Nonlinearities and Approximations
Minimization of the Sum of Absolute Deviations - transformation

1

$$\in_i = Y_i - \sum_j X_{ji} b_j$$

2. A constraint set is formed by moving the $\sum X_{ji}b_j$ term to the left side of the equation.

$$\in_i + \sum_j X_{ji} b_j = Y_i$$

3. The basic problem of minimizing the summed absolute values of all ε_i is:

$$\begin{aligned} Min \quad & \sum_i |\in_i| \\ s.t. \quad & \in_i + \sum_j X_{ji} b_j = Y_i \quad for\ all\ i \\ & \in_i \ \overset{<}{\underset{>}{-}}\ 0 \quad b_j \ \overset{<}{\underset{>}{-}}\ 0 \quad for\ all\ i\ and\ j \end{aligned}$$

4. Conversion to LP problem: $\in_i = \in_i^+ - \in_i^-$

5. The resultant problem is:

$$\begin{aligned} Min \quad & \sum_i |\in_i^+ - \in_i^-| \\ & \in_i^+ - \in_i^- + \sum_j X_{ji} b_j = Y_i \quad for\ all\ i \\ & \in_i^+, \in_i^- \geq 0 \quad b_j \ \overset{<}{\underset{>}{-}}\ 0 \quad for\ all\ i\ and\ j \end{aligned}$$

Which is still nonlinear

Nonlinearities and Approximations

Minimization of the Sum of Absolute Deviations

- transformation

6. $\in_i^+ * \in_i^- = 0,$ which can be written as:

$$\left|\in_i^+ - \in_i^-\right| = \left|\in_i^+\right| + \left|\in_i^-\right| = \in_i^+ + \in_i^-$$

$$\textit{whenever} \quad \in_i^+ * \in_i^- = 0$$

7. The formula then becomes:

$$\begin{aligned}
Min \quad & \sum_i \left(\in_i^+ + \in_i^-\right) \\
s.t. \quad & \in_i^+ - \in_i^- + \sum_j X_{ji}\, b_j = Y_i \quad \textit{for all } i \\
& \in_i^+ * \in_i^- = 0 \quad \textit{for all } i \\
& \in_i^+ , \in_i^- \geq 0 \qquad b_j \ \substack{\leq \\ - \\ >} \ 0 \quad \textit{for all } i \textit{ and } j
\end{aligned}$$

This is an LP formulation except for the constraint on the product of ε_i^+ and ε_i^-. However, this constraint can be dropped.

Nonlinearities and Approximations

Minimization of the Sum of Absolute Deviations

- transformation

Reduced Problem:

$$Min \quad |\in| = \in^{+} + \in^{-}$$

$$\in^{+} - \in^{-} = Y$$

$$\in^{+}, \in^{-} \geq 0$$

Rearranging the first constraint: $\in^{+} = Y + \in^{-}$

Y = 4			Y = -6		
ε^{+}	ε^{-}	$\varepsilon^{+} + \varepsilon^{-}$	ε^{+}	ε^{-}	$\varepsilon^{+} + \varepsilon^{-}$
4	0	4*	0	6	6*
16	12	28	14	20	34
Z + 4	Z	2Z + 4	Z	Z + 6	2Z + 6

* These cases are the only ones in which $\varepsilon^{+} * \varepsilon^{-}$ equals zero.

Nonlinearities and Approximations

Minimization of the Sum of Absolute Deviations

- transformation An Example:

Price of Raw Oranges	Quantity of Oranges Sold	Quantity of Juice Sold
10	8	5
5	9	1
4	10	9
2	13	8
6	15	2
9	17	3

$$Min \quad \sum_i |\in_i|$$

$$s.t. \qquad \in_i = Y_i - b_0 - X_{il} b_i - X_{i2} b_2 \quad \textit{for all } i$$

$$\in_i \begin{matrix} \geq \\ \overline{<} \end{matrix} 0 \qquad b_0, b_1, b_2 \begin{matrix} \leq \\ \overline{>} \end{matrix} 0 \qquad \textit{for all } i$$

