1.6 Momentum

Momentum

- An object with **mass** that is in **motion** has **momentum**

The momentum equation

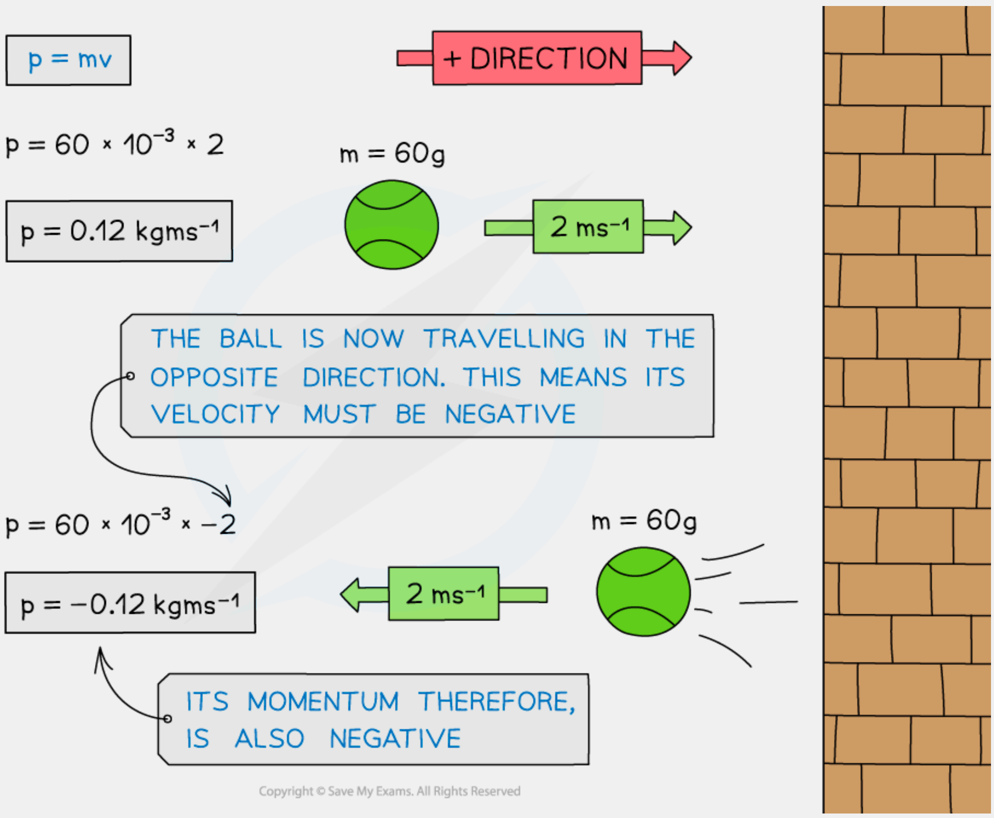
- Momentum is defined by the equation:

$$momentum = mass \times velocity$$

$$p = mv$$

- Where:
  - p = momentum, measured in kilogram metres per second (kg m/s)
  - m = mass in kilograms (kg)
  - v = velocity in metres per second (m/s)
- This means that an object at rest (i.e. $v = 0$) has **no** momentum
- Momentum keeps an object moving in the same direction
  - It is difficult to change the direction of an object that has a large momentum
- Velocity is a **vector** with both magnitude and direction
  - This means that the momentum of an object also depends on its **direction** of travel
  - Therefore, momentum can be either **positive** or **negative**
- If an object has positive momentum, then an object travelling in the opposite direction will have negative momentum

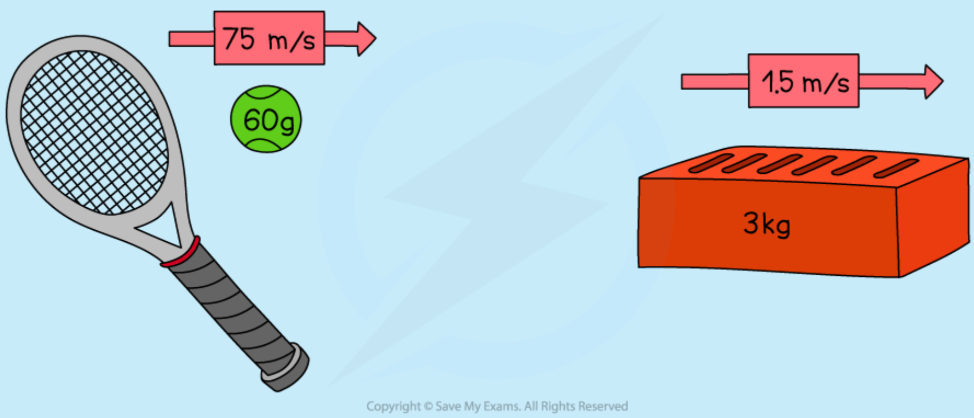
How does the momentum of a ball change after a collision?



