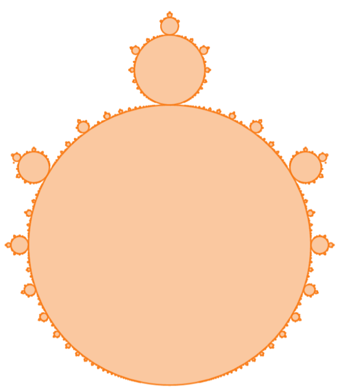
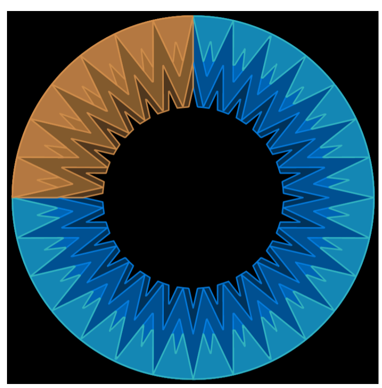
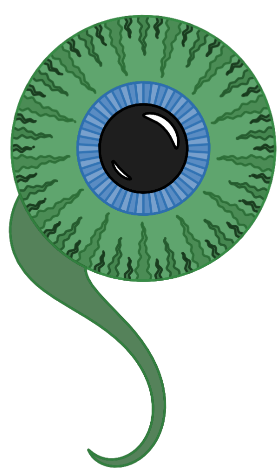
Arts and Graphs

In our modern world, we don't even need to use physical tools to make beautiful art pieces. With the advent of computers, many have been able to express their artistic talents through graphic design. Personally, I've found a way to use a program called Desmos Graphing Calculator to make some unique art pieces. Imagine you've got an array of pixels on a screen and each pixel is labeled with an x coordinate (What columb it's in) and a y coordinate (What row it's in). When talking about a specific pixel, you might just give it's coordinates in the form (x,y). When you type an equation into Desmos, say $x + y = 5$, it finds all the pixels whose coordinates match the equation and lights them up. So, in this example, (2,3) would light up but (3,4) wouldn't. Here are some examples of things I've made:

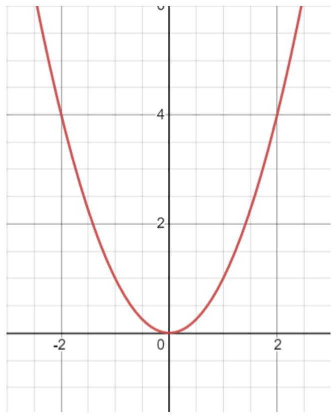



Now, you may have seen Desmos art before. Again, checking Google images, there's loads of awesome things people have made. If you've been following 3Blue1Brown for a while, you may remember that he was involved in judging a math competition where participants submitted pieces they had made using Desmos. But something I've noticed is that people tend to limit themselves to using fairly basic shapes: lines, circles, sine waves, and parabolas. The idea is that you start with a short line segment, then put another line segment at the end of the first one at a slight angle, then another at the end of that, then another, and eventually all the lines put together make the appearance of a more complex curve.

The problem is I'm incredibly lazy and I get really bored using the simple equation for a line over and over again. So to combat my laziness, I try to find a single equation that can create the whole curve in one go. I've been doing this for a while now and I've come up with some pretty good techniques over the years. I could honestly talk all day about all the things I've learned from using Desmos, but today I'm going to talk specifically about something I call "Inconsistent Transformations". I don't know if this concept already has an official name, this is just what I call it. I may have come up with the name as I was writing this.