An Alternative Formulation would be:

$$Min \quad \sum_i \left(\in_i^+ + \in_i^-\right)$$

$$\begin{aligned} s.t. \qquad & \in_1^+ - \in_1^- = 10 - b_0 - 8b_1 - 5b_2 \\ & \in_2^+ - \in_2^- = 5 - b_0 - 9b_1 - 1b_2 \\ & \in_3^+ - \in_3^- = 4 - b_0 - 10b_1 - 9b_2 \\ & \in_4^+ - \in_4^- = 2 - b_0 - 13b_1 - 8b_2 \\ & \in_5^+ - \in_5^- = 6 - b_0 - 15b_1 - 2b_2 \\ & \in_6^+ - \in_6^- = 9 - b_0 - 17b_1 - 3b_2 \\ & \in_i^+, \quad \in_i^- \geq 0 \qquad b_0, b_1, b_2 \begin{matrix} \leq \\ \overline{>} \end{matrix} 0 \quad \textit{for all } i \end{aligned}$$

Nonlinearities and Approximations

Minimization of the Sum of Absolute Deviations

- transformation An Example:

Table 9.1. Minimization of Sum of Absolute Deviations Formulation

	ε_1^+	ε_1^-	ε_2^+	ε_2^-	ε_3^+	ε_3^-	ε_4^+	ε_4^-	ε_5^+	ε_5^-	ε_6^+	ε_6^-	b_0	b_1	b_2	
Obj	1	1	1	1	1	1	1	1	1	1	1	1				Min
1	1	-1											1	8	5	=10
2			1	-1									1	9	1	=5
3					1	-1							1	10	9	=4
4							1	-1					1	13	8	=2
5									1	-1			1	15	2	=6
6											1	-1	1	17	3	=9

Nonlinearities and Approximations

Minimization of the Sum of Absolute Deviations

- transformation An Example:

Table 9.2. Solution of Minimization of Absolute Deviation Sum Example

Objective function = 11.277

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
ε_1^+	5.787	0	Obs 1	0	1
ε_1^-	0	2.000	Obs 2	0	-0.660
ε_2^+	0	1.66	Obs 3	0	0.191
ε_2^-	0	0.340	Obs 4	0	-1
ε_3^+	0	0.809	Obs 5	0	-0.532
ε_3^-	0	1.191	Obs 6	0	1
ε_4^+	0	2.000			
ε_4^-	2.723	0			
ε_5^+	0	1.532			
ε_5^-	0	0.468			
ε_6^+	2.766	0			
ε_6^-	0	2.000			
b_0	3.426	0			
b_1	0.191	0			
b_2	-0.149	0			

Nonlinearities and Approximations

Minimization of Largest Absolute Deviation

- transformation

$$Min \quad \left\lfloor \underset{i}{Max} \left| \in_i \right| \right\rfloor$$

$$s.t. \qquad \in_i = Y_i - \sum_j X_{ji} \, b_j \quad \text{for all } i$$

$$\in_i, \quad b_j \begin{array}{c} < \\ - \\ > \end{array} 0 \qquad \text{for all } i \text{ and } j$$

Min ε

$$\in \geq \quad Y_i - \sum_j X_{ji} \, b_j$$

$$\in \geq -\left(Y_i - \sum_j X_{ji} \, b_j \right)$$

The Composite Linear Program is:

$$\begin{array}{llllll}
Min & \in & & & & \\
s.t. & -\in & - & \sum_j X_{ji} \, b_j & \leq & -Y_i \quad \text{for all } i \\
 & -\in & + & \sum_j X_{ji} \, b_j & \leq & Y_i \quad \text{for all } i \\
 & \in \geq 0 & & b_j \begin{array}{c} < \\ - \\ > \end{array} 0 & & \quad \text{for all } j
\end{array}$$

Nonlinearities and Approximations

Minimization of Largest Absolute Deviation

- transformation Example:

Rows	ε	b_0	b_1	b_2		
Objective	1				Minimize	
1^+	-1	-1	-8	-5	$\leq$	-10
1^-	-1	1	8	5	$\leq$	10
2^+	-1	-1	-9	-1	$\leq$	-5
2^{--}	-1	1	9	1	$\leq$	5
3^+	-1	-1	-10	-9	$\leq$	-4
3^-	-1	1	10	9	$\leq$	4
4^+	-1	-1	-13	-8	$\leq$	-2
4^-	-1	1	13	8	$\leq$	2
5^+	-1	-1	-15	-2	$\leq$	-6
5^-	-1	1	15	2	$\leq$	6
6^+	-1	-1	-17	-3	$\leq$	-9
6^-	-1	1	17	3	$\leq$	9
			1		$\leq$	0
				1	$\geq$	0

