

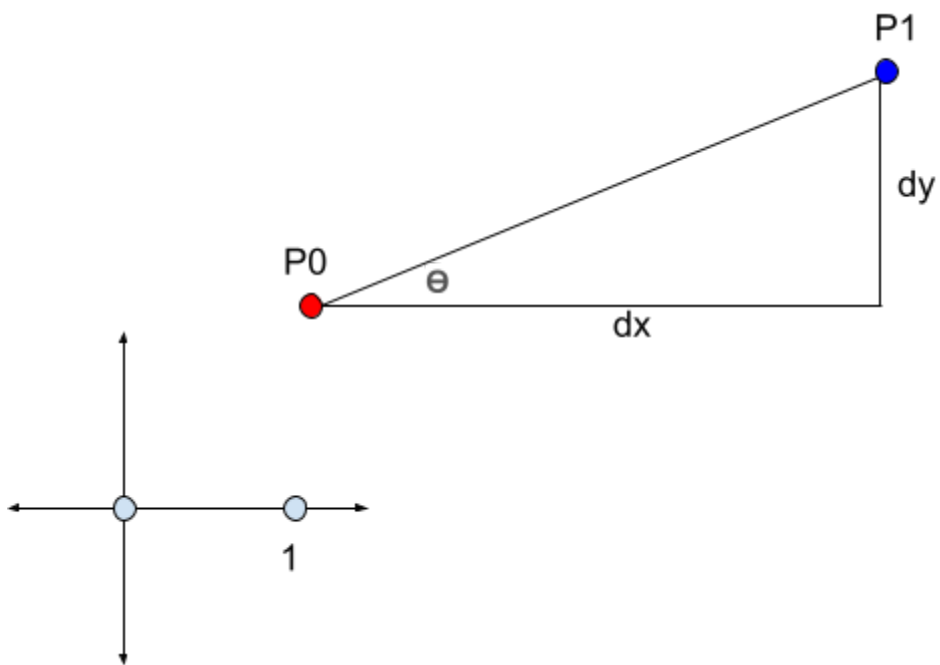
# Linear Gradient Transformation

We define our simple Linear Gradient as follows:

Two points : P0 and P1

Two colors: C0 and C1

How can we create a transformation (matrix) that maps from the unit interval (0, 0) ... (1, 0) to our gradient points?



We will create the mapping in 3 steps.

1. scale the unit interval to be the same length as the line between P0...P1

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dx = P1.x - P0.x  
dy = P1.y - P0.y  
D = sqrt( dx^2 + dy^2 )
```

```
[ D 0 0 ]  
[ 0 D 0 ]  
[ 0 0 1 ]
```

2. rotate the line  $(0, 0) \dots (D, 1)$  to be parallel to the line  $P0 \dots P1$

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We haven't computed  $\theta$  yet, but we don't need to, since we don't really want  $\theta$  but we want its sine and cosine.

Remember that cosine and Sine are defined as ratios of our triangle

$$\begin{aligned} \cos(\theta) &= dx / D \\ \sin(\theta) &= dy / D \end{aligned}$$

Hence we can write this matrix as:

$$\begin{bmatrix} dx/D & -dy/D & 0 \\ dy/D & dx/D & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. translate so that the origin maps to  $P0$

$$\begin{bmatrix} 1 & 0 & P0.x \\ 0 & 1 & P0.y \\ 0 & 0 & 1 \end{bmatrix}$$

We now concatenate these 3 separate transformations in this order, to create a single matrix.

$$\begin{bmatrix} 1 & 0 & P0.x \\ 0 & 1 & P0.y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dx/D & -dy/D & 0 \\ dy/D & dx/D & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which concatenates into

$$\begin{bmatrix} dx & -dy & P0.x \\ dy & dx & P0.y \\ 0 & 0 & 1 \end{bmatrix}$$

This is the transformation that maps the unit X-axis  $(1, 0)$  onto our line-segment between  $P0$  and  $P1$ .