# Educational case study

(work in progress)

By Olga Laurenty Fall 2022

Abstract

#### Context

This study is about a 12 years old girl in 7th grade, of hispanic origins, speaking spanish at school, which I will call Dess. Dess is struggling in math and I'm her math teacher. Dess is in 7th grade at a private Montessori school in South Bay, California, USA. Des joined the school in 6th grade coming from a local public school. She said that she doesn't have any help for math at home.

#### Description of the classroom

There are 10 students in 7th (4) and 8th grades (6). There are 2 teachers: myself responsible for math and a science-humanities teacher. There are 4 hours of math per week. There are 4 group levels. Teacher (myself) personalizes for each student or group of students. The main curriculum used is Core Connections, College Preparatory Math.

#### Assessment

Dess was assigned to her grade level by the previous teacher who left at the end of August. Dess was working with 2 peers on the chapter 1 of Course 2 (grade 7) from August to the beginning of October. In her notebook, the work didn't show any particular difficulty but when she sat for the chapter 1 test. She couldn't do the problems.

#### Experimentation

During september, I presented to the 3 students in the Course 2 level a lesson on <u>area of triangles and trapezoid</u>, as I noticed errors on those, as well as the <u>classification of numbers</u>, The test had a large section on areas and perimeter and Dess had a very low success rate. After the test I repeated the lesson on area and perimeter and assigned more practice. Then she took a retake test on chapter 1 and it got only slightly better.

I sat with her in a one on one (1:1) session to try to understand what was missing. I saw that she mastered the multiplication tables, could easily compute a long division and long multiplication but with errors which she didn't see. We went step by step together on how to calculate area and perimeter. So what I discovered is:

- She didn't know that you can drop the multiplication sign. So a formula: hb/2 did not make any sense to her because she didn't know what to do with h (height) and b (base). I explained to her that when we have multiplication we can use 3 x 4 or 3 · 4 or

$$a \cdot b = ab$$

- The second difficulty was that **she didn't seem to understand the concept of decimals**.

So I decided to move her on and work on identifying the blocking points. I gave a lesson on decimals part 1 to the 3 students. I really insisted that they should ask if something is not clear and Dess asked about the 1 to 9 that I wrote under each decimal place value. I praised that she asked the question and explained that this is how we count: when we count up we group quantities in 10th: 10 units, 10 tens, 10 hundreds, etc. we do the same with the parts of 1 so we have 10 tenths (1/10), inside each 1/10 there are 10 hundredth (1/100) etc. So the countring goes like 0.01, 0.02 — 0.09, 0.10, 0.11, etc. We drop the zeros in front of . and at the end of . I'm not sure that she got the concept but was able to continue following the rest of the lesson. She did understand well the concept of least common denominator and can easily do addition of fractions with different denominators.

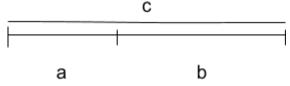
Then I assigned to her the chapter 2 exercises. The 2 other students went forward and Dess was alone on her work. She could not do most of it. Especially there was this exercise:

Mario ordered a pizza for dinner. When it arrived, Mario quickly ate ½ of the pizza. While Mario was getting napkins, his pet poodle ate ½ of the pizza.

- a. Draw a model of the pizza that shows the portion that has been eaten.
- b. Write a numerical expression to show the fraction of the pizza that is left.
- c. About what percept of the pizza is left?

Dess for a. draw a circle divided in .... Parts and didn't do b. nor c. I sat with her 1:1 to go through that problem. We read the problem and tried to solve it. I noticed difficulties in:

- Fundamental concept of addition, subtraction, multiplication and division. Dess answered when I asked if you have 4 apples and I have 2, how many do we have? She said 6. Then I ask, what operation did you use? She didn't know. She eventually answered 4 + 2. Then I asked if you have 4 apples and we together have 10, how many apples do I have. She answered 6 right away. Then I asked what operation you use. She said I divided 10 by 4. We worked together through it and she said 10 - 4 = 6. Then we went to the fractions problem (1/8 + 1/3) and she struggled first to understand what operation she needed to use. Once stated the equation she easily solved 1/8 + 1/3 = 3/24 + 8/24=11/24 Then she couldn't find the operation to find what is left. We went back to the whole example and that it is a pizza. I say that it means 1. Than because she



struggles I draw a line

I go through c=, she hesitates, but answers a+b. then I say a= c-b, so b=?. Dess answers a-c. I give a concrete example and she corrects herself: b=c-a
I help her to translate that to the fractions and we get to 1 - 11/24 then she replaces 1 with 1/24, but corrects herself rapidly when I say to pay attention.

