Comma-Predecessor Theorem
Michael S. Branicky
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Here, we use the following characterization of comma successor k' of number k in base b from https://oeis.org/A121805:

> Let x be the least significant digit of k; then k' = k + x*b + y where y is the most significant digit of k' and is the smallest such y, if such a y exists. If no such y exists, [there is no successor]. (*)

Note that k' - k <= b^2 - 1.

Theorem. All terms of A367600 (numbers that are not the comma-successor of any number) greater than 98 are of the form

    c*10^i for i >= 2 and 2 <= c <= 9. (**)

Corollary. The theorem and proof are for b = 10, but replacing 10 with b, 9 with b-1 and 8 with b-2 gives the general case for base b > 2.

Proof. Let k' have i + 2 digits, where i >= 1. Then

    k' = f' s w z (#)

where |f'| = |w| = |z| = 1 and |s| >= 0.

Since f' is the first digit of k', we had to have
(1) produced k' by y = f' in (*)
(2) no smaller y would have produced f' using (*)

From (2), we have that k' cannot be equal to f' * 10^i for i >= 2, and 2 <= f' <= 9, else adding 1 <= f' - 1 <= 8 would have produced a first digit of f' - 1, satisfying (*). Thus, terms of the form (**) cannot have predecessors.

It remains to show that all other terms > 98 have predecessors.

    90 -> 90 + 10*0 + 9 = 99, so 99 has a predecessor

Also note that if k' = 10^i for i >= 2, then (10^i - 1) - 9*10 = 10^i - 91 is its predecessor.

All other terms > 100 have predecessor

    k' = n - f' - 10*x > 0, where x = (n-f') mod 10 and f' is the first digit of n. ###

Corollary. These terms are precisely the greater successors (> 99) of terms in A367346.