PRACTICE   6.3/6.4 – Extrapolation and Piecewise Functions

*Full, worked solutions can be found in the folder linked on the Course Website ☺

Exercise 6G

1 The CEO of a publishing company wants to know whether there is a linear association between the number of pages in a book (x) and the number of errors (y) found in the book. Ten books were chosen at random, and the information is shown in the table.

x	100	130	170	80	220	260	290	300	200	150
y	8	10	13	10	12	13	15	16	9	10

a Plot the data points on a scatter graph. Label the axes.

b Describe the correlation. Hence, explain why it is appropriate to find the regression line.

c Find the regression line of y on x.

d Hence estimate the number of errors in a book that has 280 pages.

e Comment on whether it would be reliable to use this equation to estimate the number of errors in a book that has 400 pages.

2 A group of 10 employees at a factory were given a number (x) of training sessions. They were then asked to complete a task. The times taken to complete this task (y) were recorded, measured in minutes. The results are shown in the table.

Number of sessions (x)	3	4	5	3	7	7	8	9	9	8
Time taken (y min)	10	15	14	12	7	12	6	5	6	4

a Find the correlation coefficient, r.

b Comment on the relationship between the number of training sessions and the time taken to complete the task.

c Find the equation of the regression line of y on x.

d Hence estimate how long it would take an employee to complete the task if they were given six training sessions.

3 There is a moderate positive linear correlation between variables x and y. The regression line for y on x is $y = 2x + 1$, and it was determined from a table of values where x is such that $3 \le x \le 12$.

a Explain why it is appropriate to find the equation of the regression line.

b Explain why it is not reliable to use the regression line to estimate the value of y when $x = 0$.

c Explain why it is reliable to use the regression line to estimate the value of y when $x = 10$.

d Explain why it is not valid to use the regression line to estimate the value of x when $y = 20$.
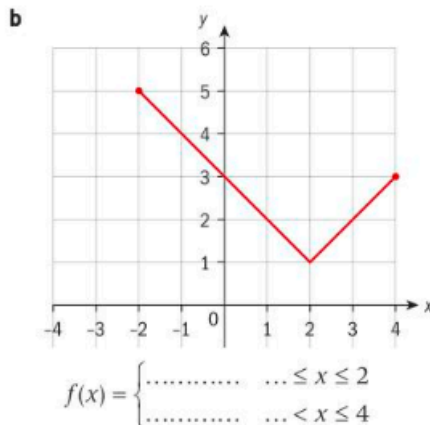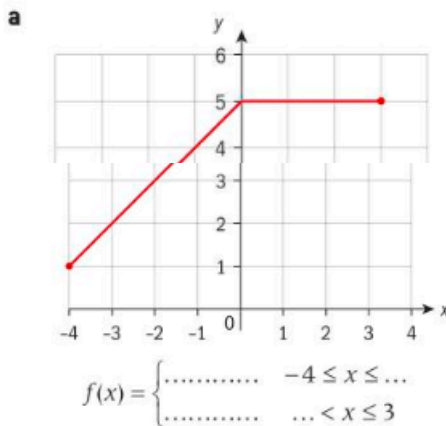
1 For each of the following piecewise linear functions:

    i State the domain.