#### 2022/11/02

I assigned to her some exercises that I had from the russian method in mathematics from 3rd grade and checked with her.

- The short, simple word problems were done correctly
- She didn't see **smart computations** such as 152 + 88 152 she got 74. Most of that exercise was wrong.
- She didn't know the **distributive property** 2\*6+2\*5=2 \*(6+5) . she wrote: in a following exercise: calculate a perimeter of a rectangle 5\* 6: 2\*.....+ 2\*....= 2\* (....+ ....) = ... Dess solved like this: **2\*5+2\*6=2\*(10+12)=34** I gave her a short lesson on distributive property.

#### Reflections of the day:

 It seems like I need to work on the very foundations of the operations as she seems not to know when to use one or another.

#### 2022/11/04

Here is the assessment I wrote for our parent/ student / teacher meeting in a week

### Dess

Dess is orderly and structured. She is hardworking and motivated. Dess is strong in procedural calculations; multiplication facts, long division, least common denominator and addition of fractions with different denominators. She is also able to see patterns in numbers which is the foundation of mathematical thinking. We have identified some areas of improvement. Operations:

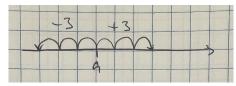
- 1. The first is being able to identify the calculations that she is able to do, this means knowing how she gets to the answer and showing her work.
- 2. The second step is to understand the meaning of operations at an abstract / logical level which will allow her to identify when we use the four operations and how. For example, how to reverse addition or multiplication
- 3. Third step is to perform smart calculations, not busy ones. This skill allows her to look at the operations and see patterns and different ways of arriving at a result, then select the most efficient one. This will allow Dess to reduce the number of calculation errors. For example: 29+34+11-24 can be solved in a smart way if we group operations 29 + 11 = 40 and 34 24=10 and then add 40 + 10=50. These strategies allow to increase computational speed and accuracy.
- 4. Fourth to critically reason instead of guessing, building strong logic and structure in problem solving. This means that the focus is not in getting the answer right, and continuing guessing until you succeed, but to start by posing the problem and then finding a logical path to arrive at a conclusion. This involves what we call cognitive flexibility, meaning the capacity to try a different path when we get stuck on a problem. For example, a decimal 0.75 becomes 75/100 and the reasoning steps that lead to that

- conclusion can be 0.75 is 75 hundredths which is 75 \* 1/100 = 75/100 or another way can be 0.75 = 0.7 + 0.05 = 7\*0.1+5\*0.01=7\*1/10+5\*1/100=7/10+5/100=75/100
- 5. Fifth, to know how to check her work, build a collection of strategies to use in reversing or choosing different ways of arriving at a result to verify the logic and calculations. For example: 100 45=55 and I can check by calculating 45 +55 = 100; another example: 9+10+10.3=29.3 I can check by seeing that I can rewrite the operation as 10+10+10-1+0.3= 30.3-1=29.3
- 6. Last at this point of time, to master foundational mathematical concepts such as the distributive property: 2\*5+2\*6=2\*(5+6), the concept of decimals, and mathematical symbols and conventions (for example that we can drop the multiplication sign and base b \* height h can be written as bh

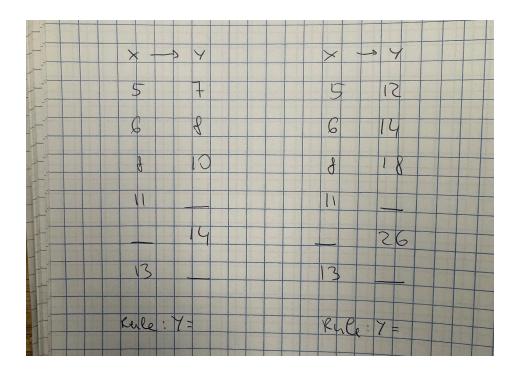
#### 2022/11/08

Dess did a lot of work over the weekend on the russian math worksheets I found (I'll make them available online later). We went over the work with her in 1 on 1 setting. She was attentive, focused and guick in answers.