If you’ve taken a calculus course, you’ve probably learned the basic transformations you can apply to a curve like Translation and Stretching/Squishing. For those of you that haven’t, here’s a quick run down. Let’s say you start with a simple parabola:

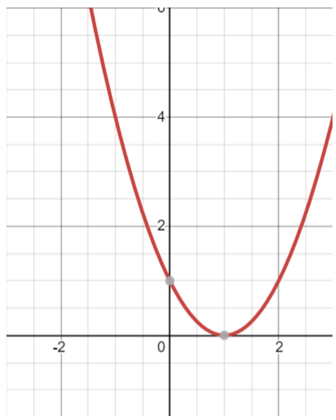


$y = x^2$

We can look at the coordinates of some of these points using this table:

x	y
-2	4
-1	1
0	0
1	1
2	4

And, doing a bit of math in our head, this all looks right: $4 = (-2)^2$, $1 = (-1)^2$, $9 = 3^2$, everything looks good. Now imagine we replace the “x” in our parabola equation with “x - 1”, so our new curve looks like this:



$y = (x - 1)^2$

Now, when we’re putting in our x inputs, before we do anything we subtract 1. If we have the input x = 3, we have to subtract it down to 2, then we carry on as before. It’s almost like we put x = 2 into our original equation. And this happens for any input of x. Again, pulling out our coordinate table:

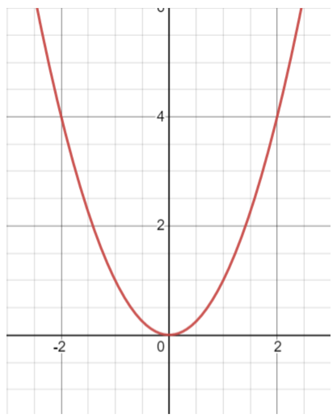
x	y
-1	4
0	1
1	0
2	1
3	4

You can see that the x inputs have been shifted up by 1. The input x = 0 used to have the output y = 0, now x = 1 has that output. Originally, x = -2 gave us y = 4, now that's what $x = -1$ gives us. So if the x coordinate of each point goes up by one, that means the whole curve will move to the right. This is called a translation: Moving a curve around without changing anything about its shape, orientation, or proportions.

Try to think of a transformation as a sort of action or operation that you're performing on all the points of a curve. In this case, subtracting 1 from all the x's in the equation shifts the x coordinate of every point up by one. You could also add 7 to all the y's in the equation, which would shift the y coordinate of every point down by one. Try plugging this into Desmos and see what happens.

In fact, you aren't even limited to arithmetic. What happens if we multiply the x by 2 [$y = (2x)^2$]? Well, if we plug some value in for x, before we do anything, we will have to multiply it by 2, so it will be as if we input a number twice as big into the original equation. So, $x = 3$ would have the y output that $x = 6$ would otherwise have had; $x = 153$ would have the y output that $x = 306$ normally does; and so on. This is how you Stretch and Squish a curve. You can even do some weirder things like: $y = (\sin(x))^2$; $\log(y) = x^2$; $y = (x^3 + 4x^2 - x - 2)^2$. Technically, any function can be used as a transformation, though you tend to just stick with Translations and Stretching/Squishing since they're more predictable. Plug something like this into Desmos and try to work out what the transformation is doing.

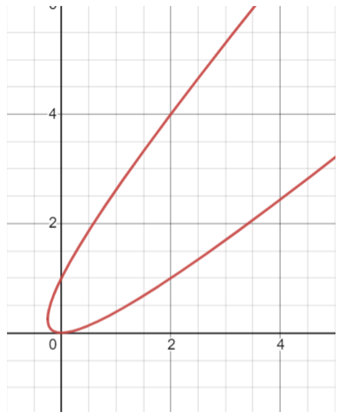
Hopefully you now understand how transformations work, but these have all been what I call “Consistent Transformations”. Again, there might be real names for these concepts that I don’t know about. With some of these transformations, different points on the curve are affected differently, but the function acting on the curve never changes. In that way, the transformation is consistent. To show you what an Inconsistent Transformation looks like, let’s go back to our parabola for a minute:



$y = x^2$

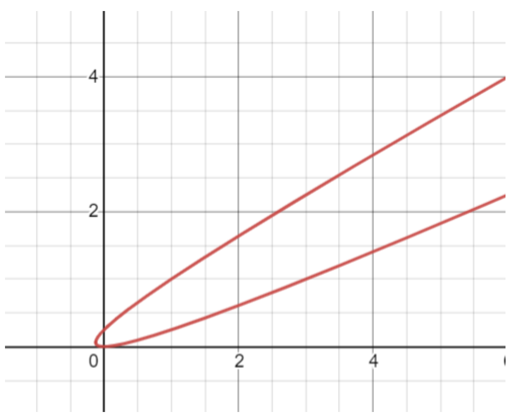
So what if this time we changed the equation to $y = (x - y)^2$? This means we don’t have a function anymore, but remember, Desmos just plots the points that line up with the equation. It doesn’t need to be a function. Before we plug this into Desmos to see what it gives us, let’s try to work it out for ourselves.

When we looked at $y = (x - 1)^2$, the -1 shifted the x coordinate of every point up by one. So maybe if we have $y = (x - y)^2$, the $-y$ will shift the x coordinate of every point up by that point’s y coordinate. So the point (0,0) has a y coordinate of 0 so it wouldn’t move at all. The point (1,1) has a y coordinate of 1 so it would move to (2,1). (3,9) has y coordinate 9 so it would move to (12,9). The higher up you go, the more things move to the right. Imagine drawing our parabola on a flat wooden panel made of really thin horizontal planks. Now imagine pulling one plank a little to the right, then the one above it a little more, then a little more with the one above that, and so on. That's basically the effect here. Now let's put this into Desmos and see what we get:



$y = (x - y)^2$

This is what I mean when I say the transformation is inconsistent. In a way, it's just a translation, but not every point on the curve is translated by the same amount. How much the point moves depends on what it's y coordinate is. Since y is the variable that makes the transformation inconsistent, I'm going to call it "the Inconsistent Variable". This is significant because meddling with this variable changes the extent to which each point is transformed. For example, let's look at what happens if we change our equation to $y = (x - 2y)^2$:



$y = (x - 2y)^2$

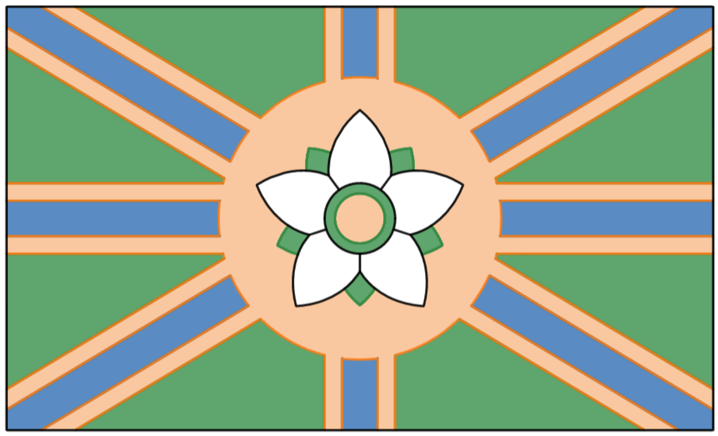
Multiplying the y by 2 doubles the amount that each point is translated, so the tilting effect we got last time is even more prominent.

Now to be honest, it's not quite as simple as I've made it out to be. You can use variables in a transformation, but you have to make sure you're doing it in the right way. For example, let's take a look at the equation $y = (x - x)^2$. If we treat that $-x$ as an inconsistent transformation, you would expect that the x coordinate of each point would go up by… well… the x coordinate. So the point (1,1) would go to (2,1); (2,4) would go to (4,4); (3,9) would go to (6,9) and so on. But if you know anything about algebra, you can easily see that $x - x = 0$. So this equation simplifies down to $y = 0$. Just a horizontal line. That's very different from what we predicted, so what's going on here?

