The Gallagher Index: A Measure of Disproportionality

The Gallagher index is a measure of the degree of distortion that exists between the share of votes obtained by each party and the share of seats that it obtains. It is a measure of proportionality. The Gallagher Index is the measure of electoral disproportionality most commonly used by political scientists.

As explained in more detail below, the index is roughly equivalent to the seat share premium of the winning party in percentage terms. If the winning party won 55% of the seats with 50% of the vote (a winner's premium of 5%), the index would usually be equal to 5 or less, depending on the structure of distortions faced by the other political parties.

Most of us don't need to worry about how the index works. We know that in practice, it goes from less than 1 (very proportional) to as high as 20 or more (very disproportional). It varies depending on the results of each election.

Canada's Gallagher index federally in 2015 was 12. Prof Byron Weber Becker of Waterloo University has calculated Gallagher indexes for all sorts of electoral system options for Canada.

The mathematical formula for the Gallagher index, reproduced below from Wikipedia is somewhat intimidating, but can be taken one step at a time:

1. Calculate the difference between each party's vote percentage (V) and its percentage of seats (S) where "n" is the number of political parties (i = 1 to n).
2. Take the square of each of these numbers and calculate the sum.
3. Divide by two.
4. Calculate the square root to get the value of the index.

$$\mathrm{LSq} = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(V_i - S_i)^2}$$

Getting a sense of how this formula works is not as difficult as it first seems.

To understand it most easily, imagine that there are two parties. Party A wins the election with 50% of the vote and 55% of the seats. Party B loses the election with 50% of the vote and 45% of the seats. You take V-S squared for Party A (5*5=25) and add V-S squared for Party B (also 25), but since the two numbers represent the same distortion (the winner's premium is matched by the loser's deficit), you divide that by two. This ensures that the same distortion is counted only once. Then you take the square root, and you get back a number that corresponds to the winner's 5% seat premium that you started with: 5. The Gallagher index is 5.

This is why it makes sense to speak of the Gallagher index as a "percentage" even if that is not **exactly** true. It is very closely related to the percentage premium of seats that goes to the winning party.

Why do we square each deviation instead of just taking the percentages and adding them up? One reason is that the deviations consist of both positive and negative numbers, so just adding them up would not serve much purpose. Squaring the numbers eliminates this problem. The Gallagher index is based on the method of "least squares," as noted in the simple-language blog "Gallagher Index Made Easy"

Squaring the numbers also gives larger distortions more weight. If you take the same total of distortion percentages and spread them out in small ways over a large number of parties, you'll end up with a lower Gallagher index than if there is a large distortion in favour of the winning party. This gives you a more accurate picture of the resulting distortion in power relations.

In the case of Canada's 2015 election the biggest distortion was in the result for the winner (54% of seats minus 39.5% = 14.5. By the time you take the squares of the distortions for each party and apply the formula, the Gallagher index is 12. This represents a quite high level of disproportionality by international standards. (Raellerby's Wikipedia file shows details).

As Ryan Campbell points out, a Gallagher index of 5 still allows for quite a bit of distortion. The leading party could win a majority with 45% of the vote. Indeed, they could win with even less than 45% in a multi-party setting in which the 5% disadvantage is spread out over many parties.

You can see a detailed listing of the Gallagher index by country and by election here. Walking one's way through this data is interesting, because it shows that not all PR countries are equally proportional. Among the lowest are the Netherlands and the Nordic countries with Gallagher indices between 2 and 3. You can also see changes over time. New Zealand went from 18 just before it adopted MMP to about 4 today. The above link is to a web archive, and it appears that Gallagher himself may no longer be updating that listing. However it provides a useful historical record.

Click here for a Google Sheet that you can use to calculate the Gallagher index.

FVC supporter Mercédez Roberge has also done a compilation that shows the average Gallagher index over multiple elections in the following file (see the third tab).

One issue with the Gallagher index is that it completely ignores regional disproportionalities. Alberta can go disproportionately blue while Toronto and the Atlantic Provinces go disproportionately red, but the Gallagher does the calculation at a national level and so they cancel out. This is a crucially important consideration in a country as geographically and culturally diverse as Canada. Prof. Byron Weber Becker from the University of Waterloo thus recommends the use of a "composite Gallagher index" that computes the Gallagher index for each province/territory and computes their average weighted by the number of MPs for each region. The composite Gallagher for the 2015 election was 17.2 (rather than 12.0), better reflecting the regional disproportionality.

It gets even worse if you look at specific regions within a province. [Here's a table of results by region in the 2022 Quebec election](#).

Technical Annex on limitations of the Gallagher Index

(contribution by Antony Hodgson, president of Fair Voting BC)

How it characterizes deviations

There are at least two other indices that have been widely used to measure deviations between a party's vote share and seat share: the Loosemore-Hanby index (https://en.wikipedia.org/wiki/Loosemore–Hanby_index) and the Rae index. The LHI measures the total absolute deviation (ie, it cares just as much about a 1% deviation for a party that wins a seat share close to its vote share as about an extra 1% deviation for a party that is already 10% away from its fair share, whereas the GI tends to discount deviations that are closer to zero). E.g., if Party A has 10% more seats than their vote share would give them, and parties B and C each have 5% under, then the GI would be sqrt((10^2 + 5^2 + 5^2)/2) = sqrt(75) = 8.7%, whereas the LHI would be (10+5+5)/2 = 10%. Another way of thinking of this is that the Gallagher Index tends to understate the total deviation.

What it measures

Both the GI and the LHI are 'party proportionality' metrics, which means that they focus exclusively on party vote vs seat share, and so ignore the question of whether voters end up being represented by a candidate they prefer. In the extreme, it would be possible for a pure party list system to deliver a perfect GI or LHI without giving any voter the ability to vote for a candidate at all. For this reason, the GI or LHI has to be complemented by a metric that assesses how many voters have a representative that they have specifically voted for. Fair Voting BC calls this a 'direct representation' metric, which ranges from 0% for a pure party list system to upwards of 90-95% for candidate-focused systems such as STV and RU-PR (MMP tends to have a value in the range of 50-65%, depending on whether they are closed or open list systems).