Shoddy Splitting

How to fold into thirds…approximately.

Inspired by this video by Dr. James Tanton. Here's a longer explanation. And here's Stand Up Maths celebrating Thirdsday.

You'll need a bunch of string, cut into pieces that are a couple-few feet long, enough so that each participant can have their own string.

1. Show how to fold into thirds. You could show the video of James Tanton doing it, but it's more fun to do it yourself and have participants follow along.
2. Participants practice doing it themselves so everyone has an understanding of how it goes.
3. How does it work?? Participants explore.
    a. This takes a bunch of time. In the video, Tanton says each time it "halves the error", but even with that hint I spent some time embracing productive struggle.
    b. Here's some things participants can do if they are stuck
        i. Exaggerate - even if your first guess at ⅓ is absurd, it will still work. Even if you start with exactly ⅓, it will still work.
        ii. Measure the error each time. You can mark where ⅓ is actually on the string by tying a knot, and then measure how much you're off by after doing the trick once, twice, three times. There's some off-ness in the measuring, but it should be about half each time.
    c. Our participants came up with two explanations. Each depends on a bit of algebra - if your first length of string is called x, then your "new guess" is $\frac{1-x}{2}$.
        i. Here's a sort of "numerical analysis-y" approach. Say your first guess, AKA x is also $\frac{1}{3} + \epsilon$ where ϵ represents some error term. Then your next guess is $\frac{1-(\frac{1}{3}+\epsilon)}{2} = \frac{\frac{2}{3}-\epsilon}{2} = \frac{1}{3} - \frac{\epsilon}{2}$. What's important to note is that you are now off by $\frac{\varepsilon}{2}$, which is half of what you were off by before.
        ii. Another participant thought of it like when you do integration by substitution with trig functions, and sometimes cycle back to the same integral in Calc 2. It also reminds me of a continued fraction. Let y be what happens when you do this process infinitely. Then $y = \frac{1-\frac{1-\frac{1-\dots}{2}}{2}}{2}$ where the … stands for go on forever. Then this means $y = \frac{1-y}{2}$. Solving this yields $y = \frac{1}{3}$.

4. What other fractions can you make?

a. You could focus this on using only folding in halves.
b. Participants might notice that ½ , ¼, ⅛, etc are “easy”. For ⅙, you can get your best guess at ⅓ and then fold that in half. What other fractions can they get like this?
c. To make other fractions, you can start with a guess, and keep track of your foldings in ½. E.g., make a guess at ⅕. Then the other end is ⅘. Folding the long end in half for a guess at ⅖. Then fold the short end in half for a new guess at ⅕. If you keep this process going, you’ll get pretty close to ⅕.