The momentum of the tennis ball is positive as it approaches the wall and negative after the collision, as it moves in the opposite direction

Worked Example

Determine which object has the most momentum.



Answer:

Step 1: Determine the momentum of the tennis ball using the momentum equation

$$p = mv$$

$$p = 0.06 \times 75$$

$$p = 4.5 \; kg \; m/s$$

Step 2: Determine the momentum of the brick using the momentum equation

$$p = mv$$

$$p = 3 \times 1.5$$

$$p = 4.5 \; kg \; m/s$$

Step 3: Compare the momentum of each object

- Both the tennis ball and the brick have the same momentum
- Even though the brick is much heavier than the ball, the ball is travelling much faster than the brick
- This means that on impact, they would both exert a similar force (depending on the time it takes for each to come to rest)

Examiner Tips and Tricks

You can remember momentum as mass in motion. The units of momentum are **kg m/s** which is the product of the units of mass (kg) and velocity (m/s).

Which direction is taken as positive is completely up to you in the exam, as long as you are consistent throughout a question. In general, the right and upwards are taken as positive, and down or to the left as negative.

Conservation of momentum

- The principle of conservation of momentum states that:

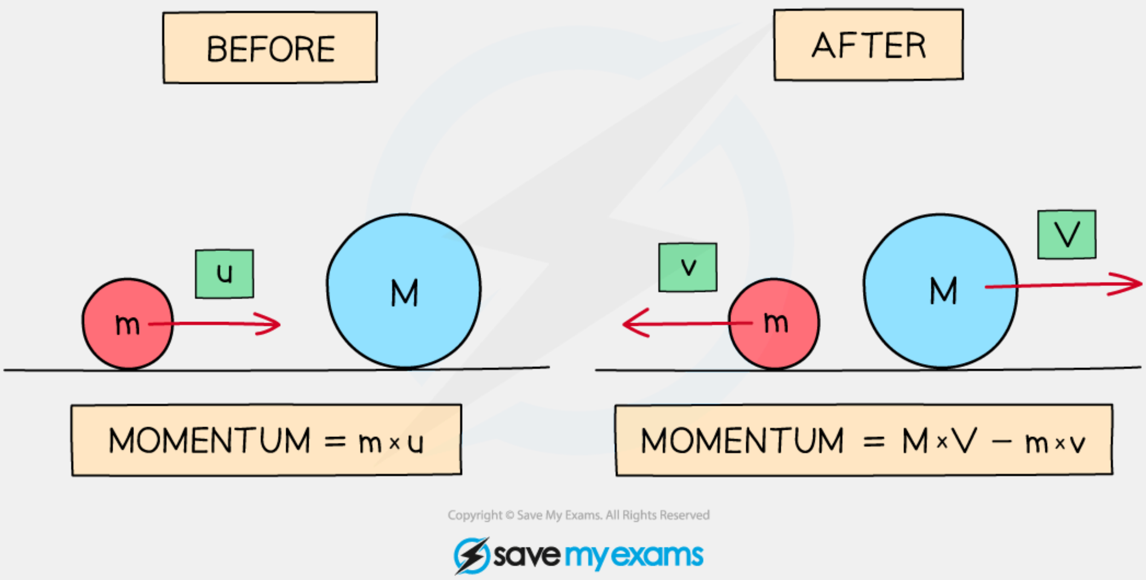
In a closed system, the total momentum before an event is equal to the total momentum after the event

- A **system**, in physics, is an object or group of objects
- A **closed system** means that no energy is transferred into or out of the system and there are no external forces acting
- The principle of conservation of momentum can also be written as:

The total momentum before a collision = The total momentum after a collision

- Since momentum is a **vector** quantity, a system of objects moving in opposite directions (e.g. towards each other) at the same speed will have an overall momentum of 0 since they will **cancel out**
    - Momentum is **always conserved** over time
- The diagram below shows two masses *m* with velocity *u* and *M* at rest (ie. zero velocity)