Table 9.3. Solution of Largest Absolute Deviation Example

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
ε	3.722	0	1^+	0	-0.222
b_0	7.167	0	1^-	7.44	0.0
b_1	-0.111	0	2^+	4.89	0.0
b_2	0.000	2.056	2^-	2.56	0.0
			3^+	5.78	0.0
			3^-	1.67	0.0
			4^+	7.44	0.0
			4^-	0	-0.5
			5^+	3.22	0.0
			5^-	4.22	0.0
			6^+	0	-0.278
			6^-	7.44	0.0

Nonlinearities and Approximations

Optimizing a Fraction

- transformation

$$\begin{array}{llll}
\text{Max} & \dfrac{C_0 + \sum_j C_j X_j}{d_0 + \sum_j d_j X_j} & & \\
\text{s.t.} & \sum_j a_{ij} X_j & \leq \; b_i & \text{for all } i \\
 & X_j & \geq \; 0 & \text{for all } j
\end{array}$$

where: $$d_0 + \sum_j d_j X_j \neq 0$$

Transformation to LP:

$$y_0 = \left[d_0 + \sum_j d_j X_j \right]^{-1}$$

$$y_0^{-1} = d_0 + \sum_j d_j X_j$$

$$d_0 y_0 + \sum_j d_j X_j y_0 = 1$$

$$\begin{array}{llllll}
\text{Max} & C_0 y_0 & + & \sum_j C_j X_j y_0 & & \\
\text{s.t.} & & & \sum_j a_{ij} X_j y_0 / y_0 & \leq \; b_i & \text{for all } i \\
 & d_0 y_0 & + & \sum_j d_j X_j y_0 & = \; 1 & \\
 & y_0, & & X_j & \geq \; 0. &
\end{array}$$

Nonlinearities and Approximations

Optimizing a Fraction

- transformation

More Transformations:

Let $y_j = X_j y_0$

$$
\begin{array}{lllll}
\text{Max} & C_0 y_0 + \sum_j C_j y_j & & & \\
\text{s.t.} & \sum_j a_{ij} y_j / y_0 & \leq & b_i & \text{for all } i \\
 & d_0 y_0 + \sum_j d_j y_j & = & 1 & \\
 & y_0, \quad y_j & \geq & 0. &
\end{array}
$$

$$\sum_j a_{ij} y_j \leq b_i y_0$$

$$
\begin{array}{lllll}
\text{Max} & C_0 y_0 + \sum_j C_j y_j & & & \\
\text{s.t.} & -b_0 y_0 + \sum_j a_{ij} y_j & \leq & 0 & \text{for all } i \\
 & d_0 y_0 + \sum_j d_j y_j & = & 1 & \\
 & y_0, \quad y_j & \geq & 0 & \text{for all } j
\end{array}
$$

Nonlinearities and Approximations

Optimizing a Fraction / Example

- transformation

Suppose that it is desirable to solve the following problem

$$\frac{1.8X_1+1.7X_2}{10+4X_1+4.1X_2} \quad s.t. \quad 1.5X_1 + X_2 \le 6 \quad 3X_1 + 4X_2 \le 30 \quad X_1, X_2 \ge 0$$

Then the transformed problem is

$$\begin{array}{lccccccc} \text{Max} & & & 1.8y_1 & + & 1.7y_2 & & \\ \text{s.t.} & -6y_0 & + & 1.5y_1 & + & y_2 & \le & 0 \\ & -20y_0 & + & 3.0y_1 & + & 4y_2 & \le & 0 \\ & 10y_0 & + & 4.0y_1 & + & 4.1y_2 & = & 1 \\ & y_0 & & y_1, & & y_2, & \ge & 0 \end{array}$$

Once a solution to this problem is obtained, the values of the original variables are recovered using the formulas

$$X_1 = y_1 / y_0$$
$$X_2 = y_2 / y_0$$

Nonlinearities and Approximations

Optimizing a Fraction / Example

- transformation

Table 9.4. Solution to the Example for Optimizing a Fraction

Objective function = 0.2899					
Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
y_0	0.032	0	1	0	0.342
y_1	0.042	0	2	0	0.042
y_2	0.126	0	3	0	0.290