In an exercise a+3 ... a-3 which is >, < or =, she said that a number +3 might be = to the same number -3, I drew a number line, put a on it and said: what does + 3 means? (I forgot what she answered), I drew 3 hops on the right of the number, then I asked what -3 means (I forgot what she answered here as well) and drew the 3 hops on the left. She said that she understands that it can't be the same number and she answered correctly that a+3 > a-3



- She correctly did number lines and finding wrong patterns
- She didn't know what double means she thought it meant + 2 not \*2
- In a word problem, such as (not this one, but same type. I'm using a simplified version) John has 38 apples and Paul 16 more, how many do they have in total, Dess did 38+16=54, but saw her mistake immediately when looked at her solution before I said anything. She wrote 38+16=54+38=92. I said that in mathematics, equal means that both sides are equal and we should separate: John=38a, Paul=38+16=54, total=38+54=92
- She solved correctly a simple multiplication word problem and filled in correctly multiplication facts
- She was not familiar to the use of letters for numbers and equations and I introduced it to her:



At the end of the 15/20 minutes, Dess said that it was much more fun than the textbook CPM exercises and she said that in a happy and confident tone of voice.

#### 2022/11/20

Today I spent about one hour with Dess working together on the booklet I gave her to do last week.

#### Observations:

- Multiplication facts are mastered
- Addition and subtraction of 3-digit numbers is mastered
- Using algebraic rules such as y=5x and y=x+5 to apply to numbers mastered
- Mastered finding x for algebraic expressions such as x+9=28, x-9=28, 23 -x=11, x+3=8 (note from later: Dess didn't do correctly this type of exercise one month later)
- For x \* 5 = 35 Dess wrote 5 / 35 = 7 I asked how much is 5 / 35 ? and then she corrected her self: 35 / 5 = 7
- When asked what is 14 less than 25 she answered 25 14 = 11
- Also answered correctly what number is 9 greater than 65 : 65+9=74
- When asked what number is t greater than 45, she hesitated but answered 45 +t, answered more surely 8 greater then m = m+8
- When asked what number is t less than 40 answered t-40, I explained for addition we can do a+b=b+a but not for subtraction: a b is not b-a
- Was able to do afterwards "what number is a less than b": b-a and "c greater than d": d+c
- We did together simplification of equations:
  - x+x+x-x=2x
  - 12+x+x-x+12 = x+24
  - -2x-x+16-12=x+4

- x+x+14+x-x-14=2x
- -4x-x-x-x=0
- -12+x-x+x-x+3=15
- Dess after the first 2 was able to do the simplifications
- Dess solved correctly the following problems
  - "There were 8 green balloons and twice as many red balloons at james's birthday party. How many red balloons were there at james's party?
    - Dess wrote 8\*2=16
  - "Hammy the hamster stored 6 grains of rice in his right cheek. He stored 3 times as many grains in his left cheek. How many grains of rice did Hammy store in his left cheek?
    - Dess wrote: "6 \* 3 = 18 He store 18 grains in his left cheek"
- Dess was not able to solve the following problems
  - Stacy solved 18 problems from her math homework. She has half as many left to solve. How many problems did Stacy's teacher assign for homework?
    - She did 18 / 2 = 9
    - I wrote for her:
      - Solved: (how many?) 18p
      - Left: (how many) 18/2=9 (she did it but said 18/9 not 18/2)
      - Total?
    - Then Dess said that total is 36, 18\*2 = 36
    - I made up a similar example: if you run a loop, you run 50 and what is left is 20 what is the total, she answered 70, but when asked: so what is total number of problems assigned, she still hesitated but ended up saying 18+9=27
  - Bella and Camilla are gathering leaves for their nature collection. Bella has 18 leaves. Camilla has twice as many as Bella. How many leaves should Camilla give Bella so that they have the same number of leaves?
    - Dess started guessing which operation to use. She said subtraction. I said yes camilla has to give to bella therefore camilla has to subtract from hers.
    - I draw 3 apples and 5 apples and said how much do you need to move from one pile to the other so they both are equal? She said one. "How do you know? " She couldn't tell. So I ended up telling her: B = 18, C= 18\*2=36 B+C = 18 + 36=(Dess calculated 54) Then Dess wrote 54/2 = 27 (calculating division and long division is mastered)
  - A visual problem (a drawing) where scales showed
    - A flog and a chick weighting 3
    - 2 chicks waiting 4
    - A chick and a bee weighting 5
    - A frog and a bee weighting as a mouse
    - The question was what is the weight of the mouse.

