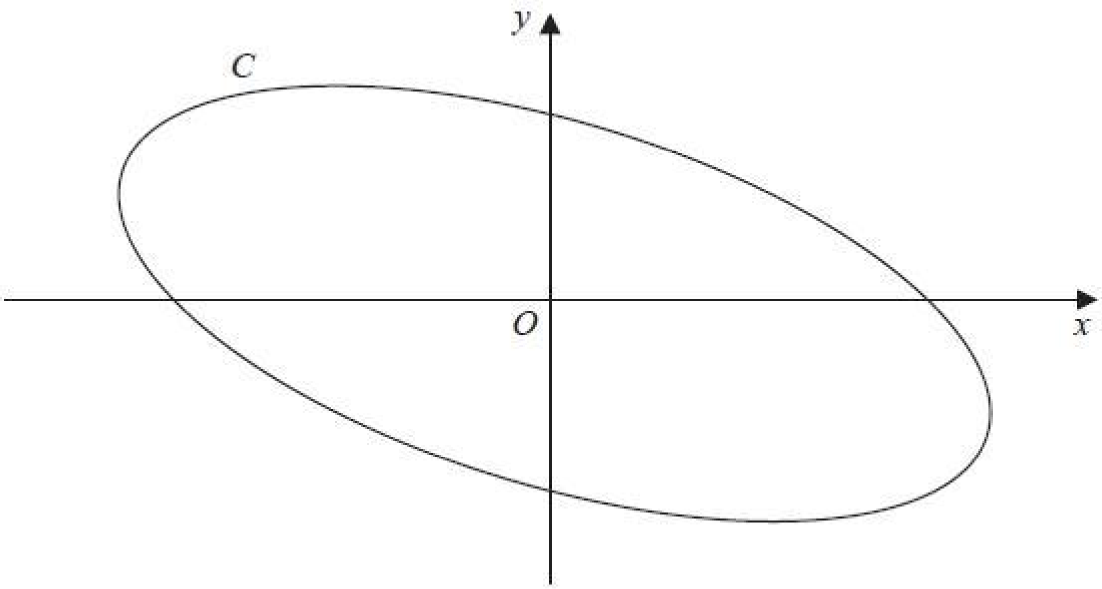

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = \sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

(Total 5 marks)

Q9.

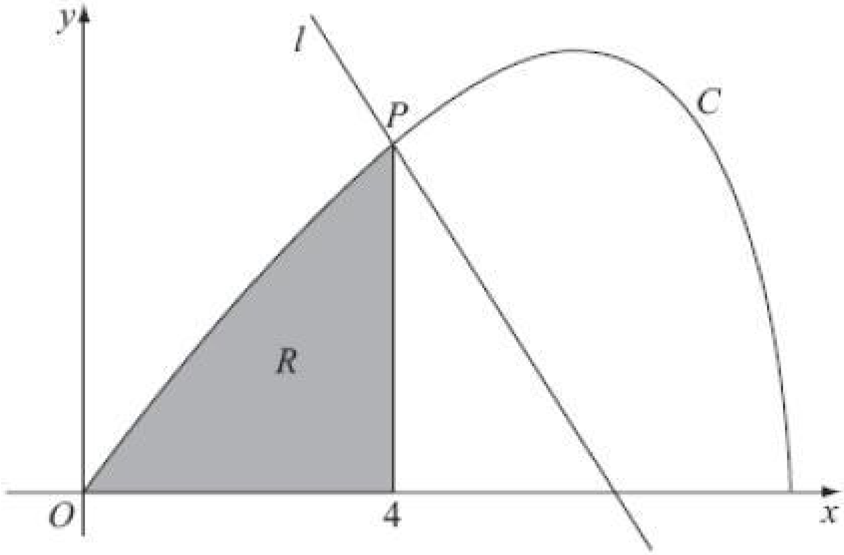

Figure 3

Figure 3 shows the curve *C* with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point *P* lies on *C* and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of *t* at the point *P*.

(2)

The line *l* is a normal to *C* at *P*.

(b) Show that an equation for *l* is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region *R* is enclosed by the curve *C*, the *x*-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of *R* is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of *R*, giving your answer in the form $a + b\sqrt{3}$, where *a* and *b* are constants to be determined.

(4)

(Total 16 marks)

Q13.

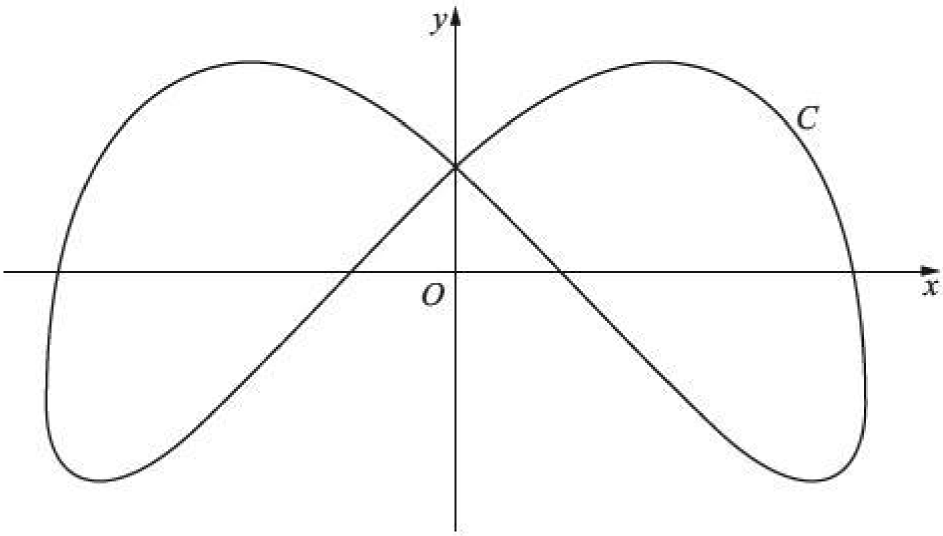

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(3)

Find the coordinates of all the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(5)

(Total 8 marks)