Nonlinearities and Approximations

Grid Point Approximations- Separable Programming -

$$f(X) \cong F(X) = f\left(\hat{X}_k\right) + \frac{f\left(\hat{X}_{k+1}\right) - f\left(\hat{X}_k\right)}{\hat{X}_{k+1} - \hat{X}_k}\left(X - \hat{X}_k\right)$$

$$\begin{array}{lllllll} X & = & \lambda_k \hat{X}_k & + & \lambda_{k+1} \hat{X}_{k+1} & & \\ & & \lambda_k & + & \lambda_{k+1} & = & 1 \\ & & \lambda_k , & & \lambda_{k+1} & \geq & 0 \end{array}$$

$$F(X) \cong \lambda_k f\left(\hat{X}_k\right) + \lambda_{k+1} f\left(\hat{X}_{k+1}\right)$$

$$\begin{array}{llll} \text{Max} & \sum_j f_j\left(X_j\right) & & \\ \text{s.t.} & \sum_j g_{ij}\left(X_j\right) & \leq & b_i, \quad \text{for all } i \\ & X_j & \geq & 0 \end{array}$$

$$\begin{array}{lllll} \text{Max} & \sum_j \sum_\mu \lambda_{j\mu} f_j\left(\hat{X}_{j\mu}\right) & & & \\ \text{s.t.} & \sum_j \sum_\mu \lambda_{j\mu} g_{ij}\left(\hat{X}_{j\mu}\right) & \leq & b_i, & \text{for all } i \\ & \sum_\mu \lambda_{j\mu} & = & 1 & \text{for all } j \\ & \lambda_{j\mu} & \geq & 0 & \text{for all } j \text{ and } \mu \end{array}$$

$$X_j = \sum \lambda_{j\mu} \hat{X}_{j\mu}$$

Nonlinearities and Approximations

Separable Programming / Example 1

Suppose we approximate the problem.

$$
\begin{array}{llllllll}
\text{Max} & (4-.25X) & X & & - & (1+.25Z) & Z & \\
 & & X & - & 3Y & & & \leq & 0 \\
 & & & & 2Y & - & Z & \leq & 0 \\
 & & X, & & Y, & & Z & \geq & 0
\end{array}
$$

To set this problem up, suppose we use values of X equal to 1,2,3,4,5,6 and the same values for Z. The separable programming representation is in Table 12.

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	Y	β_1	β_2	β_3	β_4	β_5	β_6	
max	3.75	7	9.75	12	13.75	15		-1.25	-3	-5.25	-8	-10.125	-15	
constraint 1	1	2	3	4	5	6	-3							≤ 0
constraint 2							2	-1	-2	-3	-4	-5	-6	≤ 0
convex 1	1	1	1	1	1	1								≤ 1
convex 2								1	1	1	1	1	1	≤ 1

Note that λ_2 stands for the amount of the grid point X=2 utilized having an objective value equal to the nonlinear function of X evaluated at X=2. The GAMS formulation is called SEPARABL and the resultant solution is shown in Table 9.5. The objective function value is 7.625. The model sets $\lambda_4 = \lambda_5$ = 0.5 amounting to 50% of grid point X_4 and 50% of X_5 or X=4.5. The value of Y = 1.5. Simultaneously β_1 = 1 implying Z = 3.

Solution:

Table 9.5. Solution to the Step Approximation Example

Objective function = 7.625					
Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
λ_1	0	-3.000	1	0	1.750
λ_2	0	-1.500	2	0	2.625
λ_3	0	-0.500	3	0	5.000
λ_4	0.5	0	4	0	2.625
λ_5	0.5	0			
λ_6	0	-0.500			
Y	1.5	0			
B_1	0	-1.250			
B_2	0	-0.375			
B_3	1	0			
B_4	0	-0.125			
B_5	0	-0.750			
B_6	0	-1.875			

Nonlinearities and Approximations

Separable Programming / Example 2

Suppose we wish to approximate the following problem

$$
\begin{array}{llclcl}
\text{Max} & 3X & - & 3Y & & \\
\text{s.t.} & X & - & (20 + 2Y - .2Y^2) & \leq & 0 \\
 & X, & & Y & \geq & 0
\end{array}
$$

Selecting a grid for Y of 0, 1, 2, 3, 4 and 5, the separable programming formulation becomes

$$
\begin{array}{lllllllllllllllll}
Max & 3X & - & 0\lambda_1 & - & 3\lambda_2 & - & 6\lambda_3 & - & 9\lambda_4 & - & 12\lambda_5 & - & 15\lambda_6 & & \\
s.t. & X & - & 20\lambda_1 & - & 21.8\lambda_2 & - & 23.2\lambda_3 & - & 24.2\lambda_4 & - & 24.8\lambda_5 & - & 25\lambda_6 & \leq & 0 \\
 & & & \lambda_1 & + & \lambda_2 & + & \lambda_3 & + & \lambda_4 & + & \lambda_5 & + & \lambda_6 & = & 1 \\
 & X, & & \lambda_1, & & \lambda_2, & & \lambda_3, & & \lambda_4, & & \lambda_5, & & \lambda_6 & \geq & 0
\end{array}
$$

