Classical Mechanics Comprehensive Exam June 29, 2009

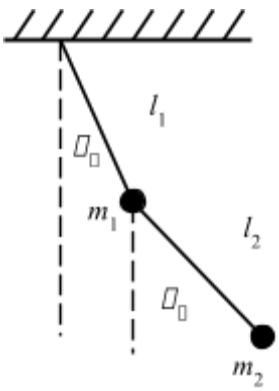
(Vectors are indicated with boldfaced characters.)

1. A Frisbee, which is axially symmetric disk with the moment of inertia along the symmetry axis I_3 greater than $I_1 = I_2$, is thrown into the air so that the rotation vector $\boldsymbol{\omega}$ is not aligned with the symmetry axis. As a consequence, the disk wobbles. Air resistance on the disk exerts a frictional torque on the disk given by $\tau_f = -c\omega$.

   a. How does the component of $\boldsymbol{\omega}$ in the direction of the symmetry axis change with time?

   b. Show that degree of wobble steadily diminishes as a result of the air resistance. Hint: examine the time dependence of the angle between $\boldsymbol{\omega}$ and the symmetry axis.

2. A double pendulum, which is shown in the diagram, consists of a mass m_1 suspended from a massless rod of length l_1 from a fixed point. Attached to the mass m_1 is a second massless rod of length l_2 with a second mass m_2 at the end of the rod.



   a. Derive the equations of motion for the double pendulum using Lagrange's method. Assume the amplitudes are small.

   b. If $l_1 = l_2$ and $m_1 = m_2$, what are the eigenfrequencies?

   c. If $l_1 = l_2$ and $m_1 = m_2$, what are the normal modes, η_1 and η_2, in terms of θ_1 and θ_2?