Principle of conservation of momentum for a collision

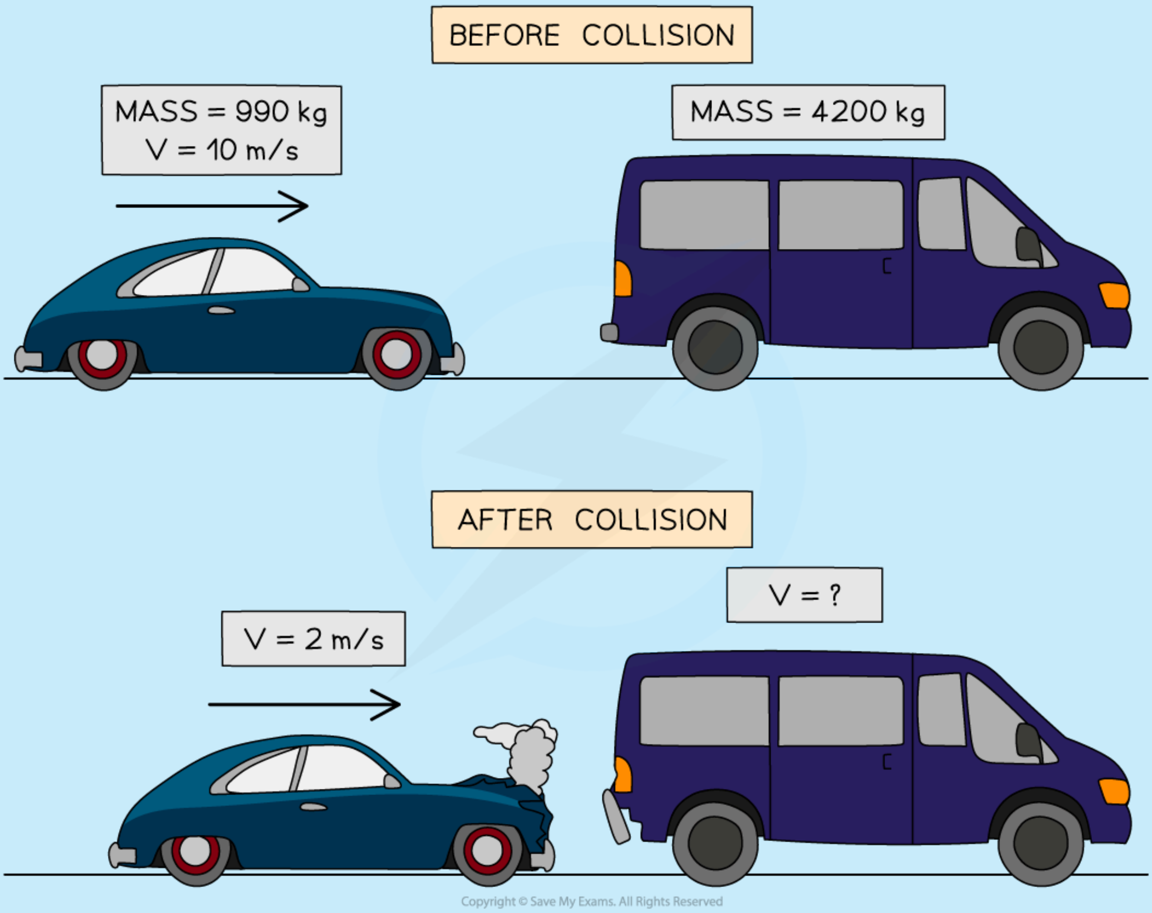


The momentum of a system before and after a collision is constant

- Before the collision:
    - The momentum is only of mass *m* which is moving
    - If the right is taken as the positive direction, the total momentum of the system is ***m* × *u***
- After the collision:
    - Mass *M* also now has momentum
    - The velocity of *m* is now -*v* (since it is now travelling to the left) and the velocity of *M* is *V*
    - The total momentum is now the momentum of *M* + momentum of *m*
    - This is ($M \times V$) + ($m \times -v$) or **($M \times V$) – ($m \times v$)**

Worked Example

The diagram shows a car and a van, just before and just after the car collided with the van, which is initially at rest.



Use the idea of conservation of momentum to calculate the velocity of the van when it is pushed forward by the collision.