- Dess confidently said 2 saing that she did 5 -3=2. I asked why do you do this operation? She said: I don't know. I asked her to think and observe the picture to find clues to build her reasoning. She got it pretty quickly once she noticed the pattern, finding easily the weight of the chick: 2 and then frog, bee and mouse.
- Dess calculated correctly the area and the perimeter of rectangles
- Dess couldn't understand what was asked from her in the following exercise:
  - 3\*8-6= \_\_\_\_\* 3=  $\rightarrow$  Dess wrote 3\*8-6=18\*3=54 (I didn't see this during class so I'll need to talk to her)
- Dess was mistaken on this type of exercise:

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- b * 1 b+ 1; Dess wrote >
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- c + 0 c 0
- A+6\_\_a-6 (she did this correctly)
- m-0 \_\_\_ m \*0
- k\*1 k+1
- n / 5 \_\_\_\_n\*5 (this one was correct but it is probably random luck)
- d + d + 2d \_\_3d +d
- k+2k+3k\_k\*6
- 0\*y \_\_y / 0
- With my help when asked each term how much it is, Dess could do most of it. **She did say that k / 1 = 1** several times. I asked how much is 2 /1? And Dess said 1. I explained that 2/2 = 1 and drew 2 apples divided in 1 person and that person having 2 apples. So I concluded 2/1=2. Will see if it sticks.

#### 2022/12/02

Today I checked Dess's work but didn't have time to go over everything so I asked another 7th grade student who is at Algebra level (9th grade) to help Dess check her work and that Dess. The other girl was very happy to help out and Dess seemed content as well.

#### 2022/12/06

Dess participated in the lesson for her math group (2 other girls) on operations with negative decimal numbers. She seemed confident and at ease.

I called the 3 girls to the board to do the following operation on the board (3.71+(-5.25))(4+(-9)) Dess correctly identified the order of operations. To do 3.71 - 5.25, as Dess seemed to struggle, I wrote on the board **how much is "3 - 5 = " Dess said 8**, then I said: what does it mean to do 3 - 5? I drew a number line, circled 3 and then hopped 5 times on the left. When I started doing that Dess said the answer is -2.

Dess did solve correctly a word problem such as: Carol has 19 leaves and Mary has 34, how many should Mary give to Carol so they would have the same amount.

Today I started to prepare Dess for the catchup on the CPM course 2 (7th grade book). I asked the 3 girls of the group to sit on the floor around me and I wanted to cover the foundations of **probability**. I was surprised that the other 2 girls also didn't know that probability goes from 0% to 100%, not more, not less. We chatted, the other 2 girls found some silly and funny examples of things we can be certain to happen and others will never happen. (all 3 girls where not able to generate further inquiry questions about probability when I asked them to). Dess was not engaged, was looking at her feet, struggling to answer my questions.

After the lesson I sat with Dess and asked her what was wrong. She refused to share at first. I insisted asking: Is it linked to math or that is about something outside of math? She said math as well. I asked how did she feel about math and she answered: "not very well". So I said: "I believe that you are very smart and that you will catch up. You didn't have the same support that others had and at your last school they didn't work on what other girls worked at our school. That is hard and now you have to work harder than others. But working hard is what makes people strong and I believe that you are strong. I believe in you and I want you to believe in yourself too. I'll help you. We can spend more time together, for example on zoom during christmas vacations, and I believe that you can catch up by January with the work I prepared for you." I printed a collection of exercises with the topics covered in the first 2 chapters of the 8th grade book. She nodded and said: ok. Later she asked how much we are going to work during vacation as she is planning to travel. I answered: whatever amount works for you and me.

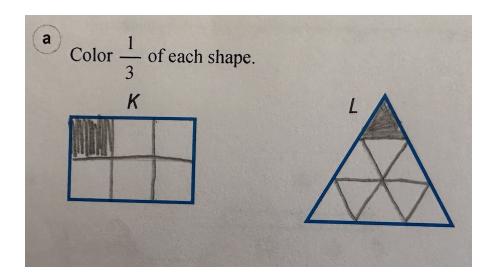
After the discussion, we went over the problems I assigned and I talked about the sign = that what it means is that we have the same thing on both sides. She didn't seem to know that.