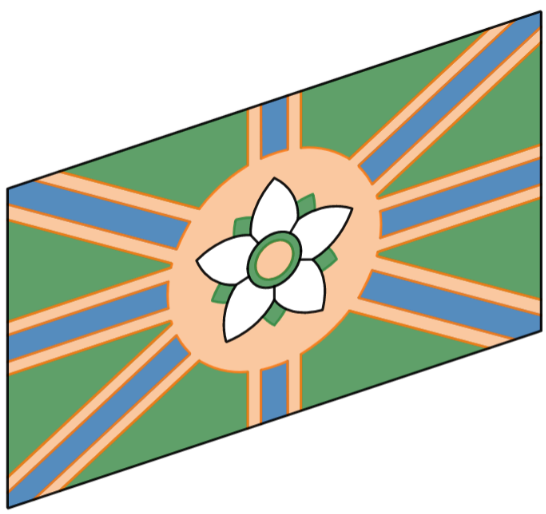
Going back to $y = (x - y)^2$, let's take a closer look at what happens to an individual point. Say, the point (3,9). This point's y coordinate is 9 so the point should slide 9 units to the right. But importantly, this doesn't require the point to go up or down, so the inconsistent variable, y, doesn't change as the transformation occurs. Compare that to $y = (x - x)^2$. Again looking at the point (3,9), the x coordinate starts at 3, so the point should move 3 to the right. But as it does that the x coordinate changes, so how much does the point need to move now?

This kind of reveals inconsistent transformations for what they really are. At the end of the day, this is just a conceptual trick that helps to predict what's going to happen if you tweak an equation in some way. But the trick stops working if you use it in situations where the logic doesn't apply anymore. Basically, we're pretending the inconsistent variable is a constant, and as long as the variable doesn't change as the point moves from its original position to its new position, this is basically true. But if the variable does change, then we can't treat it as a constant anymore and the trick stops working.

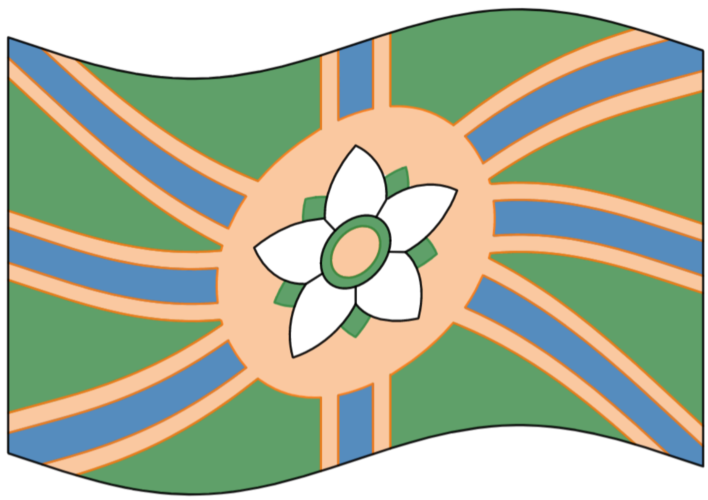
But as long as you follow that rule, again, you can go a bit crazy with this. I, as a proud citizen of Abbotsford, British Columbia, made the Abbotsford flag a few years ago:



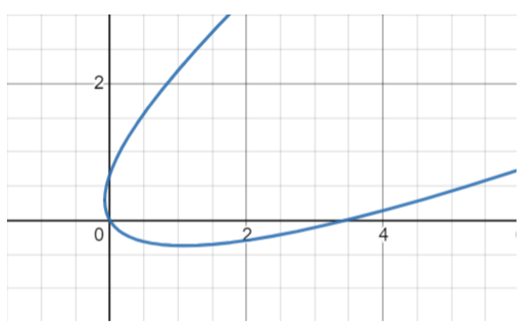
I think this looks pretty good, but what if I want the flag to look like it's waving in the wind? If we do a simple Inconsistent Transformation on this flag, replacing the y's in each equation with $y - x$, this is what we get:



That's not quite what we're looking for. Remember, in this vertical translation, each point gets moved up or down depending on our inconsistent variable, x in this case. In this first attempt, we subtracted x, so we had a direct, linear relationship between a point's x coordinate and the shift in its y coordinate. The further to the right a point was the further it was moved up, and the further to the left a point was the further it was moved down. But like we did before, we can medal with the inconsistent variable a bit. We could instead substitute the y's with $y - \sin(x)$. Sine is a function that's output fluctuates between 1 and -1. This gives us the exact waviness we're looking for:



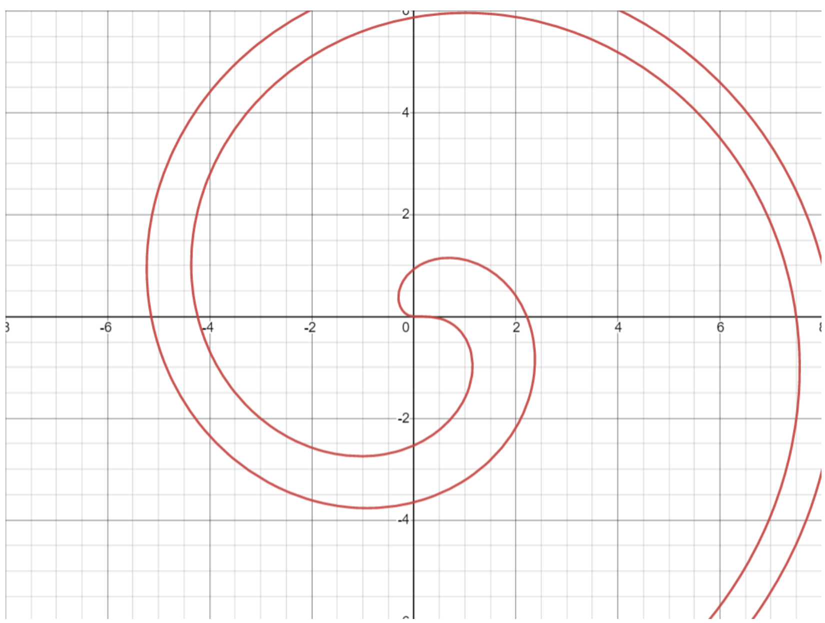
Another good example of this is "twisting" a curve. I'm not going to go too deeply into an explanation here, but suffice it to say it is possible to rotate a curve by some fixed angle. For example, if we wanted to rotate our parabola clockwise by 60°, that would look something like this:



$$y \cos(60) + x \sin(60) = [x \cos(60) - y \sin(60)]^2$$

I know that equation might look a bit intimidating, but you really don't need to understand all the moving parts. One thing you should notice is all the 60's. Whatever number we put in there, that's how much the curve will be rotated by.

So that would be a consistent rotation, but what would an inconsistent rotation look like? To try to visualize what would happen here, we can go back to the wooden panel analogy. This time, our wood panel is made of thin concentric rings of wood. If you've ever seen Pirates of the Caribbean 3, this is kind of like the map to Davy Jones's Locker. Now imagine we draw our parabola on this wooden panel. We'll let the central circle stay where it is, but the first ring around it we'll turn just a tiny bit; then we'll turn the next one a little bit more; then the next one a bit more; then the next one a bit more; and so on. When we step back and look at it, we'll get something that looks like this:



It doesn't even look like a parabola anymore. This is why I love challenging myself to come up with more creative ways to draw complex curves. It always leads to some interesting new perspective, or a new technique I'd never considered before. I'm probably not going to do any other sort of 'explainer' thing. Starting up a whole YouTube channel just seems way too intimidating. But there are plenty more things to learn from making art on Desmos. I would highly encourage you to try your hand at making something. Even if it's just a simple logo or a famous YouTuber's icon, you might have more fun that you expect.