CHAPTER 5

DAY 14 – DERIVATIVES OF IMPLICIT, INVERSE TRIGNOMETRIC EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Implicit functions

- It is not always possible to write a function in the form $y = f(x)$. For example consider the function $x + \sin sin\, xy - y = 0$. Such functions that cannot be solved for y area called ***implicit functions.*** So, to find $\frac{dy}{dx}$, we differentiate both sided of the implicit functions with respect to x and then solve for $\frac{dy}{dx}$.

  - E.g.: Consider the function $y + \sin sin\, y = \cos cos\, x$. To find $\frac{dy}{dx}$ we differentiate both sides of the equation with respect to x. Thus we have,

$$\frac{d}{dx}(y + \sin sin\, y) = \frac{d}{dx}(\cos cos\, x)$$

$$\frac{dy}{dx} + \frac{d}{dx}(\sin sin\, y) = -\sin sin\, x$$

$$\frac{dy}{dx} + \cos cos\, y \frac{dy}{dx} = -\sin sin\, x$$

$$\frac{dy}{dx}(1 + \cos cos\, y) = -\sin sin\, x$$

$$\frac{dy}{dx} = \frac{-\sin sin\, x}{1+\cos cos\, y}$$

Exponential function

- The ***exponential function*** with positive base $b > 1$ is the function $y = b^x$.
- When $b = e$, (where e is a transcendental number between 2 and 3), we get the function $y = e^x$ and it is called the ***natural exponential function***.
- $\frac{d}{dx}(e^x) = e^x$

Logarithmic functions

- The function $f: R^{+} \to R$ defined by $f(x) = x$ is called the ***logarithmic function.***
- $x = y$, if $b^{y} = x$.
- If $b = e$, the function is called the ***natural logarithmic function*** and is denoted by $\ln ln\, x$. For over convenience we denote it as $\log log\, x$.
- $\frac{d}{dx}(\log log\, x) = \frac{1}{x}$.

Questions

1. Find $\frac{dy}{dx}$ for the following functions
   a. $2x + 3y = \sin sin\, y$
   b. $x^{2} + y^{2} = 25$
   c. $xy + y = \tan tan\, x + y$
   d. $x^{2} + xy + y = 0$
   e. $xy^{2} + x^{2}y = 2$
   f. $\sqrt{x} + \sqrt{y} = 1$

2. Find $\frac{dy}{dx}$ for the following functions
   a. $y = sin^{-1}\left(\frac{1-x^{2}}{1+x^{2}}\right)$
   b. $y = cos^{-1}\left(\frac{2x}{1+x^{2}}\right)$
   c. $\left(\frac{3x-x^{3}}{1-3x^{2}}\right)$

3. Differentiate the following functions with respect to x.
   a. e^{2x}
   b. e^{-x}
   c. $\frac{\cos cos\, x}{e^{x}}$
   d. $e^{x^{3}}$

e. e^{x}

f. $\sqrt{e^{\sqrt{x}}}$

g. $e^{2x}.\cos cos\, e^{x}$

4. Differentiate the following functions with respect to x.

a. $\log log\,(x+5)$

b. $\cos cos\,(\log log\, x)$

c. $\frac{\cos cos\, x}{\log log\, x}$

d. $\log log\,(\log log\, x)$

e. $e^{x}.\log log\, x$

More questions must be practiced.