Here is what I think at this point of time. I see that she really needs to fit in with the others and what I'm doing (including others in the lessons) is not sufficient. I think my mistake was to give to her only the worksheets and that I didn't continue assigning to her the problems in the 7th grade math textbook. So I'm planning to do more combinations of both for the next week. Another strategy is that I might give her the answers for self-check. She might benefit from seeing how to write the answers.

#### 2023/01/06

I worked with Dess over zoom twice over the christmas break and on the first week after vacations, I spent about 30 minutes with her going through her goals for the next 6 months. She said that she wanted to catch up with the others and I redirected to have a more fixed goal such as complete the grade level material by and of the school year. She agreed to work on the handout lessons (foundations) with 1 lesson per week and CPM with one review & preview per day. It is probably too ambitious but why not.

She turned in the lesson 8 of foundational math (Advanced 3rd grade level) and she correctly did 16 out of 38 exercises. Strangely she did correctly some exercises but then made reasoning mistakes on the variations of the concept. She didn't use the strategies that I suggested such as to draw and even couldn't draw fractions.



#### 2023/01/11

I've explained the case to a friend with a PhD in Neuroscience and she immediately indicated a probable mild intellectual disability.

So I consulted the school administration and we had our first meeting. My co-teacher also thinks that there is a cognitive limitation especially in abstraction, inference, association and she supports the hypothesis of a mild intellectual disability. We all agreed as well that Dess has good executive functions, good social and emotional skills and is a pleasure to work with.

Her good executive functions and good memory allowed her to memorize math facts and calculating procedures but underdeveloped the problem solving skills.

So I started searching scientific literature on teaching math to intellectually disabled children and found this one:

https://files.eric.ed.gov/fulltext/EJ1166253.pdf

#### And this:

https://www.ldonline.org/ld-topics/teaching-instruction/number-sense-rethinking-arithmetic-instruction-students-mathematical

Thinks I want to try with her:

- Tip calculation
- Sales tax calculation
- Systematic use of a number line (vertical)
- Systematic use of drawings
- Writing to explain what she did

#### 2023/02/01

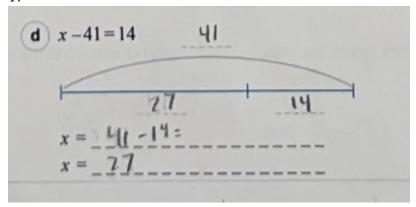
We started having weekly 1h one on one sessions after school with Dess. I do see progress and I started doubting that Dess has a mild intellectual disability. She does struggle in some parts but she is learning.

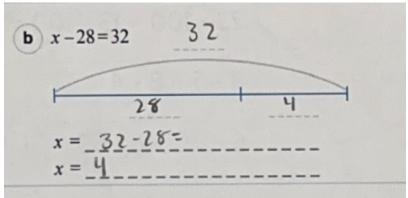
How she can correctly solve most of the calculations with integers and fractions. She has good creativity with splitting shapes in equal parts. Yesterday during tutoring we worked together on the fractions and she was not able to put the fractions on a number line. With my guidance she was able to split the shapes in equal parts and color the fractions indicated.

I will give her a test tomorrow to validate some of this progress.

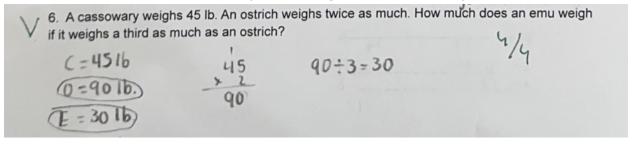
#### 2023/02/14

She didn't do as well on the test, as I thought. She still struggles with subtraction. Here is the type of mistake that I see.





She did correctly the area calculations but mislabeled the sides. She did one of the 2 problems correctly. That was great! She did it all by herself.



I gave her a C. Maybe that was a mistake.

Today she said she couldn't do the following calculation: 40\*8-80. She answered that it equals 40 because **320 - 80=40** and she showed how she did the calculations in column ignoring the 2 hundreds left. So when I asked to her to check, she didn't know how and when I said that if 320 -80=40 then 80+40=320 she said no.

She also struggled and was not able to find a solution to the following problem:

If one chick and 2 cups equal 35 grams, and 2 chicks equal (in weight) 1 cup, what is the weight of one chick? (the problem was a drawing with scales) In particular she couldn't check her work. She initially said 15, then 18, then 17 and was not able to trace back to check if it makes sense to have one chick weighting 17 grams.

