Name: ID Number:

VMT Problem Solving Round

Do not open the test until instructed to do so.

Information:

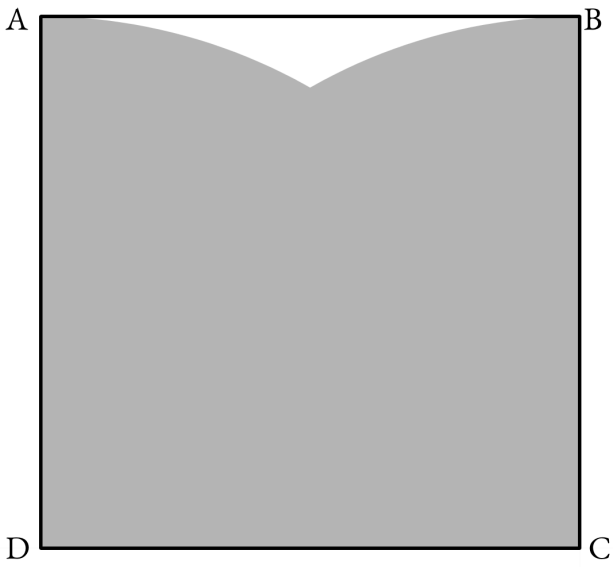
- The Problem-Solving Round contains 10 questions that you will be given 40 minutes to answer.
- There will be an 11th estimation question that will be used as a tiebreaker. There is no penalty for making an educated guess.
- Do not expect to be able to solve every question. Likewise do not be afraid to skip questions. The questions are only roughly but not strictly in order of difficulty.
- Please write legibly and in the specified form; all answers that cannot be read or are in the incorrect form will be considered wrong.

Instructions:

- You will receive one answer sheet.
- On the answer sheet, make sure to LEGIBLY write your name, Competition ID, and the school that you attend.
- Only answers on the answer sheet will be graded, not answers in the test booklet.
- Only basic writing implements (i.e. pencils, pens, erasers) are allowed. All other tools (e.g. calculators, compasses, rulers, etc…) or external help is prohibited.

Good luck and have fun! If you have any questions, raise your hand.

1. Given the square ABCD in the following figure, with side lengths of 1, and two quarter circles, with radii of 1 centered at point C and D, what is the area of the unshaded area? (your answer should be in simplified radical form, and as a single fraction)



2. Find the area between the triangle formed by line A, whose equation is given by $3x - 2y = 4$; the y-axis; and line B, which passes through the point (1.5, 2), and is perpendicular to $3x + 2y = 4$. Answer as a decimal rounded to the nearest tenth.

3. Two 5 sided die, with faces 1, 2, 3, 4, 5 are modified such that the number on each face represents the likelihood that that face will be rolled. That is, the face with the number 5 on it is 5 times more likely to be rolled than the face with the number 1, the face with the number 4 on it is 2 times more likely to be rolled than the face with the number 2, etc. What is the probability that the resulting product of the numbers rolled on each of the two die is odd?

4. Consider all 5 letter "words" made from the letters A through H. (In this problem, words are just strings of letters, not necessarily actual English words.) How many of the words do not contain the sub-word "BAD"? (letters may not be repeated).

5. Suppose that x and y are integers such that $x \geq 5$, $y \geq 3$, and $\sqrt{x-5} + \sqrt{y-3} = \sqrt{x+y}$. Compute the maximum possible value of xy.

6. One fifth of a class prefers the color blue, a sixth of the class prefers red, and those who prefer green constitute one half of the sum of those who prefer blue and red. The remaining 27 students prefer yellow. How many students are in the class?

7. Colin runs a snow cone stand. His ice machine can produce 6000 cm^3 of ice every 30 minutes. A complete snow cone consists of ice filling a conical paper cup with a hemisphere of ice on top. His snow cone cups are 15 centimeters deep and have a circular top with a diameter of 6 centimeters. How many complete snow cones can he make in an hour? (Approximate pi as 3.14)

8. In year N, the 300th day of the year is a Tuesday. In the year N+1, the 200th day is also a Tuesday. On what day of the week did the 100th day of year N-1 occur?

9. A point is chosen uniformly at random in square ABCD. What is the probability that it is closer to one of the 4 sides than to one of the 2 diagonals?

10. Twenty five of King Peter's knights are seated at their customary round table. Three of them are chosen - all choices being equally likely - and are sent off to slay a troublesome goat. Let P be the probability that at least two of the three had been sitting next to each other. If P is written as a fraction in lowest terms, what is the sum of the numerator and denominator?

Tiebreaker:
The Narayana's Cows sequence is an integer sequence based on the special breeding patterns of cows, in which a cow gives birth to one calf every year, but each calf begins reproducing only when it reaches its third year. The sequence is defined recursively as $a_n = a_{n-1} + a_{n-3}$. Estimate the 2025th term in scientific notation.