Answer:

Step 1: State the principle of conservation of momentum

$$total\ momentum\ before\ =\ total\ momentum\ after$$

Step 2: Calculate the total momentum before the collision

$$p\ =\ mv$$

- Momentum of the car

$$p\ =\ 990\times 10$$

$$p = 9900\ kg\ m/s$$

- Momentum of the van
    - The van is at rest
    - Therefore, *v* = 0 m/s
    - Therefore, *p* = 0 kg m/s
- Total momentum before collision

$$p_{before} = 9900 + 0 = 9900\ kg\ m/s$$

Step 3: Calculate the total momentum after the collision

- Momentum of the car

$$p = 990 \times 2$$

$$p = 1980\ kg\ m/s$$

- Momentum of the van

$$p = 4200 \times v$$

- Total momentum after collision

$$p_{after} = 1980 + 4200v$$

Step 4: Rearrange the conservation of momentum equation to solve for *v*

$$p_{before} = p_{after}$$

$$9900 = 1980 + 4200v$$

$$4200v = 9900 - 1980 = 7920$$

$$v = \frac{7920}{4200}$$

$$v = 1.9\ m/s$$

Examiner Tips and Tricks

If it is not given in the question already, drawing a diagram of before and after helps keep track of all the masses and velocities (and directions) in the conversation of momentum questions.

Impulse

- When an external resultant force acts on an object for a very short time and changes the object's motion, we call this **impulse**
    - For example:
        - Kicking a ball
        - Catching a ball
        - A collision between two objects

The impulse equation

- Impulse is the **product** of the **force** applied and the **time** for which it acts

$$impulse = F\Delta t$$

- Where:
    - F = resultant force, measured in newtons (N)
    - Δt = change in time, measured in seconds (s)
- Therefore, the **units** of impulse are **newton seconds** (Ns)
- The **impulse** of the resultant force is **equal to** the **change in momentum** of the object

$$impulse = F\Delta t$$

$$impulse = (ma) \times \Delta t$$

$$impulse = m \times \frac{v-u}{\Delta t} \times \Delta t$$

$$impulse = m \times (v - u)$$

$$impulse = mv - mu$$

$$impulse = \Delta p$$

- Where:

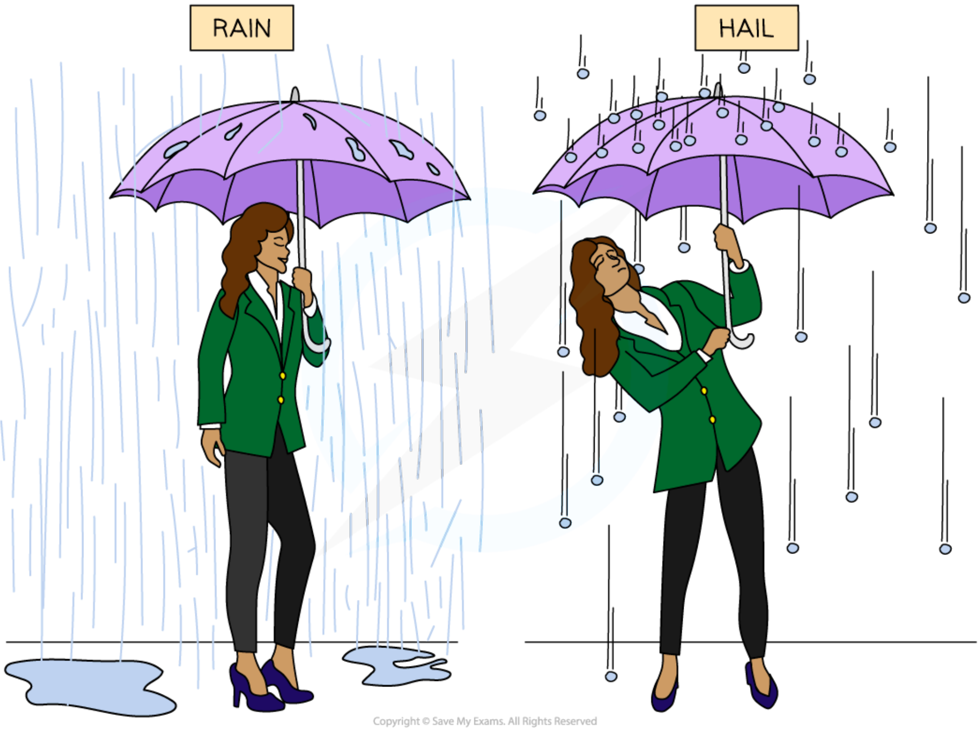
- Δp = change in momentum, measured in kilogram metres per second (kg m/s)
- m = mass, measured in kilograms (kg)
- v = final velocity, measured in metres per second (m/s)
- u = initial velocity, measured in metres per second (m/s)