She is still struggling with a number line. She can't place any fraction on a number line correctly without help.

I feel that her confidence is getting lower. What can I do to make her more successful?

#### 2023/03/03

In the last 2 weeks, Denisse has worked on the same material as before. She is getting better at placing the fractions on the number line. Also I wrote for her 3 little post-its:

1. Addition / subtraction

a+b=c

b+a=c

c-a=b

c-b=a

2. Multiplication / division

a \* b = c

 $b^* a = c$ 

c/a=b

c/b=a

3. Distributive property

a\*b+a\*c=a\*(b+c)

Dess still struggles with those concepts. We have started tutoring sessions of 1h per week since December 2022 and I think I have shown many methods and use cases of these 3 mathematical concepts but somehow they don't stick.

One day I asked Dess what she wants to do when she grows up and she said: "Nothing to do with math". Then "I was thinking about occupational therapy for children." I see her very well in that type of work. I hope she will be successful.

I also asked her to do a khan academy "get ready for 7th grade" test. It took her about 60 minutes instead of 30/45 minutes and here is the result:

### Get ready for 7th grade

## Get ready for 7th grade

SKILL	CHANGE	LEVEL	SKILL	CHANGE	LEVEL
Dividing fractions	•	Attempted	Dividing fractions	•	Attempted
Tables from equations with 2 variables	<b>^</b>	Familiar	Tables from equations with 2 variables	<b>^</b>	Familiar

## Get ready for 7th grade

## Get ready for 7th grade

Multiplying fractions	<b>^</b>	Familiar	Relationships between quantities in equations	<b>4</b>	Attempted
Add fractions with unlike denominators	•	Attempted	Understand equivalent ratios in the real world	<b>^</b>	Familiar
Nets of polyhedra	<b>↑</b>	Familiar	Multiply mixed numbers and whole numbers	<b>↑</b>	Familiar
Area of triangles	•	Attempted	Negative fractions on the number line	<b>^</b>	Familiar
Unit rates	<b>↑</b>	Familiar	Percent word problems	<b>\</b>	Attempted
Write whole numbers as fractions	<b>↑</b>	Familiar	Shape of distributions	<b>\</b>	Attempted
Surface area	•	Attempted	Translate one-step equations and solve	<b>\</b>	Attempted
Statistical questions	•	Attempted	Fractions on the number line	<b>↑</b>	Familiar
Interpreting negative numbers	<b>↑</b>	Familiar	Factor with distributive property (variables)	<b>\</b>	Attempted
Combining like terms	•	Attempted	Basic multiplication	<b>↑</b>	Familiar

## Get ready for 7th grade

## Get ready for 7th grade

Independent versus dependent variables	<b>^</b>	Familiar	Multiply mixed numbers	<b>^</b>	Familiar
Inequalities word problems	•	Attempted	Independent versus dependent variables	<b>^</b>	Familiar
Ratios on coordinate plane	•	Attempted	Inequalities word problems	<b>4</b>	Attempted
Unit fractions on the number line	<b>↑</b>	Familiar	Ratios on coordinate plane	<b>+</b>	Attempted
Comparing rates	<b>^</b>	Familiar	Unit fractions on the number line	<b>^</b>	Familiar
Equivalent ratios with equal groups	•	Attempted	Comparing rates	<b>^</b>	Familiar
Decompose area with triangles	•	Attempted		<b>→</b>	
Compare and order absolute values	<b>4</b>	Attempted	Equivalent ratios with equal groups		Attempted
Fractions as division	<b>↑</b>	Familiar	Decompose area with triangles	•	Attempted
			Compare and order absolute values	•	Attempted
Model with one-step equations	•	Attempted	Fractions as division	<b>^</b>	Familiar
Find the mistake in one-step equations	<b>4</b>	Attempted			

In total 10 out of 30 questions were answered correctly. She is **good at multiplication facts** and unit rates (A box of 15 cookies costs \$. What is the cost for 1 cookie?).

Then I assigned some more work at khan academy on **Dividing fractions and Add fractions** with unlike denominators as I know she knows how to do them and after doing some more she was successful.

At this point I have a lot of thinking going on as I see that what I'm doing is not working as I see her demotivated and putting little work in math. So I have a few ideas to try:

- Identify her strengths in math with khan academy
- Switch to mastery goals instead of assignments
- Increase motivation/ engagement by introducing real-life math