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Total No. of Printed Pages: 1

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B.Sc. (Non-Medical) (Semester – 3rd)

REAL ANALYSIS-II

Subject Code: BSNMS1307

Paper ID: [22131417]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a. Give an example of a bounded function that is not Riemann Integrable on $[0, 1]$.
- b. Define point-wise convergence and uniform convergence.
- c. State Dini's Theorem.
- d. State Cauchy's criterion of uniform convergence.
- e. Define dense set. Give example..
- f. Describe open spheres for usual metric on $\mathbb{R}$.
- g. Give an example to show that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$.
- h. State Fundamental Theorem of Integral Calculus.
- i. Write a note on exponential functions.
- j. Can a discontinuous function be Riemann Integrable? Explain.

Section – B

(5 marks each)

Q2. If $f(x) = 2x + 3$, show that f is Riemann Integrable on $[0, 2]$ and $\int_0^2 f dx = 10$.

Q3. Show that the sequence of functions $\langle f_n \rangle$, where $f_n(x) = \frac{x}{1+nx^2}$, $\forall x \in \mathbb{R}$ converges uniformly on any closed interval.

Q4. Test for uniform convergence of series $\frac{1}{(1+x)^3} + \frac{2}{(2+x)^3} + \frac{3}{(3+x)^3} + \dots$, $x \geq 0$.

Q5. (i) Prove that the arbitrary union of open sets is open.

(ii) Prove that finite intersection of open sets is open.

Q6. Prove that every continuous function is Riemann Integrable.

Section – C

(10 marks each)

Q7. (a) If f is Riemann Integrable on $[a, b]$, then prove that $|f|$ is also Riemann Integrable on $[a, b]$, and $|\int_a^b f(x)dx| \leq \int_a^b |f|(x)dx$.

(b) If $\langle f_n \rangle$ is a sequence of functions on E , and if $f_n \rightarrow f$ uniformly on E , then prove that f is continuous on E .

Q8. (a) Discuss the absolute convergence and uniform convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{n^p (1+x^{2n})}, \quad p > 0.$$

(b) Prove that the function $d: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by $d(x, y) = |x - y|$, $\forall x, y \in \mathbb{R}$ is a metric and (X, d) is a metric space.

Q9. (a) Define the following and give suitable examples:

(i) Bounded set

(ii) Interior point of a set

(iii) Closure of a set.

Is closure of a set closed? Explain.

(b) State and prove Bounded Convergence Theorem.