- Therefore:

$$impulse = F\Delta t = \Delta p = mv - mu$$

Examples of impulse

- When rain and hail (frozen water droplets) hit an umbrella, they feel very different due to the **impulse** of the force
- Water droplets tend to splatter and roll off the umbrella because there is only a very **small** change in momentum
- Hailstones have a **larger mass** and tend to bounce back off the umbrella, because there is a **greater** change in momentum
- Therefore, the impulse that the umbrella applies on the hail stones is **greater** than the impulse the umbrella applies on the raindrops
- This means that **more force** is required to hold an umbrella upright in hail compared to rain



The impulse exerted by the umbrella on the hail stones is greater than the impulse exerted on the raindrops.

Worked Example

A 58 g tennis ball moving horizontally to the left at a speed of 30 m s^{-1} is struck by a tennis racket which returns the ball back to the right at 20 m s^{-1}.

(i) Calculate the impulse of the force exerted by the racket on the ball
(ii) State which direction the impulse is in

Answer:

Part (i)

Step 1: **Write the known quantities**

- Taking the initial direction of the ball as positive (the left)
- Initial velocity, $u = 30\ m/s$
- Final velocity, $v = -20\ m/s$
- Mass, $m = 58\ g = 58 \times 10^{-3}\ kg$

Step 2: Write down the impulse equation

$$impulse = F\Delta t = \Delta p = mv - mu$$

Step 3: Substitute in the values

$$impulse = m(v - u)$$

$$impulse = 58 \times 10^{-3}(-20 - 30)$$

$$impulse = -2.9\ N\ s$$

Part (ii)

State the direction of the impulse

- Since the impulse is negative, it must be in the opposite direction to which the tennis ball was initially travelling
- Therefore, the direction of the impulse is to the right

Examiner Tips and Tricks

Remember that if an object changes direction, then this must be reflected by the change in sign of the velocity. For example, if the left is taken as positive and therefore the right as negative, an impulse of 20 N s to the right is equal to -20 N s

Force & momentum

- Newton's second law is defined by the equation:

$$F = ma$$

- Momentum is given by the equation:

$$p = mv$$

- Combining these equations gives Newton's second law in terms of momentum:

$$F = ma$$

$$F = m\left(\frac{v-u}{\Delta t}\right)$$

$$F = \frac{mv-mu}{\Delta t}$$

$$F = \frac{\Delta p}{\Delta t}$$

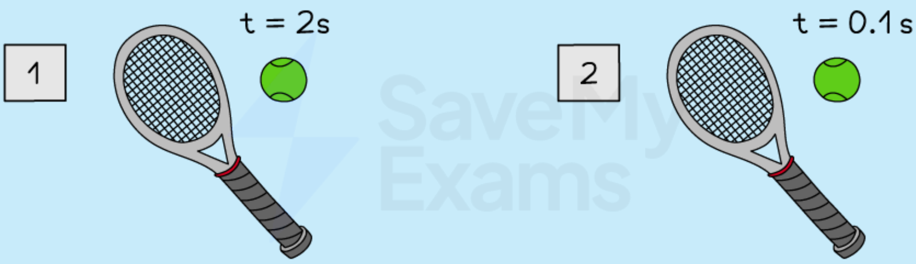
- Therefore, force can also be defined as the **rate of change of momentum** on a body
    - Or the change in momentum per unit time

Worked Example

A tennis ball hits a racket twice, with a change in momentum of 0.5 kg m/s both times.

During the first hit, the contact time is 2 s and during the second hit, the contact time is 0.1 s

Determine which strike of the tennis racket experiences the greatest force from the tennis ball.



Answer:

Step 1: Calculate the force during the first hit

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{0.5}{2}$$

$$F = 0.25\ N$$

Step 2: Calculate the force during the second hit

$$F = \frac{\Delta p}{\Delta t}$$

$$F = \frac{0.5}{0.1}$$

$$F = 5.0\ N$$

Step 3: State your answer

- The tennis racket experiences the greatest force from the ball during the **second hit**