Nonlinearities and Approximations

Separable Programming / Example 2

Table 9.6. Solution to the Constraint Step Approximation Problem

Objective function = 63.6

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
X	23.2	0	1	0	3
λ_1	0	-3.6	2	0	63.6
λ_2	0	-1.2			
λ_3	1	0			
λ_4	0	0			
λ_5	0	-1.2			
λ_6	0	-3.6			

Nonlinearities and Approximations

Functions of Multiple Variables

$$
\begin{array}{llcrcll}
\text{Max} & CX & - & \sum_j d_j Y_j & & & \\
\text{s.t.} & X & - & H(Y_1, Y_2, Y_3, \ldots, Y_n) & = & 0 & \\
 & & & Y_j & \leq & b_i & \text{for all } j \\
 & X, & & Y & \geq & 0 &
\end{array}
$$

where there are multiple inputs and one output (for simplicity). The output X is a function of the levels of the multiple inputs (Y_j). Also the function $H(Y_1 \ldots Y_n)$ has to be such that this problem has a convex constraint set.

Nonlinearities and Approximations

Homogeneous of Degree One

$$H(\alpha Y) = \alpha H(Y)$$

$$H(\alpha_u \hat{Y}_{iu}) = \alpha_u H(\hat{Y}_{ju})$$

$$\begin{array}{llllllll}
Max & CX & & & - \sum_j d_j Y_j & & & \\
s.t. & X & - \sum_u \alpha_u H(\hat{Y}_{ju}) & & & = & 0 & \\
 & & \sum_u \alpha_u Y_{ju} & - & Y_j & = & 0 & for\ all\ j \\
 & & & & Y_j & \leq & b_j & for\ all\ j \\
 & X, & \alpha_u, & & Y_j & \geq & 0 & for\ all\ u\ and\ j
\end{array}$$

Nonlinearities and Approximations
Homogeneous of Degree One / Example

Consider the problem:

$$\begin{array}{lllllll} Max & 4X & - & 20Y_1 & - & 100Y_2 & \\ s.t. & X & - & 21Y_1^{0.75}Y_2^{0.25} & & = & 0 \\ & & & Y_1 & & \leq & 50 \\ & X, & & Y_1, & Y_2 & \geq & 0 \end{array}$$

$$\begin{array}{lllllllllllll} Max & 4X & & & & & & - & 20Y_1 & - & 100Y_2 & & \\ s.t. & X & - & 29.7\alpha_1 & - & 168\alpha_2 & - & 59.4\alpha_3 & & & & \leq & 0 \\ & & & \alpha_1 & + & 8\alpha_2 & + & 4\alpha_3 & - & Y_1 & & = & 0 \\ & & & 4\alpha_1 & + & 8\alpha_2 & + & \alpha_3 & & - & Y_2 & = & 0 \\ & & & & & & & & Y_1 & & & \leq & 50 \\ & X, & & \alpha_1, & & \alpha_2, & & \alpha_3, & Y_1, & & Y_2 & \geq & 0 \end{array}$$

Table 9.8. Solution to Example Problem for Homogeneous of Degree 1

Objective function = 719.8

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
X	742.5	0	1	0	4
α1	0	-315.6	2	0	34.4
α_2	0	-403.2	3	0	100
α_3	12.5	0			
Y_1	50	14.4			
Y_2	12.5	0			

Nonlinearities and Approximations

Homogeneous of Degree Less Than One

$$H(\alpha Y) \quad < \quad \alpha H(Y)$$

$$X = aY_1^{b_1}Y_2^{b_2} \dots Y_n^{b_n} = a\prod_j Y_j^{b_j}$$

$$Y_j = \alpha_u Y_{ju}$$

$$X = a\prod_j \left(\alpha_u Y_{ju}\right)^{b_j} = a\left(\prod_j Y_{ju}^{b_j}\right)\alpha_u^{\sum_j b_j}$$

$$\sum_j b_j \leq 1$$

The problem of interest becomes:

$$\begin{array}{llllllll}
Max & CX & - & & \sum_j d_j Y_j & & & \\
s.t. & X & - & \sum_u \sum_L H(Y_{ju}\alpha_{uL})\lambda_{uL} & & = & 0 & \\
& & & \sum_u \sum_L (Y_{ju}\alpha_{uL})\lambda_{uL} & - \quad Y_j & = & 0 & for\,all\ j \\
& & & \sum_u \sum_L \lambda_{uL} & & = & 1 & \\
& & & & Y_j & \leq & b_j & for\,all\ j \\
& X, & & \lambda_{uL}, & Y_j & \geq & 0 &
\end{array}$$