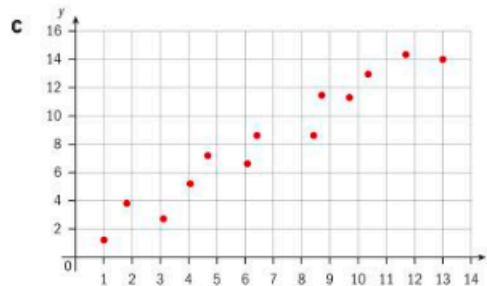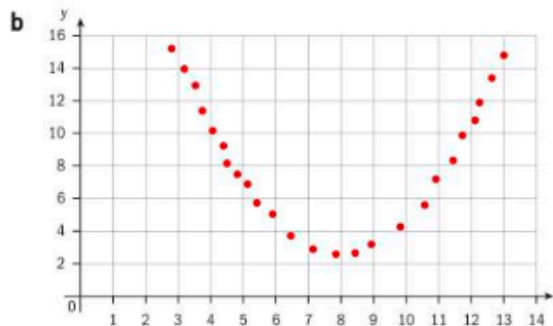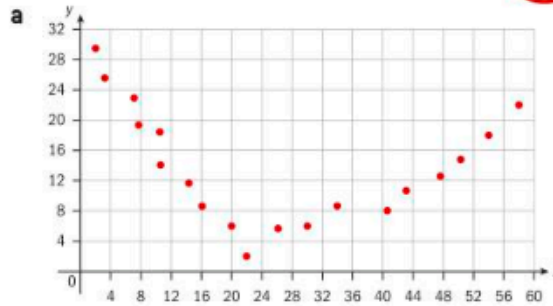   **ii** Plot the graph.

    iii Find $f(3)$ and $f(-3)$.

a $f(x) = \begin{cases} 1+x & -3 \le x \le 4 \\ 9-x & 4 < x \le 10 \end{cases}$

b $f(x) = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$

2 Determine the definition of the piecewise linear function shown in each of the graphs.

a

$f(x) = \begin{cases} \cdots\cdots\cdots & -4 \le x \le \ldots \\ \cdots\cdots\cdots & \ldots < x \le 3 \end{cases}$

b

$f(x) = \begin{cases} \cdots\cdots\cdots & \ldots \le x \le 2 \\ \cdots\cdots\cdots & \ldots < x \le 4 \end{cases}$

3 Consider the following sets of data points. Decide in which cases you would choose a piecewise linear model to fit the data points.

a

b

c

4 For any part of question **3** that was suitable for modelling with a piecewise linear function:

    a Draw this model "by eye".

    b Estimate the value of y when $x = 12$.

5 Consider the set of data points in the table.

x	1	3	4	6	7	9	10.5	12	13	15	14	17	16	18
y	5.3	4.8	4	4.2	3.4	3.1	3	2	4	7.3	8	11.5	11	15

    a Plot these points on a scatter diagram.

    b Find a piecewise linear model that best fits these data points.

    c Draw your model on the same set of axes used for part **a**.

    d Hence estimate the value of y when:

      i $x = 8$ **ii** $x = 15.5$

1 Decide whether the following statements are true or false.

a When the gradient of the regression line is positive, the correlation is strong.

b A negative linear correlation will be modelled by a line with a negative gradient.

c The independent variable, x, can never take the value zero because this would mean extrapolation.

2 For each of the following scenarios:

i State the value of the gradient of the regression line and interpret its meaning.

ii State the value of the y-intercept of the regression line and interpret its meaning if relevant, giving a reason if there is no meaning.

2. continued…

a A number of students were asked for their average grade at the end of the last year of high school, x, and their average grade at the end of their first year at university, y. On calculating the regression line for the resulting data, the result was $y = -2.50 + 1.04x$.

b It is found that the relationship between the height in centimetres, x, and the weight in kilograms, y, of a group of 15-year-old students can be modelled with the regression line $y = -70 + 0.87x$.

c A car salesman wants to study the relationship between the time in years after a particular type of car is bought, x, and the value of the car in US\$, y. The regression line is found to be $y = -250x + 9000$.

3 Different weights are suspended from a spring and the length of the spring measured. The results are shown in the table.

Weight (x g)	100	150	200	250	300	350	400
Length of spring (y cm)	26	35	32	37	48	49	52

a Find the correlation coefficient, r.

b Comment on the correlation.

c The equation of the regression line of y on x is $y = ax + b$.

i Find the value of a. Comment on its meaning.

ii Find the value of b and interpret its meaning if relevant. If not relevant, explain why.