Nonlinearities and Approximations

Homogeneous of Degree Less Than One Example

Consider the example problem

$$
\begin{array}{llllll}
\text{Max} & 0.5X & - & 2Y_1 & - \; 2Y_2 & \\
& X & - & 21Y_1^{.5}Y_2^{.25} & = & 0 \\
& & & Y_1 & \leq & 10 \\
& & & Y_1, & Y_2 \; \geq & 0
\end{array}
$$

Table 9.9. Approximations for the Homogenous of Degree Less Than One Example

X	Y_{11}	Y_{12}	X	Y_{12}	Y_{22}	X	Y_{13}	Y_{23}
21	1	1	29.7	2	1	25.0	1	2
59.4	4	4	49.9	4	2	70.6	4	8
80.5	6	6	67.7	6	3	95.7	6	12
118.1	10	10	99.3	10	5	140.4	10	20

Nonlinearities and Approximations

Homogeneous of Degree Less Than One

Example

Table 9.10. Formulation of the Homogeneous Degree Less than One Example

Rows	X	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{21}	λ_{22}	λ_{23}	λ_{24}	λ_{31}	λ_{32}	λ_{33}	λ_{34}	Y_1	Y_2	RHS
Obj	0.5													-2	-2	max
x bal	1	-21	-59.4	-80.5	-118.1	-29.7	-49.9	-67.7	-99.3	-25.0	-70.6	-95.7	-140.4			= 0
Y bal		1	4	6	10	2	4	6	10	1	4	6	10	-1		= 0
		1	4	6	10	1	2	3	5	2	8	12	20		-1	= 0
convex		1	1	1	1	1	1	1	1	1	1	1	1			≤ 1
Y lim														1		≤ 10

Nonlinearities and Approximations

Homogeneous of Degree Less Than One

Example

Table 9.11. Solution to the Homogenous of Degree Less Than One Example

Objective function = 19.651

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
X	99.3	0	x bal	0	0.500
λ_{11}	0	-5.506	Y_1 bal	0	2.850
λ_{12}	0	-0.856	Y_2 bal	0	2.000
λ_{13}	0	0.000	convex	0	11.156
λ_{14}	0	-0.606	Y lim	0	0.85
λ_{21}	0	-4.006			
λ_{22}	0	-1.581			
λ_{23}	0	-0.404			
λ_{24}	1	0.000			
λ_{31}	0	-5.519			
λ_{32}	0	-3.237			
λ_{33}	0	-4.384			
λ_{34}	0	-9.434			
Y_1	10	0.000			
Y_2	5	0.000			

Nonlinearities and Approximations

Iterative Approximations

Based on the concept of a Taylor series expansion. This method solves the problem :

$$\begin{aligned} Max \quad & f(X) \\ & g(X) \le b \\ & L_j \le X_j \le G_j \end{aligned}$$

The Approximating Problem:

$$Max \quad f(X_0) + \frac{d}{dX} f(X_0)\,(X - X_0)$$

$$s.t. \quad g_i(X) = g_i(X_0) + \frac{d}{dX} g_i(X_0)\,(X - X_0) \le b_i$$

$$L_j \le X_j \le G_j$$

$$\mu_j = X_j - X_{0j}$$

$$Max \;\; f(X_o) + \sum_j \frac{d}{dX_j} f(X_o)\mu_j$$

$$\sum_j \frac{d}{dX_j} g(X_o)\mu_j \le b_i - g_i(X_o)$$

$$-Lim_j^- \le \mu_j \le Lim_j^+$$

$$\begin{aligned} Lim_j^- &= \min\left[\beta_j,\, X_{oj} - L_j\right] \\ Lim_j^+ &= \min\left[\beta_j,\, G_j - X_{oj}\right] \end{aligned}$$

$$X_{0j}^{k+1} = X_{0j}^k + \mu_j^*$$